Analysis of first order and second order systems

In time-domain analysis the response of a dynamic system to an input is expressed as a function of time. It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.

The time response of any system has two components: transient response and the steady-state response. Transient response is dependent upon the system poles only and not on the type of input.

It is therefore sufficient to analyze the transient response using a step input. The steady-state response depends on system dynamics and the input quantity. It is then examined using different test signals by final value theorem.

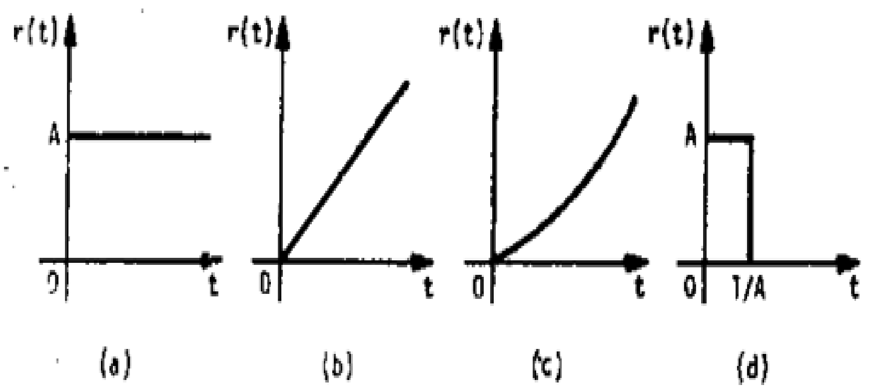
Standard test signals

a) Step signal: $r(t) = Au(t).$

b) Ramp signal: $r(t) = At; \ \ t > 0.$

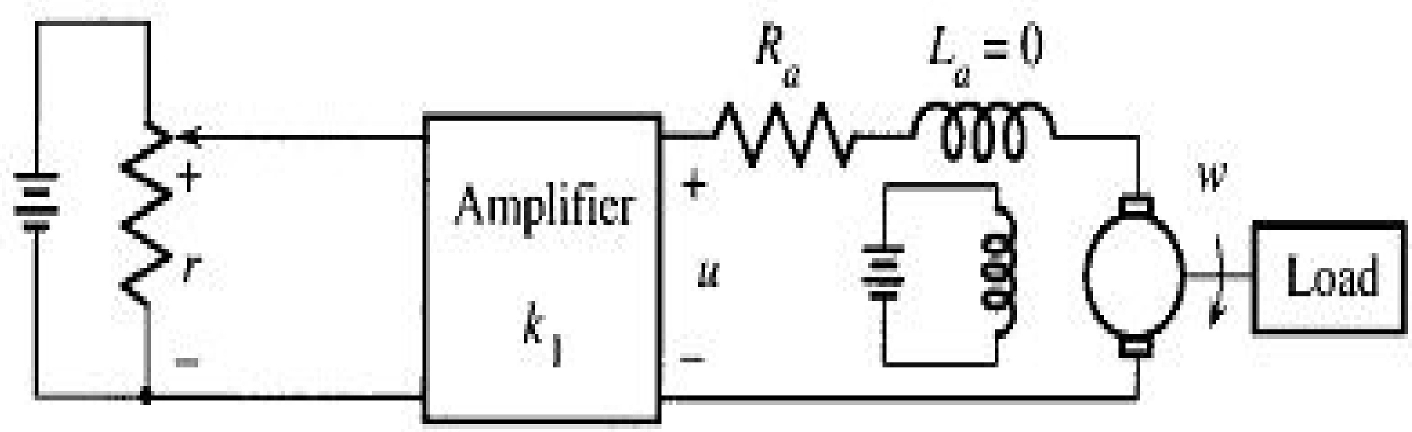
c) Parabolic signal: $r(t) = At^2/2; \ \ t > 0.$

d) Impulse signal: $r(t) = \delta(t).$



- **Time-response of first-order systems**

Let us consider the armature-controlled dc motor driving a load, such as a video tape. The objective is to drive the tape at constant speed. Note that it is an open-loop system.

