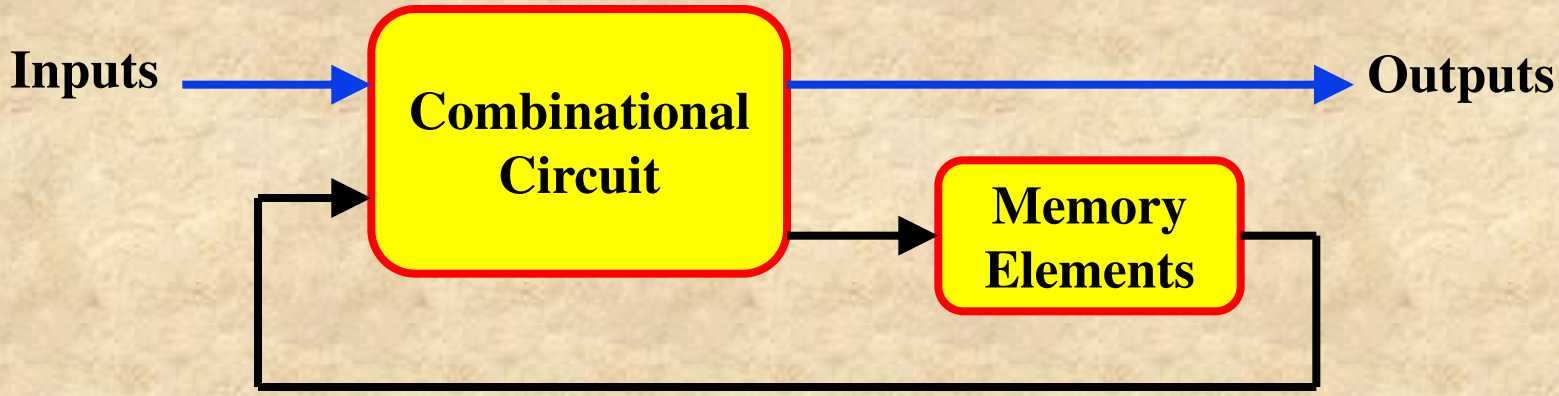
Analysis of clocked sequential circuits, state reduction and assignments & design procedure
Lecture 2

