Laplace-Transform (LT)

One-sided LT of some common signals, important theorems

and properties of LT

The Laplace Transform

The Laplace Transform of a function, f(t), is defined as;

$$L[f(t)] = F(s) = \int_{0}^{\infty} f(t)e^{-st}dt$$

Eq A

The Inverse Laplace Transform is defined by

$$L^{-1}[F(s)] = f(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} F(s)e^{ts}ds$$

Eq B

The Laplace Transform

We generally do not use Eq B to take the inverse Laplace. However, this is the formal way that one would take the inverse. To use Eq B requires a background in the use of complex variables and the theory of residues. Fortunately, we can accomplish the same goal (that of taking the inverse Laplace) by using *partial fraction expansion* and recognizing *transform pairs*.

The Laplace Transform

Laplace Transform of the unit step.

$$L[u(t)] = \int_0^\infty 1 e^{-st} dt = \frac{-1}{s} e^{-st} \Big|_0^\infty$$

$$L[u(t)] = \frac{1}{s}$$

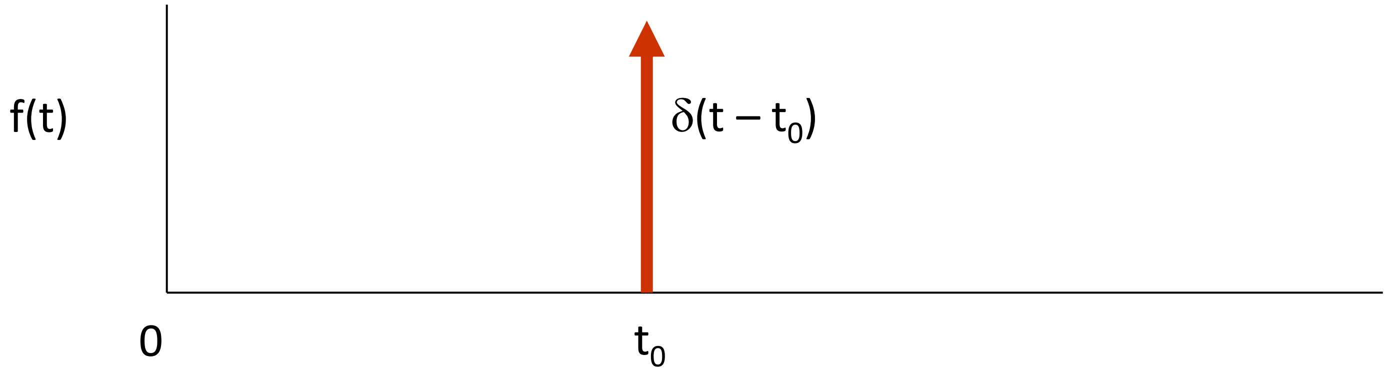
The Laplace Transform of a unit step is:

$$\frac{1}{s}$$

The Laplace Transform

The Laplace transform of a unit impulse:

Pictorially, the unit impulse appears as follows:



Mathematically:

$$\delta(t - t_0) = 0 \quad t \neq 0$$

$$\int_{t_0-\varepsilon}^{t_0+\varepsilon} \delta(t - t_0)\,dt = 1 \qquad \varepsilon > 0$$

The Laplace Transform

The Laplace transform of a unit impulse:

An important property of the unit impulse is a sifting or sampling property. The following is an important.

$$\int_{t_1}^{t_2} f(t)\delta(t-t_0)dt = \begin{cases} f(t_0) & t_1 < t_0 < t_2 \\ 0 & t_0 < t_1, t_0 > t_2 \end{cases}$$

The Laplace Transform

The Laplace transform of a unit impulse:

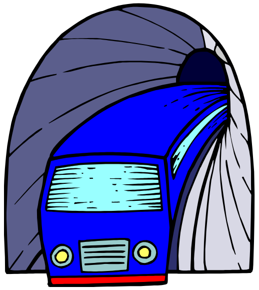
In particular, if we let f(t) = δ(t) and take the Laplace

$$L[\delta(t)] = \int_0^\infty \delta(t) e^{-st} dt = e^{-0s} = 1$$

The Laplace Transform

An important point to remember:

$$f(t) \quad \Leftrightarrow \quad F(s)$$



The above is a statement that f(t) and F(s) are transform pairs. What this means is that for each f(t) there is a unique F(s) and for each F(s) there is a unique f(t). If we can remember the Pair relationships between approximately 10 of the Laplace transform pairs we can *go a long way.*

The Laplace Transform

Building transform pairs:

$$L[e^{-at}u(t)] = \int_0^{\infty} e^{-at} e^{-st} dt = \int_0^{\infty} e^{-(s+a)t} dt$$

$$L[e^{-at}u(t)] = \frac{-e^{-st}}{(s+a)}\Big|_0^{\infty} = \frac{1}{s+a}$$

A transform pair

$$e^{-at}u(t) \Leftrightarrow \frac{1}{s+a}$$