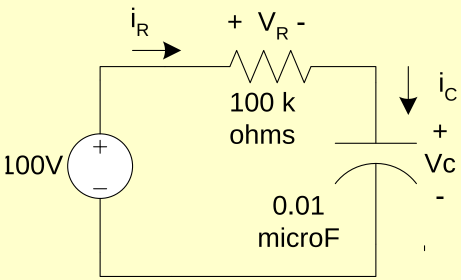
# UNIT-1

# (Lecture-12)

## Examples

# Example

**Initial condition *Vc*(0) = 0V**



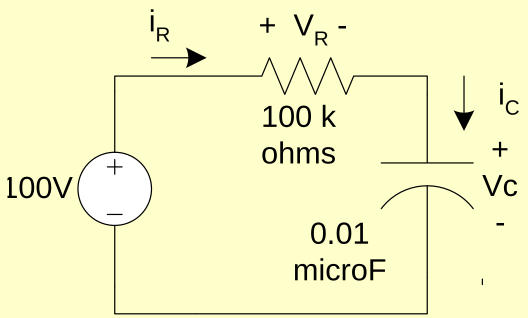

$$i_R = i_C$$

$$i_R = \frac{v_s - v_C}{R}, i_C = C\frac{dv_C}{dt}$$

$$RC\frac{dv_C}{dt} + v_C = v_s$$

$$10^5 \times 0.01 \times 10^{-6}\frac{dv_C}{dt} + v_C = 100$$

$$10^{-3}\frac{dv_C}{dt} + v_C = 100$$




**Initial condition *Vc*(0) = 0V**

$$10^{-3}\frac{dv_C}{dt}+v_C=100$$

$$\tau\frac{dx(t)}{dt}+x(t)=K_s f(t)$$

**and**

$$x=x_N(t)+x_F(t)$$
$$=\alpha e^{-t/\tau}+K_S F$$
$$=\alpha e^{-t/\tau}+x(\infty)$$

$$v_c=100+Ae^{-\frac{t}{10^{-3}}}$$

$$As\ \ v_c(0)=0,\ 0=100+A$$

$$A=-100$$

$$v_c=100-100e^{-\frac{t}{10^{-3}}}$$

# Energy stored in capacitor

$$p = vi = Cv\frac{dv}{dt}$$

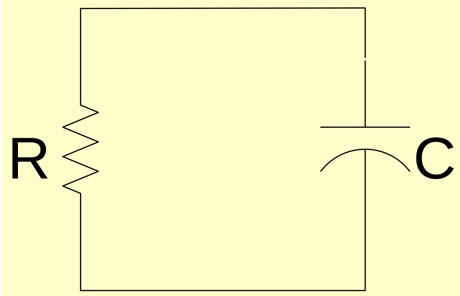
$$\int_{t_o}^{t} p\,dt = \int_{t_o}^{t} Cv\frac{dv}{dt}\,dt = C\int_{t_o}^{t} v\,dv$$

$$= \frac{1}{2}C\left\{[v(t)]^2 - [v(t_o)]^2\right\}$$

**If the zero-energy reference is selected at $t_o$, implying that the capacitor voltage is also zero at that instant, then**

$$w_C(t) = \frac{1}{2}Cv^2$$

# *RC* CIRCUIT




**Power dissipation in the resistor is:**

**$p_R = V^2/R = (V_o^2/R)\, e^{-2t/RC}$**

**Total energy turned into heat in the resistor**

$$W_R = \int_0^\infty p_R dt = \frac{V_o^2 \int_0^\infty e^{-2t/RC} dt}{R}$$

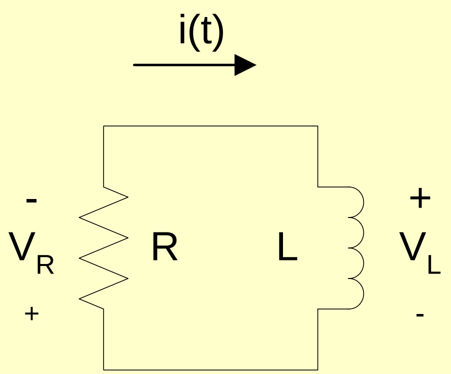
$$= V_o^2 R\left(-\frac{1}{2RC}\right)e^{-2t/RC}\Big|_0^\infty$$

