Characterization of linear time-invariant (LTI) systems

Time Invariant and Time Variant Systems

- A system is said to be *time invariant* if a time delay or time advance of the input signal leads to a identical time shift in the output signal.

$$y_i(t) = H\{x(t - t_0)\}$$
$$= H\{S^{t0}\{x(t)\}\} = HS^{t0}\{x(t)\}$$

$$y_0(t) = S^{t0}\{y(t)\}$$
$$= S^{t0}\{H\{x(t)\}\} = S^{t0}H\{x(t)\}$$

Stable & Unstable Systems

- A system is said to be *bounded-input bounded-output stable* (BIBO stable) iff every bounded input results in a bounded output.

i.e.

$$\forall t \quad |x(t)| \leq M_x < \infty \rightarrow \forall t \quad |y(t)| \leq M_y < \infty$$

Stable & Unstable Systems Contd.

Example

- $y[n]=1/3(x[n]+x[n-1]+x[n-2])$

$$|y[n]| = \frac{1}{3}\left|x[n]+x[n-1]+x[n-2]\right|$$

$$\leq \frac{1}{3}(|x[n]|+|x[n-1]|+|x[n-2]|)$$

$$\leq \frac{1}{3}(M_x + M_x + M_x) = M_x$$

Stable & Unstable Systems Contd.

Example: The system represented by

$y(t) = A\,x(t)$ is unstable ; $A>1$

Reason: let us assume $x(t) = u(t)$, then at every instant $u(t)$ will keep on multiplying with A and hence it will not be bonded.

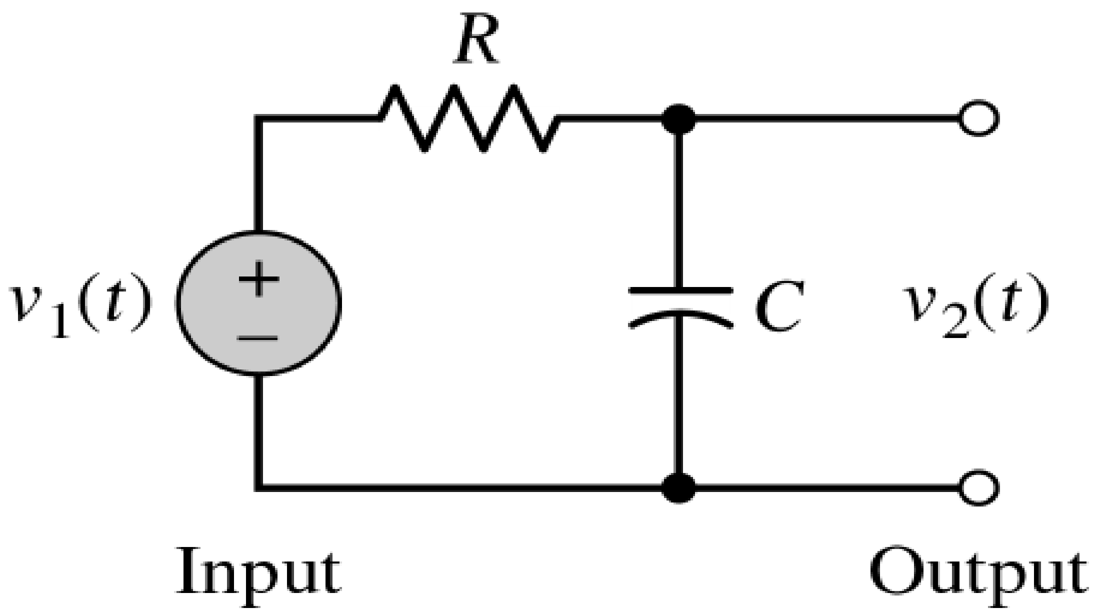
Static & Dynamic Systems Contd.

- A dynamic system possesses memory
- It has the storage devices
- A system is said to possess *memory* if its output signal depends on past values and future values of the input signal

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau$$

$$y[n] = x[n] + x[n-1]$$

Example: Static or Dynamic?



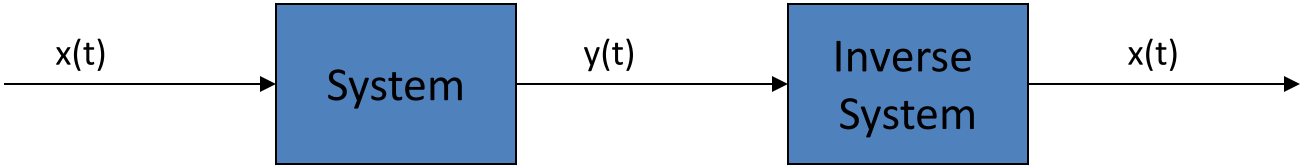
Example: Static or Dynamic?

Answer:

- The system shown above is RC circuit
- R is memoryless
- C is memory device as it stores charge because of which voltage across it can’t change immediately
- Hence given system is dynamic or memory system

Invertible & Inverse Systems

- If a system is invertible it has an Inverse System



- Example: y(t)=2x(t)
  - System is invertible→ must have inverse, that is:
  - For any x(t) we get a distinct output y(t)
  - Thus, the system must have an Inverse
    - x(t)=1/2 y(t)=z(t)