# System Function
# (Transfer Function)

A transfer function $G(s)$ is defined as the following relation between the output of the system $Y(s)$ and the input to the system $U(s)$.

$$G(s) = \frac{Y(s)}{U(s)}$$

If the transfer function of a system is known then the response of the system $Y(s)$ can be found by taking the inverse Laplace transform of $G(s) \cdot U(s)$. It is also important to note that a transfer function is only defined for linear time invariant systems with all initial conditions set to zero.

If the input to the system is a unit impulse ($U(s) = 1$), then

$$G(s) = Y(s)$$

Therefore, the inverse Laplace transform of the Transfer function of a system is the unit impulse response of the system. This can be thought of as the response to a brief external disturbance.

# Poles & Zeros

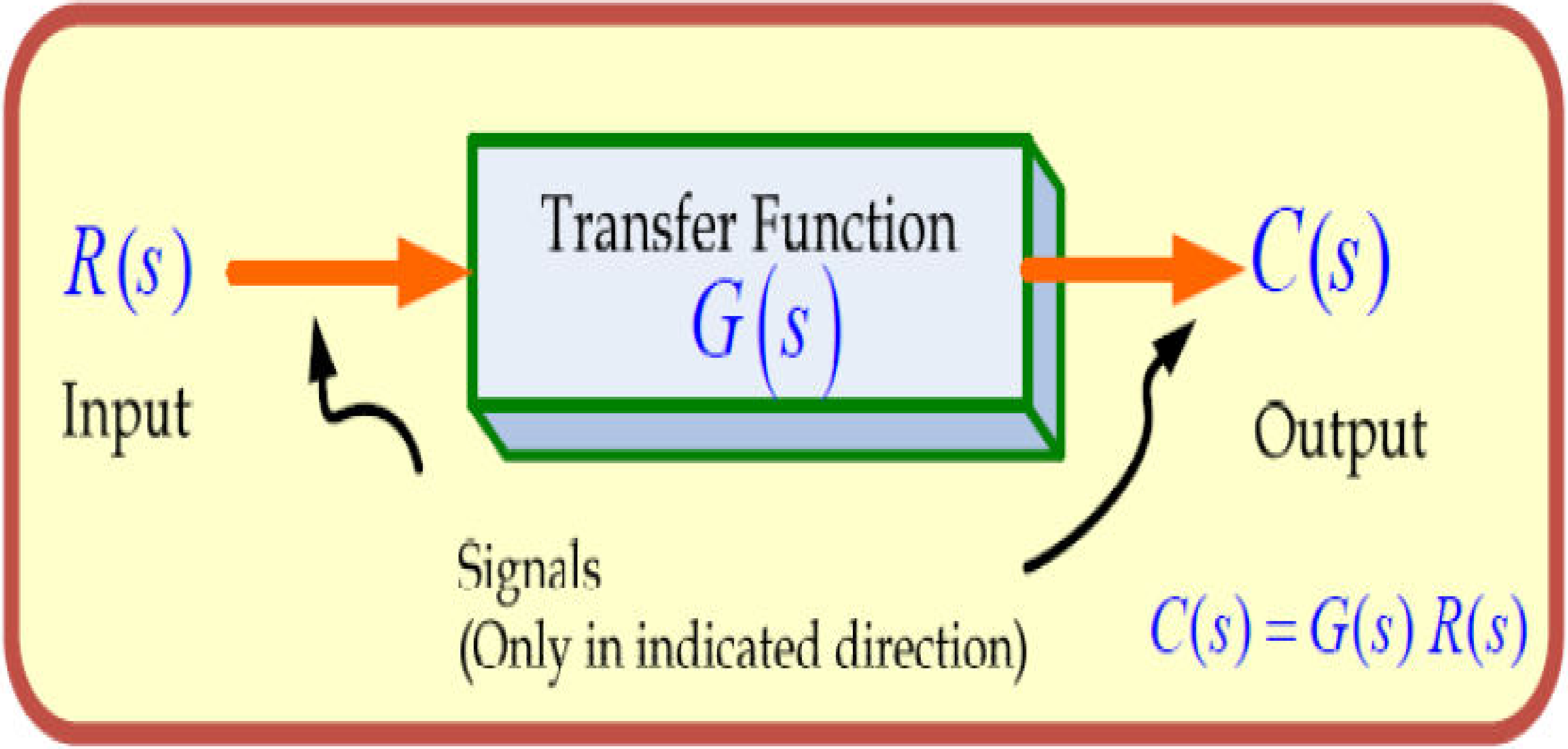
*Zeros* are defined as the roots of the polynomial of the numerator of a transfer function and *poles* are defined as the roots of the denominator of a transfer function. For the generalized transfer function