$$= \frac{1}{2} C V_o^2$$

# *RL* CIRCUITS

**Initial condition**

$\boldsymbol{i(t = 0) = I_o}$

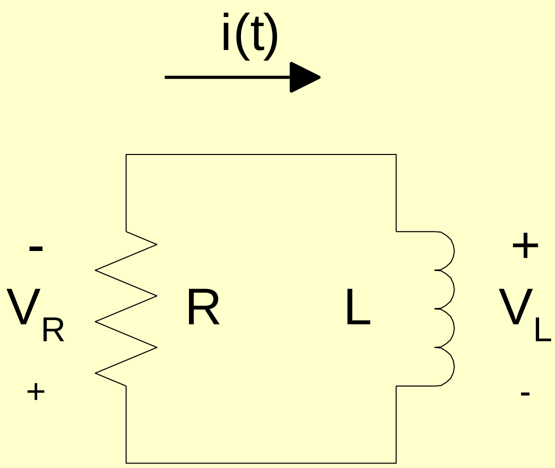



$$v_R + v_L = 0 = Ri + L\frac{di}{dt}$$

$$\frac{L}{R}\frac{di}{dt} + i = 0$$

*Solving the differential equation*

# *RL* CIRCUITS




$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$\frac{di}{i} = -\frac{R}{L} dt, \quad \int_{I_o}^{i(t)} \frac{di}{i} = \int_{o}^{t} -\frac{R}{L} dt$$

$$\ln i \,\Big|_{I_o}^{i} = -\frac{R}{L} t \,\Big|_{o}^{t}$$

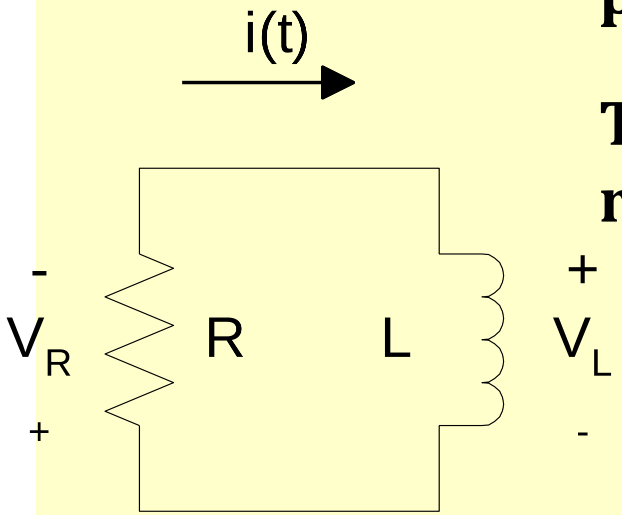
$$\ln i - \ln I_o = -\frac{R}{L} t$$

$$i(t) = I_o e^{-Rt/L}$$

**Initial condition**

$\boldsymbol{i(t = 0) = I_o}$

# *RL* CIRCUIT




**Power dissipation in the resistor is:**

$$p_R = i^2R = I_o^2 e^{-2Rt/L} R$$

**Total energy turned into heat in the resistor**

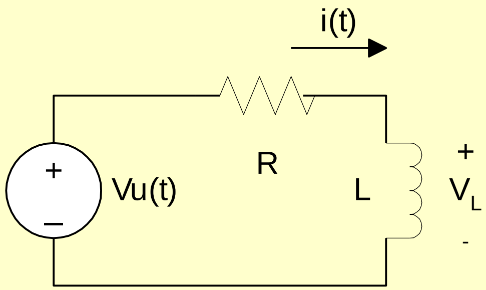
$$W_R = \int_0^{\infty} p_R dt = I_o^2 R \int_0^{\infty} e^{-2Rt/L} dt$$

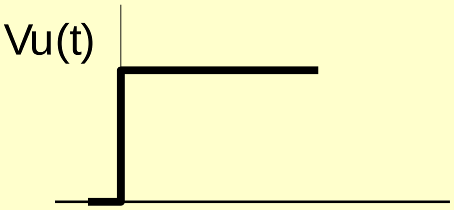
$$= I_o^2 R\left(-\frac{L}{2R}\right) e^{-2Rt/L} \Big|_0^{\infty}$$

$$= \frac{1}{2} L I_o^2$$

**It is expected as the energy stored in the inductor is** $\frac{1}{2} L I_o^2$

# RL CIRCUIT




$$Ri + L\frac{di}{dt} = V$$

$$\frac{Ldi}{V - Ri} = dt$$

*Integrating both sides,*

$$-\frac{L}{R}\ln(V - Ri) = t + k$$

$$i(0^+) = 0, thus \;\; k = -\frac{L}{R}\ln V$$

$$-\frac{L}{R}[\ln(V - Ri) - \ln V] = t$$

$$\frac{V - Ri}{V} = e^{-Rt/L} \qquad or$$

$$i = \frac{V}{R} - \frac{V}{R}e^{-Rt/L}, \;\; for\, t > 0$$

**where *L/R* is the time constant**

# DC STEADY STATE

**The steps in determining the forced response for *RL or RC* circuits with dc sources are:**

**1. Replace capacitances with open circuits.**

**2. Replace inductances with short circuits.**

**3. Solve the remaining circuit.**