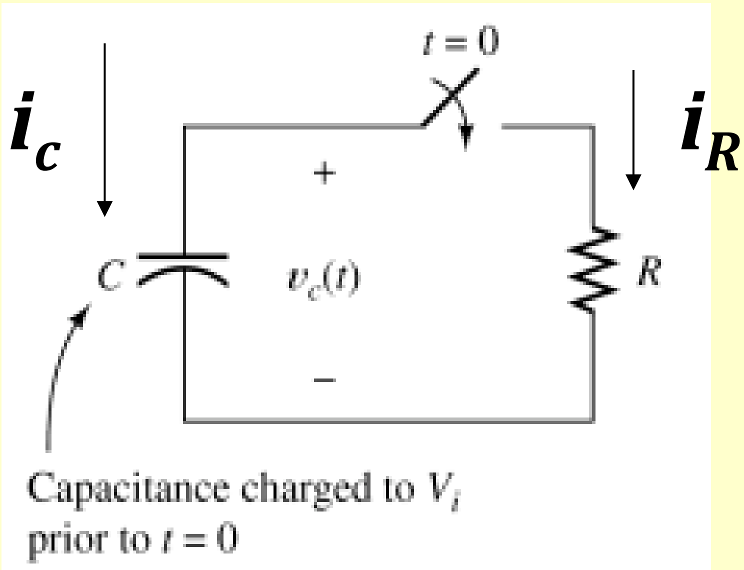
UNIT-1

(Lecture-11)

