UNIT-4

(Lecture-3)

DARLINGTON THEOREM

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Darlington's theorem may be established by writing the numerator and denominator polynomials of a one-port LCR immittance function in terms of odd to even or even to odd rational functions and making an equivalence between them and the open-circuit parameters of a two-port reactance network terminated in a l-Ω resistor. The proof is completed by demonstrating that the open-circuit parameters are realizable as a two-port reactance network.

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If Z(s) is a LCR p.r. function it may be expressed as the ratio of two Hurwitz polynomials:

$$Z(s) = \frac{P(s)}{Q(s)} \qquad \text{-----------(1)}$$

Writing P(s) and Q(s) in terms of their odd and even parts yields

$$Z(s) = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)} \qquad \text{-----------(2)}$$

The odd and even parts of P(s) and Q(s) are Hurwitz by definition.

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This relationship can be written in terms of one-port reactance functions by recalling that such parameters are the ratio of odd to even or even to odd Hurwitz polynomials. The two possible solutions are

$$Z(s) = \frac{m_1(s)}{n_2(s)} \frac{1 + n_1(s)/m_1(s)}{1 + m_2(s)/n_2(s)} \qquad \text{-------(3)}$$

$$Z(s) = \frac{n_1(s)}{m_2(s)} \frac{1 + m_1(s)/n_1(s)}{1 + n_2(s)/m_2(s)} \qquad \text{-------(4)}$$

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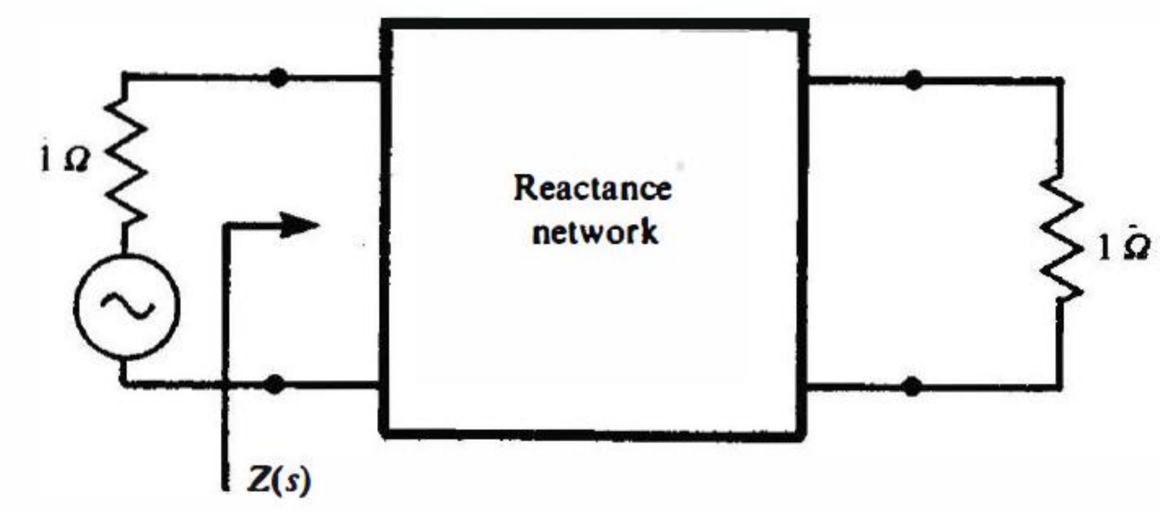
$$Z(s) = Z_{11}(s) - \frac{Z_{21}^2(s)}{1 + Z_{22}(s)}$$ ----------(5)

$$[V] = [Z][I]$$ ---------(6)