$$G(s) = \frac{(s - z_1) \cdot (s - z_2) \ldots \cdot (s - z_n)}{(s - p_1) \cdot (s - p_2) \ldots \cdot (s - p_n)}$$

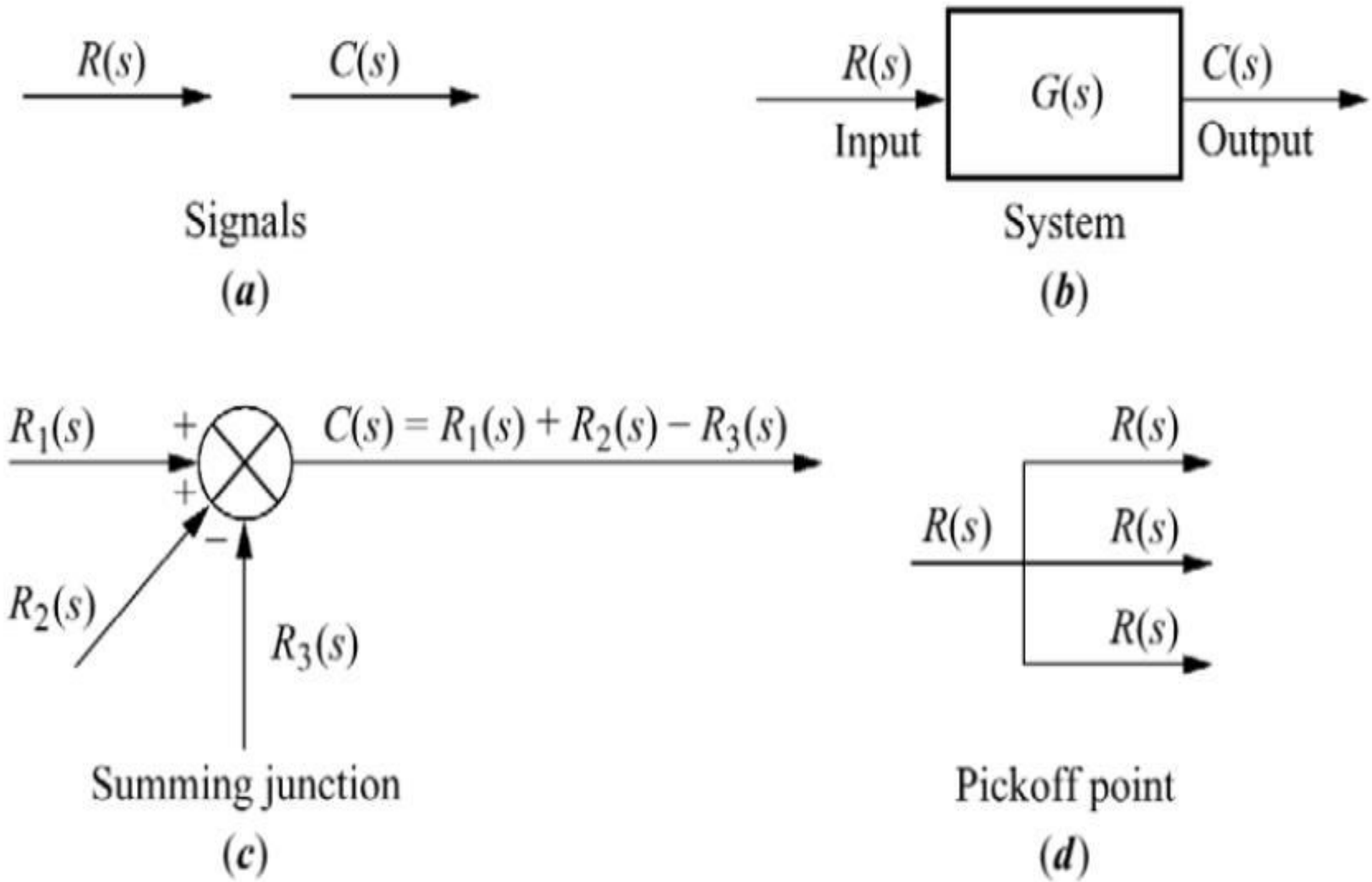
The zeros are $z_1, z_2, \ldots z_n$ and the poles are $p_1, p_2, \ldots p_n$.

