Step
response of discrete time systems

Similar to the continuous time case, transient response of a digital control system can also be characterized by the following.

1. Rise time (tr): Time required for the unit step response to rise from 0% to 100% of its final value in case of underdamped system or 10% to 90% of its final value in case of overdamped system.
2. Delay time (td): Time required for the the unit step response to reach 50% of its final value.

3. Peak time (tp): Time at which maximum peak occurs.
4. Peak overshoot (Mp): The difference between the maximum peak and the steady state value of the unit step response.
5. Settling time (ts): Time required for the unit step response to reach and stay within 2% or 5% of its steady state value.

However since the output response is discrete the calculated performance measures may be slightly different from the actual values. Figure 2 illustrates this. The output has a maximum value cmax whereas the maximum value of the discrete output is c max which is always less than or equal to cmax. If the sampling period is small enough compared to the oscillations of the response then this difference will be small otherwise c max may be completely erroneous.

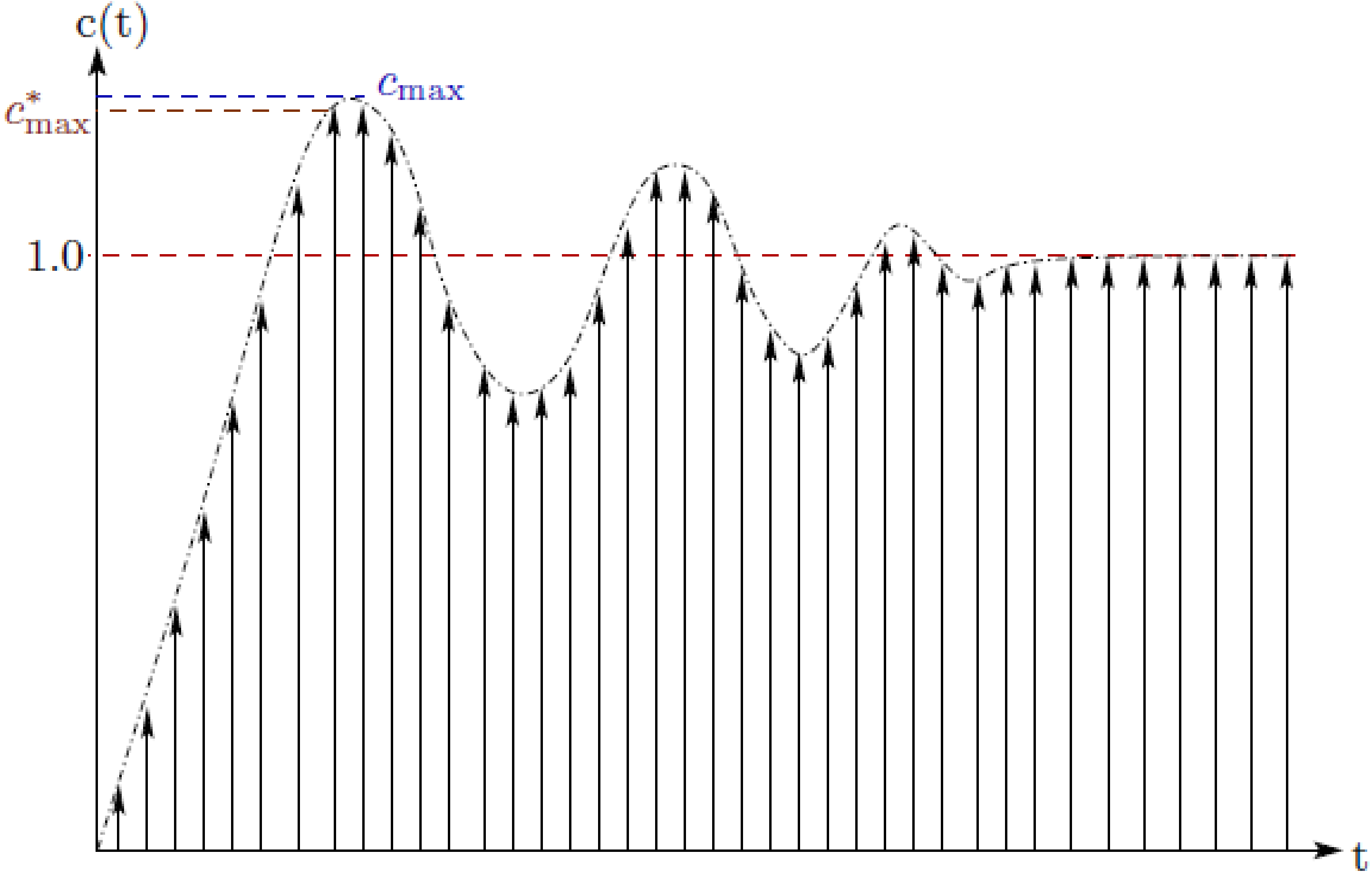

Figure : Unit step response of a discrete time system

signal energy and energy spectral density

Let $f(t)$ be an electric potential in Volt applied across a resistance of $R = 1$ ohm. The total energy dissipated in such a resistance is given by

$$E = \int_{-\infty}^{\infty} \left\{ f^2(t)/R \right\} dt.$$

Since the resistance value is unity the dissipated energy may be also be referred to as *normalized energy*. In what follows we shall refer to it simply as the dissipated energy, with the implicit assumption that it is the energy dissipated into a resistance of 1 ohm.

We recall Parseval's theorem which states that if a function $f(t)$ is generally complex and if $F(j\omega)$ is the Fourier transform of $f(t)$ then

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega.$$

The energy in the resistance may therefore be written in the form

$$E = \int_{-\infty}^{\infty} f^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega.$$

The function $|F(j\omega)|^2$ is called the *energy spectral density*, or simply the energy density, of $f(t)$. It is attributed the special symbol $\varepsilon_{ff}(\omega)$, that is,

$$\varepsilon_{ff}(\omega) \triangleq |F(j\omega)|^2 .$$

We note that its integral is equal to 2π times the signal energy

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varepsilon_{ff}(\omega)\,d\omega$$

hence the name 'spectral density'.

Given two signals $f_1(t)$ and $f_2(t)$, where $f_1(t)$ represent a current source and $f_2(t)$ the voltage that the current source produces across a resistance R of 1 ohm, we have

$$E = \int_{-\infty}^{\infty} f_1(t) f_2(t)\, dt.$$

Parseval's or Rayleigh's theorem is written

$$\int_{-\infty}^{\infty} f_1(t)\, f_2(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(-j\omega) F_2(j\omega)\, d\omega.$$

If $f_1(t)$ and $f_2(t)$ are real

$$F_1(-j\omega) = F_1^*(j\omega), \quad F_2(-j\omega) = F^*(j\omega).$$

The *normalized cross-energy* or simply *cross-energy* is therefore given by

$$E_{f_1 f_2} = \int_{-\infty}^{\infty} f_1(t) f_2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1^*(j\omega) F_2(j\omega)\, d\omega.$$

The function

$$\varepsilon_{f_1 f_2}(\omega) \triangleq F_1^*(j\omega) F_2(j\omega)$$

is called the *cross-energy spectral density*. The cross energy of the two signals is then given by

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varepsilon_{f_1 f_2}(\omega) d\omega.$$