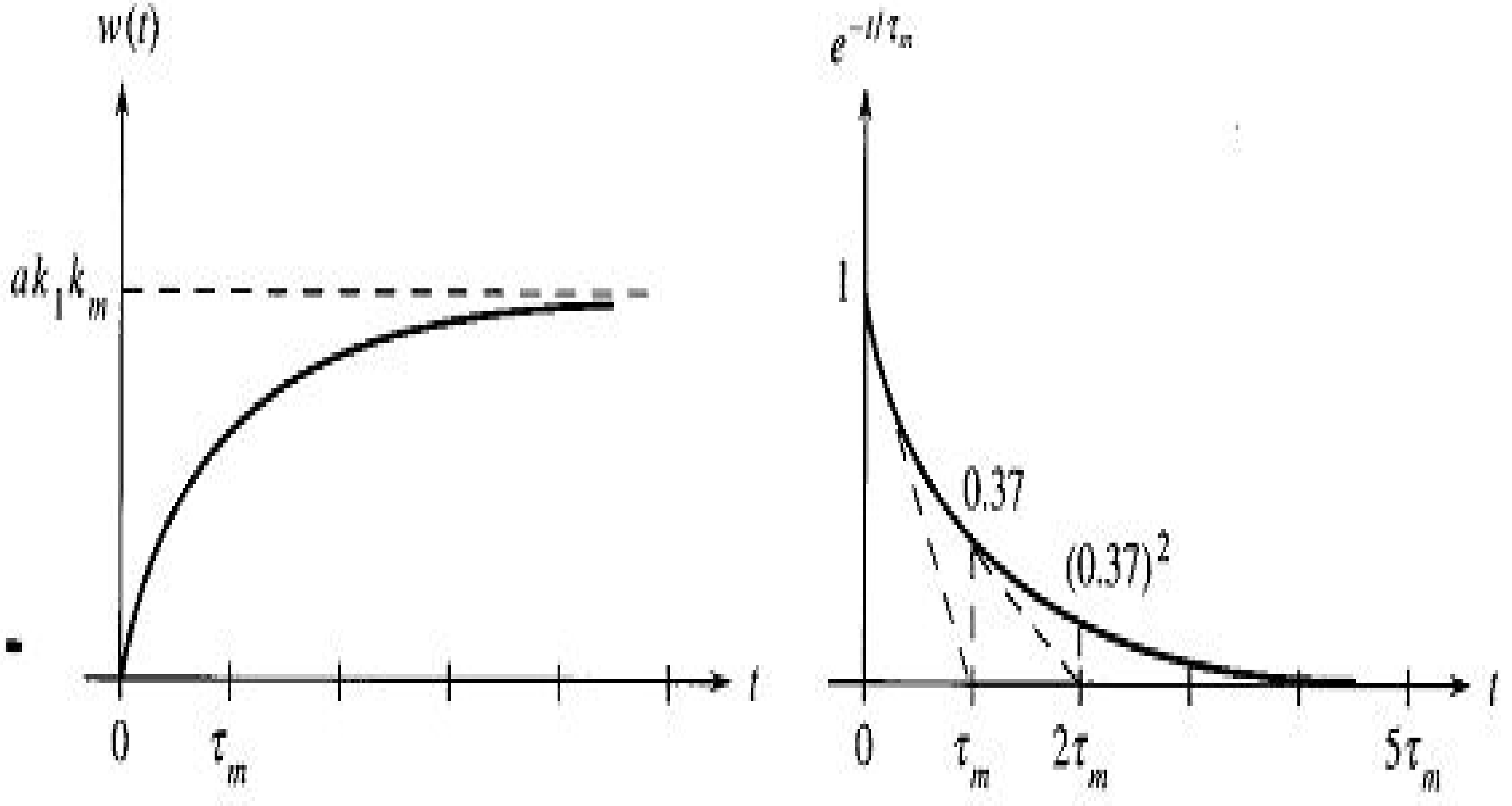


$$G(s)=\frac{W(s)}{R(s)}=\frac{k_1k_m}{\tau_m s+1}\text{; If } r(t)=au(t),\ W(s)=\frac{k_1k_m}{\tau_m s+1}\cdot\frac{a}{s}=\frac{ak_1k_m}{s}-\frac{ak_1k_m}{s+1/\tau_m}$$

