Inverse Fourier Transform

Fourier Transform and Its Inverse

- Writing the exponential function in (4) as a product of exponential functions, we have

(5) $$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(v) e^{-iwv} \, dv \right] e^{iwx} \, dw.$$

The expression in brackets is a function of *w*, is denoted by $\hat{f}(w)$ and is called the **Fourier transform** of ƒ; writing *v* = *x*, we have

(6)
$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} \, dx.$$

With this, (5) becomes

(7) $$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(w) e^{iwx}\, dw$$

and is called the **inverse Fourier transform** of $\hat{f}(w)$

- Another notation for the Fourier transform is

so that $\qquad \hat{f} = \mathscr{F}(f), \qquad f = \mathscr{F}^{-1}(\hat{f}).$

- The process of obtaining the Fourier transform
  $\mathscr{F}(f) = \hat{f}$
  from a given f is also called the **Fourier transform** or the *Fourier transform method.*

Table of Fourier Transforms

$f(x)$	$\hat{f}(\omega)$
1. $\dfrac{1}{x^2 + a^2}$ $(a > 0)$	$\dfrac{\pi}{a} e^{-a|\omega|}$
2. $H(x)e^{-ax}$ $(\operatorname{Re} a > 0)$	$\dfrac{1}{a + i\omega}$
3. $H(-x)e^{ax}$ $(\operatorname{Re} a > 0)$	$\dfrac{1}{a - i\omega}$
4. $e^{-a|x|}$ $(a > 0)$	$\dfrac{2a}{\omega^2 + a^2}$
5. e^{-x^2}	$\sqrt{\pi}\, e^{-\omega^2/4}$
6. $\dfrac{1}{2a\sqrt{\pi}} e^{-x^2/(2a)^2}$ $(a > 0)$	$e^{-a^2\omega^2}$
7. $\dfrac{1}{\sqrt{|x|}}$	$\sqrt{\dfrac{2\pi}{|\omega|}}$
8. $e^{-a|x|/\sqrt{2}} \sin\left(\dfrac{a}{\sqrt{2}}|x| + \dfrac{\pi}{4}\right)$ $(a > 0)$	$\dfrac{2a^3}{\omega^4 + a^4}$
9. $H(x + a) - H(x - a)$	$\dfrac{2\sin\omega a}{\omega}$