Solution of Network Equations

Discharge of a Capacitance through a Resistance



$$\sum i = 0, \quad i_C + i_R = 0$$

$$C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

Solving the above equation with the initial condition

$V_c(0) = V_i$

Discharge of a Capacitance through a Resistance

$$C\frac{dv_C(t)}{dt}+\frac{v_C(t)}{R}=0$$

$$RC\frac{dv_C(t)}{dt}+v_C(t)=0$$

$$v_C(t)=Ke^{st}$$

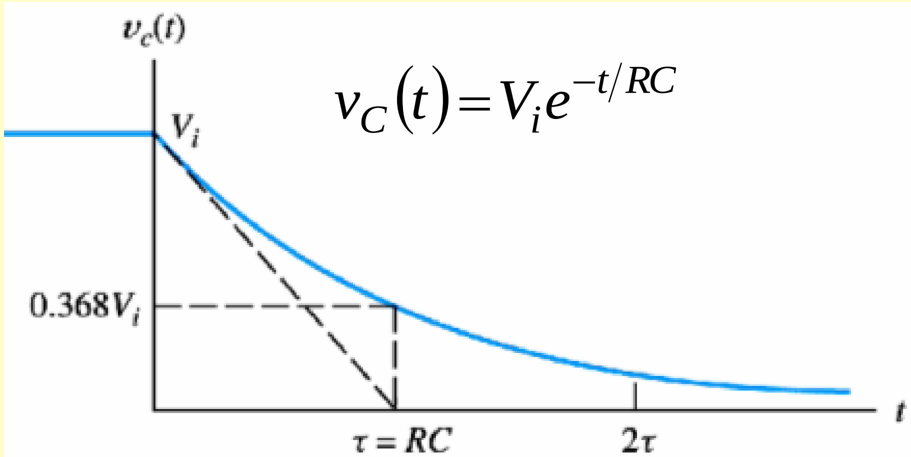
$$RCKse^{st}+Ke^{st}=0$$

$$s=\frac{-1}{RC}$$

$$v_C(t)=Ke^{-t/RC}$$

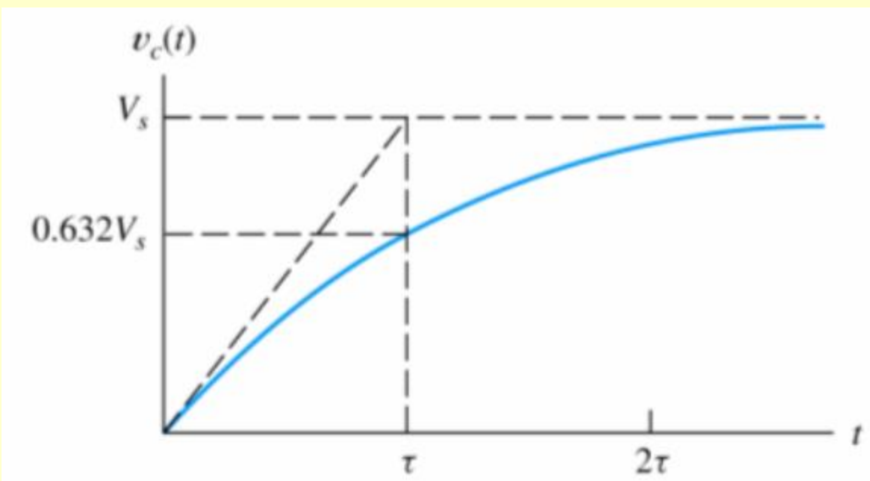
$$v_C(0^+)=V_i$$
$$=Ke^{0/RC}$$
$$=K$$

$$v_C(t)=V_ie^{-t/RC}$$



$$v_C(t) = V_i e^{-t/RC}$$

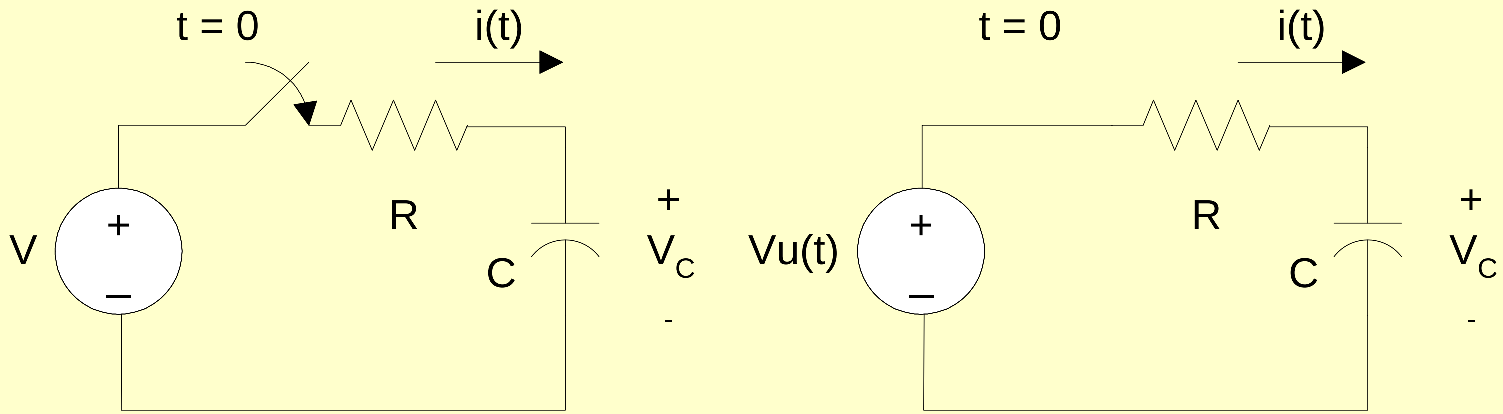
Exponential decay waveform
RC is called the time constant.
At time constant, the voltage is 36.8% of the initial voltage.



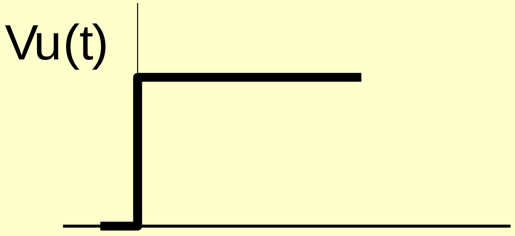
$$v_C(t) = V_i(1 - e^{-t/RC})$$

Exponential rising waveform
RC is called the time constant.
At time constant, the voltage is 63.2% of the initial voltage.

