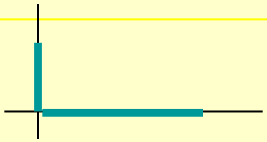
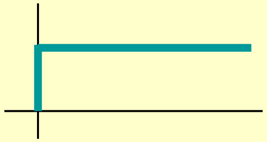
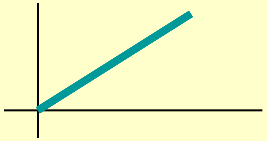
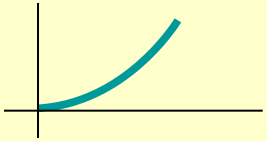
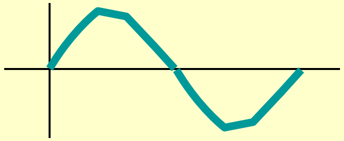
UNIT-2

(Lecture-1)

Review of Laplace Transforms

Laplace Transforms of Common Functions

Name	*f*(*t*)		*F*(*s*)
Impulse	$f(t)=\begin{cases}1 & t=0\\ 0 & t>0\end{cases}$		**1**
Step	$f(t)=1$		$\frac{1}{s}$
Ramp	$f(t)=t$		$\frac{1}{s^2}$
Exponential	$f(t)=e^{at}$		$\frac{1}{s-a}$
Sine	$f(t)=\sin(\omega t)$		$\frac{1}{\omega^2+s^2}$

Laplace Transform Properties

Addition/Scaling	$L[af_1(t) \pm bf_2(t)] = aF_1(s) \pm bF_2(s)$
Differentiation	$L\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0\pm)$
Integration	$L\left[\int f(t)dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)dt\right]_{t=0\pm}$
Convolution	$\int_0^t f_1(t-\tau)f_2(\tau)d\tau = F_1(s)F_2(s)$
Initial-value theorem	$f(0+) = \lim_{s\to\infty} sF(s)$
Final-value theorem	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

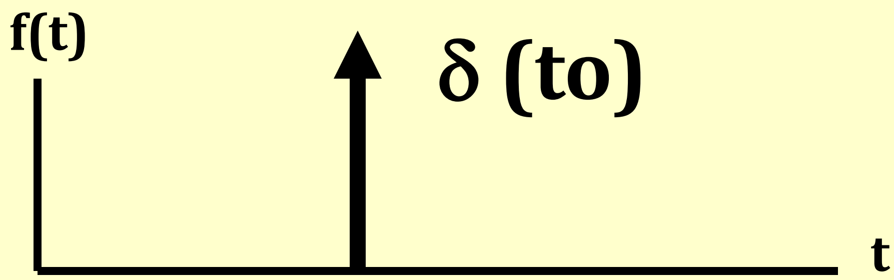
LAPLACE TRANSFORMS

- **SIMPLE TRANSFORMATIONS**

- **Impulse -- $\delta(t_o)$**

$$F(s) = \int_0^{\infty} e^{-st}\,\delta(t_o)\,dt$$

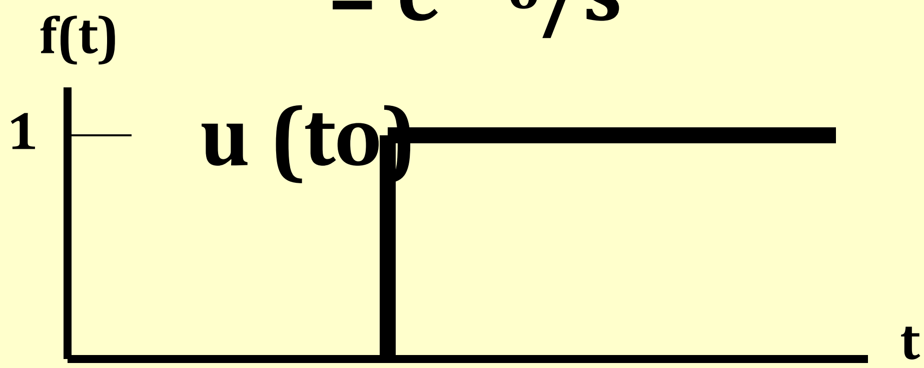
$$= e^{-st_o}$$



- **Step -- u (t_o)**

$$F(s) = \int_0^{\infty} e^{-st}\, u(to)\, dt$$

$$= e^{-st_o}/s$$



- e^{-at}

$$F(s) = \int_{0}^{\infty} e^{-st}\, e^{-at}\, dt$$

$$= 1/(s+a)$$

Linearity	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
Constant multiplication	$a\,f(t)$	$a\,F(s)$
Complex shift	$e^{at}\,f(t)$	$F(s-a)$
Real shift	$f(t - T)$	$e^{Ts}\,F(as)$
Scaling	$f(t/a)$	$a\,F(as)$

Partial Fraction Expansions

$$\frac{s+1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

- **Expand into a term for each factor in the denominator.**
- **Recombine RHS**

$$\frac{s+1}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$$

- **Equate terms in s and constant terms. Solve.**

$$A+B=1 \qquad 3A+2B=1$$

- **Each term is in a form so that inverse Laplace transforms can be applied.**

$$\frac{s+1}{(s+2)(s+3)} = \frac{-1}{s+2} + \frac{2}{s+3}$$

Example of Solution of an ODE

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 2 \quad y(0) = y'(0) = 0$$

- **ODE w/initial conditions**

$$s^2 Y(s) + 6sY(s) + 8Y(s) = 2/s$$

- **Apply Laplace transform to each term**
- **Solve for Y(s)**

$$Y(s) = \frac{2}{s(s+2)(s+4)}$$

- **Apply partial fraction expansion**

$$Y(s) = \frac{1}{4s} + \frac{-1}{2(s+2)} + \frac{1}{4(s+4)}$$

- **Apply inverse Laplace transform to each term**

$$y(t) = \frac{1}{4} - \frac{e^{-2t}}{2} + \frac{e^{-4t}}{4}$$