Two-port reactance network terminated in a l- Ω resistance



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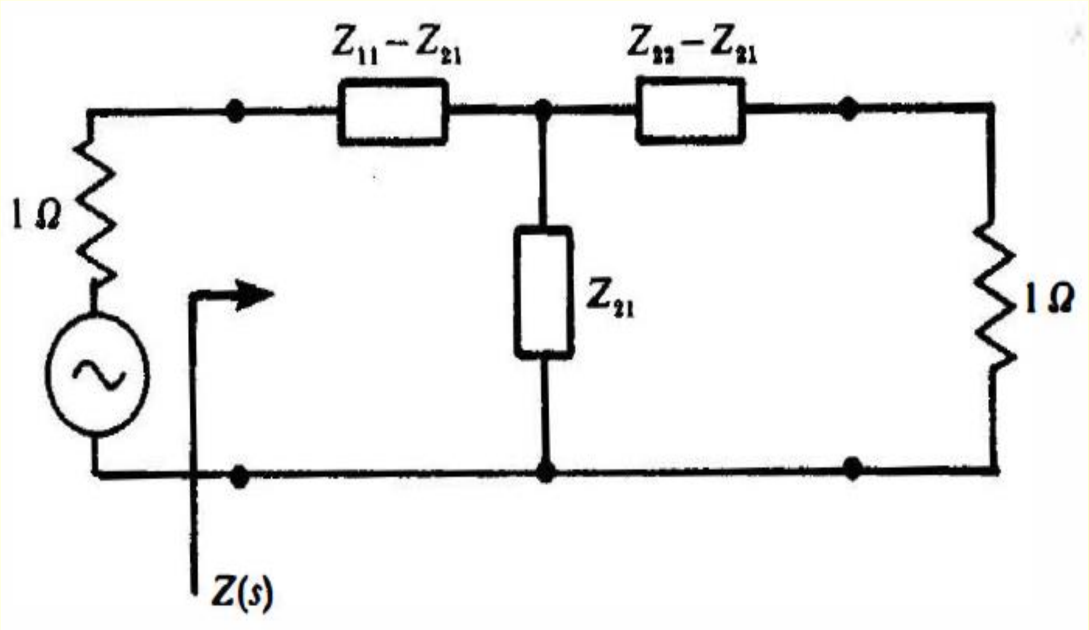
Two-port reactance network in terms of open-circuit parameters.



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The immittance of the two-port reactance network may written in the form:

$$Z(s) = \frac{Z_{11}(s) + Z_{11}(s)Z_{22}(s) - Z_{21}^2(s)}{1 + Z_{22}(s)} \quad \text{-------(7)}$$

and then extracting a common term $Z_{11}(s)$ from the numerator polynomial:

$$Z(s) = Z_{11}(s)\frac{1 + \{[Z_{11}(s)Z_{22}(s) - Z_{21}^2(s)]/Z_{11}(s)\}}{1 + Z_{22}(s)} \quad \text{-----(8)}$$

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Comparing this immittance function with that in Eq. (3) or (4) indicates that a one-to-one correspondence may be possible between each situation provided the quantity in the brace in the numerator polynomial of the preceding equation can be shown to be a reactance function. This is fortunately the case since

$$\frac{Z_{11}(s)Z_{22}(s) - Z_{21}^2(s)}{Z_{11}(s)} = \frac{1}{Y_{22}(s)} \quad \text{--------(9)}$$

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as is readily verified by making use of the relationship between [Z] and [Y]:

$$[Y] = [Z]^{-1}$$ --------(10)

The input impedance of a two-port reactance network may therefore be written in terms of one-port reactance functions as

$$Z(s) = Z_{11}(s)\frac{1 + 1/Y_{22}(s)}{1 + Z_{22}(s)}$$ -------(11)

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Equation (3) or (4) is consistent with Eq. (11) provided

$$Z_{11}(s) = \frac{m_1(s)}{n_2(s)}$$ -----(12a)

$$Z_{22}(s) = \frac{m_2(s)}{n_2(s)}$$ -----(12b)

$$\frac{1}{Y_{22}(s)} = \frac{n_1(s)}{m_1(s)}$$ -----(12c)

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$$Z_{22}(s) = \frac{n_2(s)}{m_2(s)}$$ -----(13b)

$$\frac{1}{Y_{22}(s)} = \frac{m_1(s)}{n_1(s)}$$ -----(13c)

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Writing Eq. (9) in terms of Eqs (12) or (13) gives

$$Z_{21}(s) = \frac{\sqrt{m_1(s)m_2(s) - n_1(s)n_2(s)}}{n_2(s)} \quad \text{-----(14)}$$

$$Z_{21}(s) = \frac{\sqrt{n_1(s)n_2(s) - m_1(s)m_2(s)}}{m_2(s)} \quad \text{-----(15)}$$

It is now necessary to ensure that $Z_{11}(s)$, $Z_{22}(s)$and $Z_{21}(s)$ in Eqs (12a), (12b) and (14) or (l3a), (l3b) and (15) are realizable as a two-port reactance network.