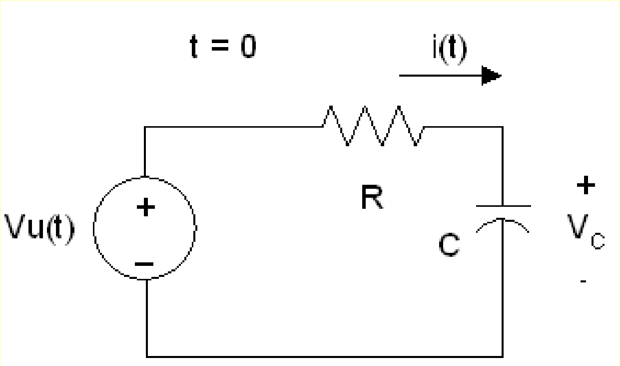
RC CIRCUIT



for $t = 0^-$, $i(t) = 0$
$u(t)$ is voltage-step function



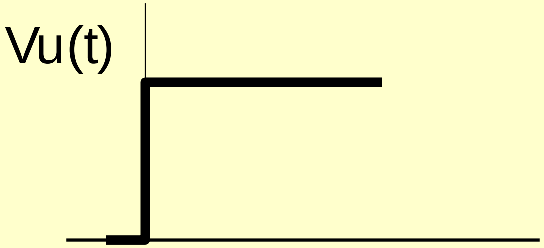
RC CIRCUIT



$$i_R = i_C$$

$$i_R = \frac{vu(t) - v_C}{R}, \; i_C = C\frac{dv_C}{dt}$$

$$RC\frac{dv_C}{dt} + v_C = V, \; vu(t) = V \; for \; t \geq 0$$



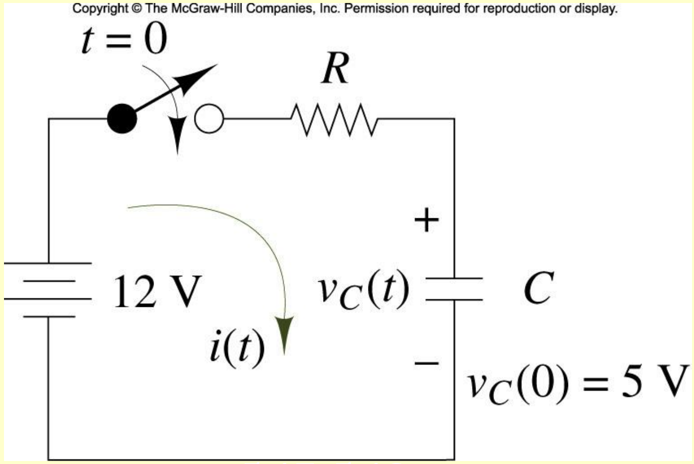
Solving the differential equation

Complete Response

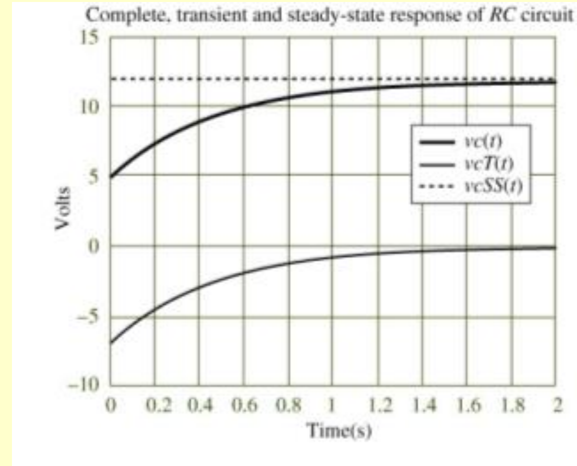
Complete response

= natural response + forced response

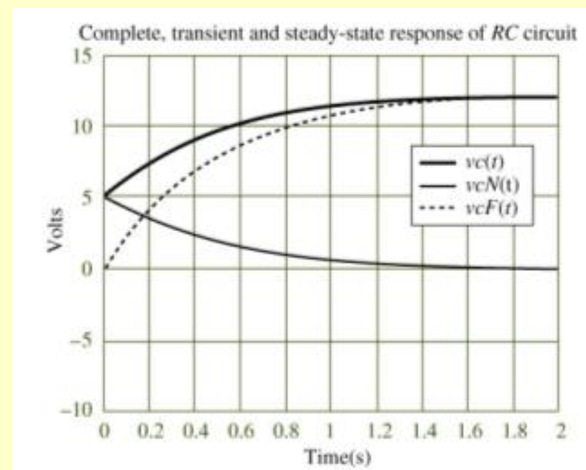
- Natural response (source free response) is due to the initial condition
- Forced response is the due to the external excitation.



5.17, 5.18



(a)

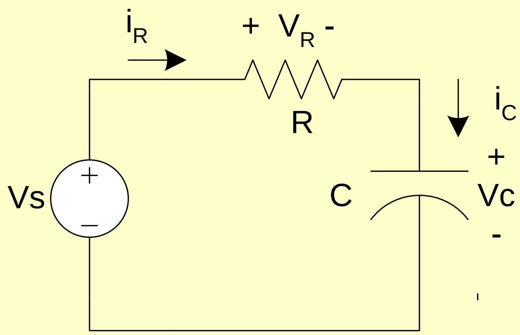


(b)

a). Complete, transient and steady state response

b). Complete, natural, and forced responses of the circuit

Circuit Analysis for RC Circuit



Apply KCL

$$i_R = i_C$$

$$i_R = \frac{v_s - v_R}{R}, i_C = C\frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} + \frac{1}{RC}v_R = \frac{1}{RC}v_s$$

v_s is the source applied.

Solution to First Order Differential Equation

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

Let the initial condition be $x(t = 0) = x(0)$, then we solve the differential equation:

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

The complete solution consist of two parts:

- **the homogeneous solution (natural solution)**
- **the particular solution (forced solution)**

The Natural Response

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

Setting the excitation *f* (*t*) equal to zero,

$$\tau \frac{dx_N(t)}{dt} + x_N(t) = 0 \quad or \quad \frac{dx_N(t)}{dt} = -\frac{x_N(t)}{\tau}$$

$$x_N(t) = \alpha e^{-t/\tau}$$

It is called the natural response.

The Forced Response

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

Setting the excitation *f* (*t*) equal to F, a constant for *t* ≥0

$$\tau \frac{dx_F(t)}{dt} + x_F(t) = K_S F$$

$$x_F(t) = K_S F \quad for \;\; t \geq 0$$

It is called the forced response.

The Complete Response

Consider the general Equation

$$\tau \frac{dx(t)}{dt} + x(t) = K_S f(t)$$

The complete response is:

- **the natural response +**
- **the forced response**

$$x = x_N(t) + x_F(t)$$

$$= \alpha e^{-t/\tau} + K_S F$$

$$= \alpha e^{-t/\tau} + x(\infty)$$

Solve for α,

$$for \;\; t = 0$$

$$x(t=0) = x(0) = \alpha + x(\infty)$$

$$\alpha = x(0) - x(\infty)$$

The Complete solution:

$$x(t) = [x(0) - x(\infty)]e^{-t/\tau} + x(\infty)$$

$[x(0) - x(\infty)]e^{-t/\tau}$ **called transient response**

$x(\infty)$ **called steady state response**