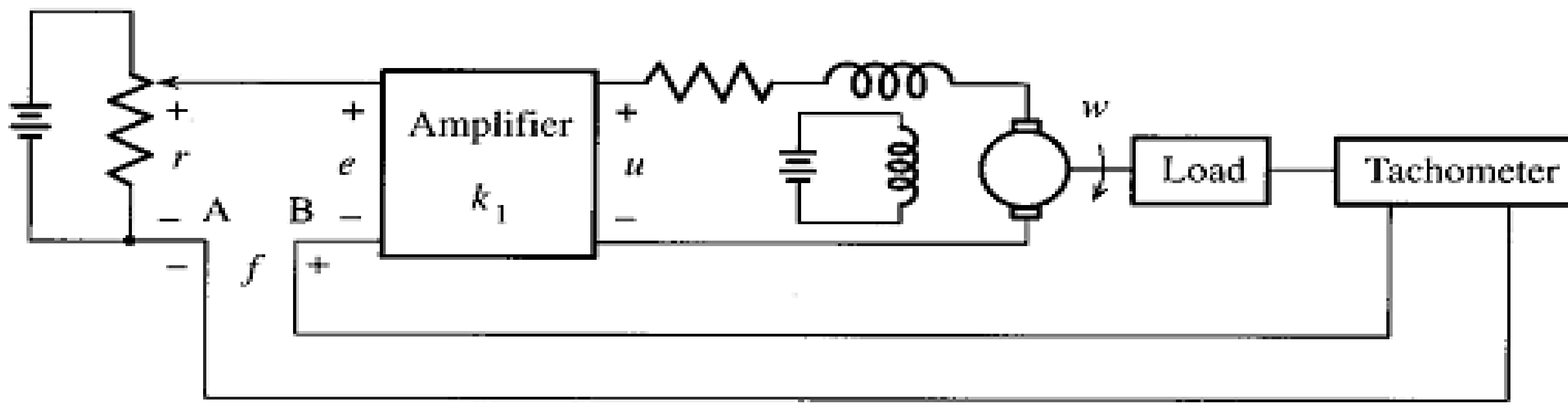
$$\Rightarrow w(t)=ak_1k_m-ak_1k_me^{-t/\tau_m}\ ;\quad \Rightarrow w_{ss}(t)=\lim_{t\to\infty}w(t)=ak_1k_m$$

$w_{ss}(t)$ is the steady-state final speed. If the desired speed is w_r, choosing $a=\frac{w_r}{k_1k_m}$ the motor will eventually reach the desired speed.

We are interested not only in final speed, but also in the speed of response. Here, τ_m is the time

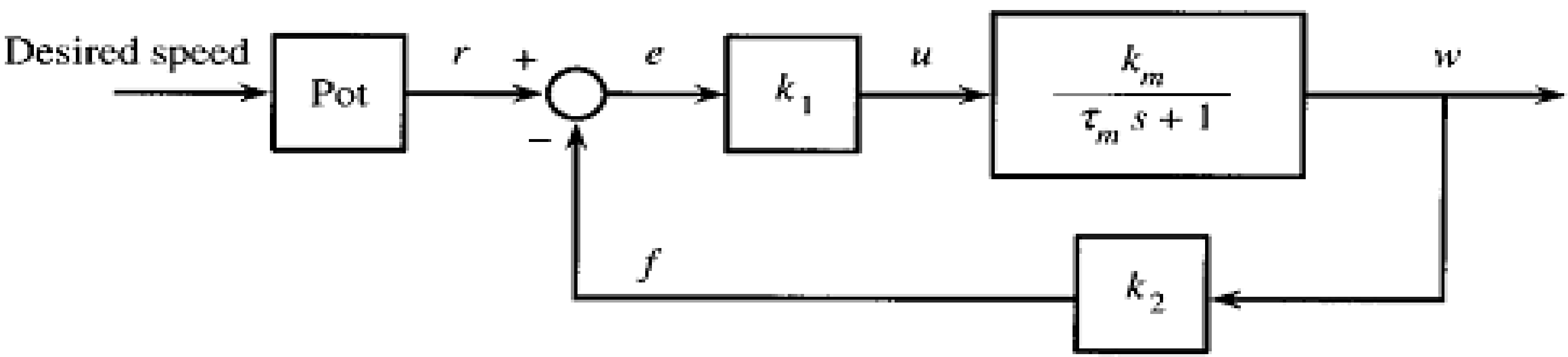


(a) (b)



Amplifier
k_1
Load
Tachometer
w
+
r
–
A
B
e
u
f

(a)



Desired speed
Pot
r
+
–
e
k_1
u
$\frac{k_m}{\tau_m s + 1}$
w
f
k_2

(b)

Here, $T(s)=\dfrac{W(s)}{R(s)}=\dfrac{k_1k_m/(\tau_m s+1)}{1+k_1k_2k_m/(\tau_m s+1)}=\dfrac{k_1k_m}{\tau_m s+(1+k_1k_2k_m)}=\dfrac{k_1k_o}{\tau_o s+1}$

where, $k_o=\dfrac{k_m}{1+k_1k_2k_m}$ and $\tau_o=\dfrac{\tau_m}{1+k_1k_2k_m}$.

If $r(t)=a$, the response would be, $w(t)=ak_1k_o-ak_1k_oe^{-t/\tau_o}$.

If a is properly chosen, the tape can reach a desired speed. It will reach the desired speed in $5\tau_o$ seconds. Here, $\tau_o \ll \tau_m$. Thus, we can control the speed of response in feedback system.

Although the time-constant is reduced by a factor $(1+k_1k_2k_m)$, in the feedback system, the motor gain constant is also reduced by the same factor. In order to compensate for this loss of gain, the applied reference voltage must be increased by the same factor.