Inverse Discrete Fourier Transform

The inverse transform of

$$F[n] = \sum_{k=0}^{N-1} f[k] e^{-j\frac{2\pi}{N}nk}$$

is

$$f[k] = \frac{1}{N} \sum_{n=0}^{N-1} F[n] e^{+j\frac{2\pi}{N}nk}$$

i.e. the inverse matrix is $\frac{1}{N}$ times the complex conjugate of the original (symmetric) matrix.

Note that the $F[n]$ coefficients are *complex*. We can assume that the $f[k]$ values are *real* (this is the simplest case; there are situations (e.g. radar) in which two inputs, at each k, are treated as a *complex pair*, since they are the outputs from $0\ ^{\text{o}}$ and $90\ ^{\text{o}}$ demodulators).

In the process of taking the inverse transform the terms $F[n]$ and $F[N-n]$ (remember that the spectrum is symmetrical about $\frac{N}{2}$) combine to produce 2 frequency components, only one of which is considered to be valid (the one at the *lower* of the two frequencies, $n \times \frac{2\pi}{T}$ Hz where $n \leq \frac{N}{2}$; the higher frequency component is at an "aliasing frequency" ($n > \frac{N}{2}$)).

From the inverse transform formula, the contribution to $f[k]$ of $F[n]$ and $F[N-n]$ is:

$$f_n[k] = \frac{1}{N}\{F[n]e^{j\frac{2\pi}{N}nk} + F[N-n]e^{j\frac{2\pi}{N}(N-n)k}\}$$

For all $f[k]$ real, $\quad F[N-n] = \sum_{k=0}^{N-1} f[k] e^{-j\frac{2\pi}{N}(N-n)k}$

But $\quad e^{-j\frac{2\pi}{N}(N-n)k} = \underbrace{e^{-j2\pi k}}_{1 \text{ for all } k} \; e^{+j\frac{2\pi n}{N}k} = e^{+j\frac{2\pi}{N}nk}$

i.e. $F[N-n] = F^*(n)$ (i.e. the complex conjugate)

Substituting into the Equation for $f_n[k]$ above gives,

$$f_n[k] = \frac{1}{N}\{F[n]e^{j\frac{2\pi}{N}nk} + F^*(n)e^{-j\frac{2\pi}{N}nk}\} \quad \text{since} \quad e^{j2\pi k} = 1$$

ie. $$f_n[k] = \frac{2}{N}\{\mathrm{Re}\{F[n]\}\cos\frac{2\pi}{N}nk - \mathrm{Im}\{F[n]\}\sin\frac{2\pi}{N}nk\}$$

$$\text{or} \quad f_n[k] = \frac{2}{N}|F[n]| \cos\{(\frac{2\pi}{NT}n)kT + \arg(F[n])\}$$

i.e. a sampled sinewave at $\frac{2\pi n}{NT}$ Hz, of magnitude $\frac{2}{N}|F[n]|$.

For the special case of $n = 0$, $F[0] = \sum f[k]$ (i.e. sum of all samples) and the contribution of $F[0]$ to $f[k]$ is $f_0[k] = \frac{1}{N}F[0]$ = average of $f[k]$ = d.c. component.