Identifying the poles and zeros of a transfer function aids in understanding the behavior of the system. For example, consider the transfer function $F_1(s) = \frac{32}{s \cdot (s+1) \cdot (s+2)}$ .This function has three poles, two of which are negative integers and one of which is zero.

# Block diagram Representation



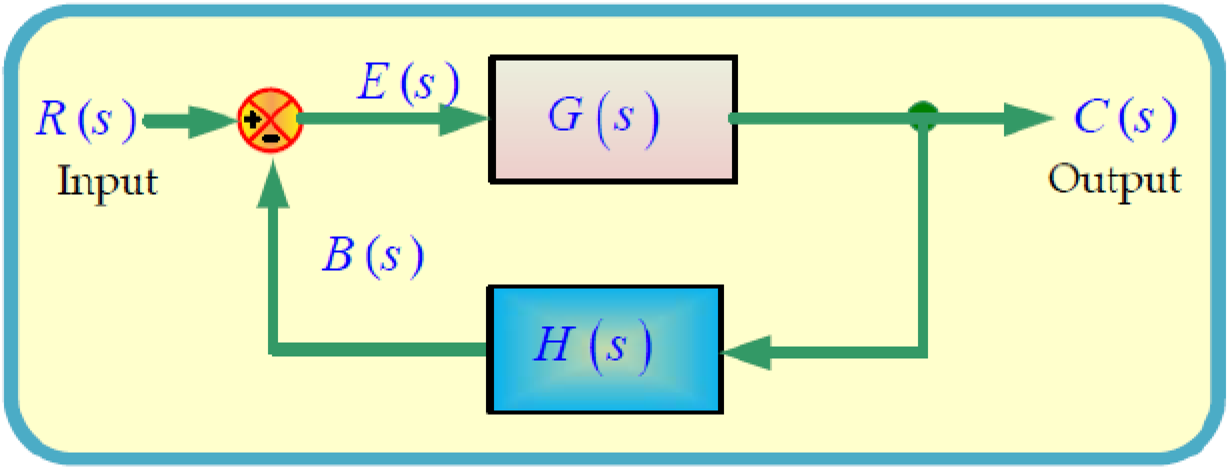
Transfer Function
$G(s)$
$R(s)$
Input
$C(s)$
Output
Signals
(Only in indicated direction)
$C(s) = G(s)\,R(s)$



R(s)
C(s)
Signals
(a)
R(s)
Input
G(s)
C(s)
Output
System
(b)
R1(s)
+
+
−
C(s) = R1(s) + R2(s) − R3(s)
R2(s)
R3(s)
Summing junction
(c)
R(s)
R(s)
R(s)
R(s)
Pickoff point
(d)

# Definitions

- $G(s) \equiv$ Direct transfer function = Forward transfer function.
- $H(s) \equiv$ Feedback transfer function.
- $G(s)H(s) \equiv$ Open-loop transfer function.
- $C(s)/R(s) \equiv$ Closed-loop transfer function = Control ratio
- $C(s)/E(s) \equiv$ Feed-forward transfer function.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$