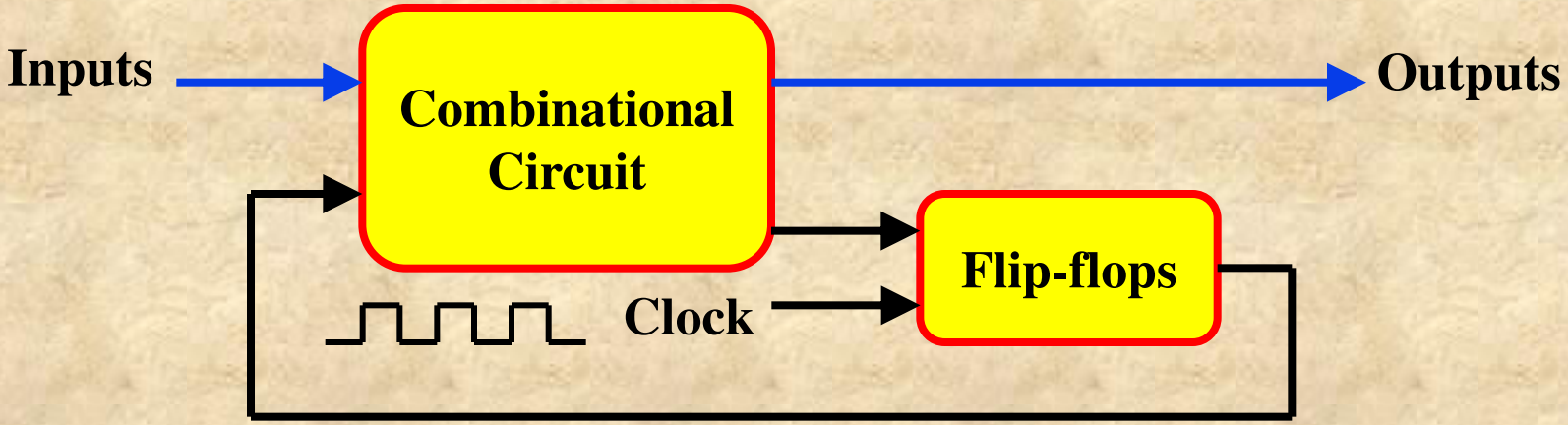
★ Dronacharya Group if Institutions

Sequential Circuits

★ Asynchronous

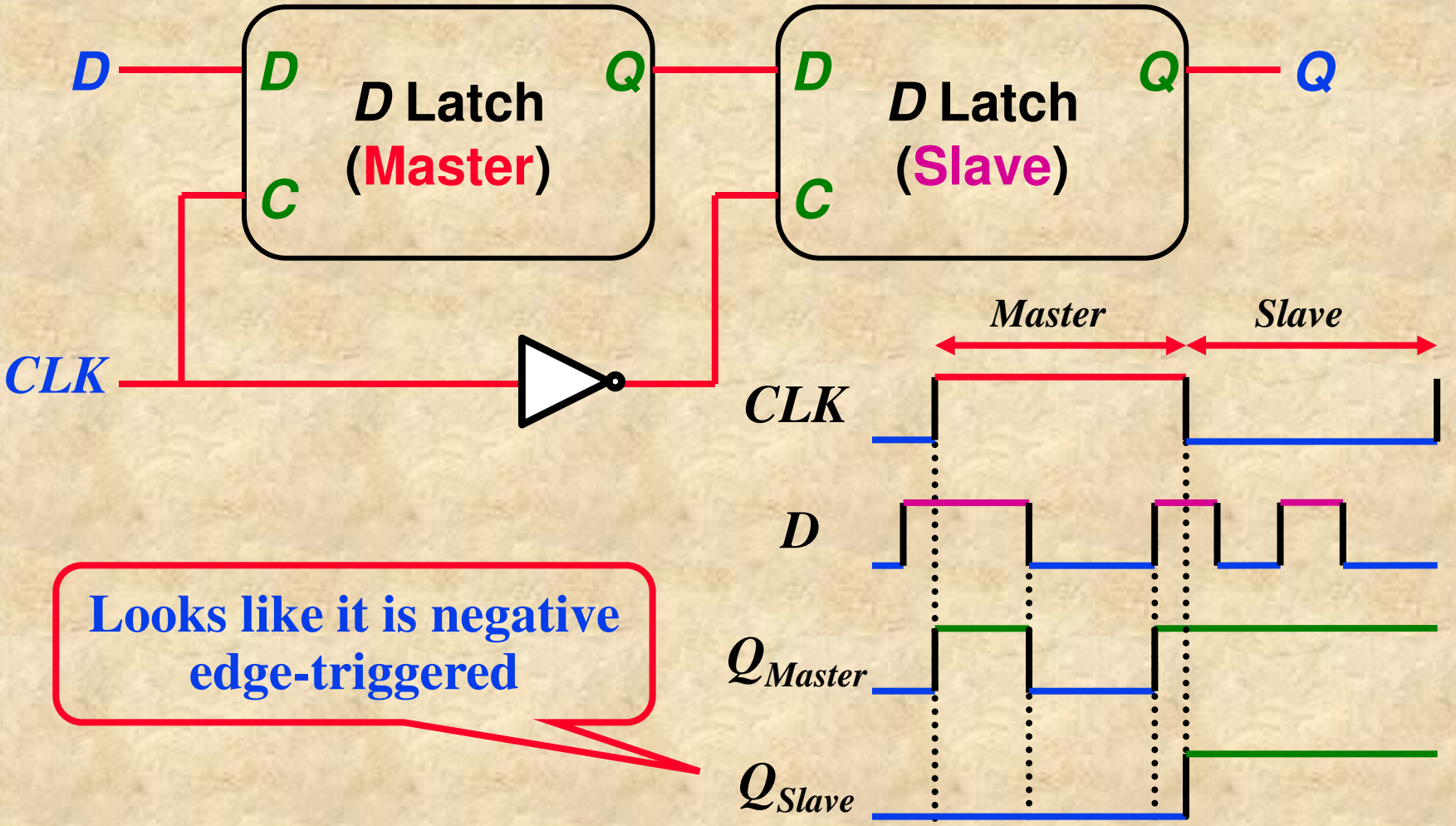


★ Synchronous

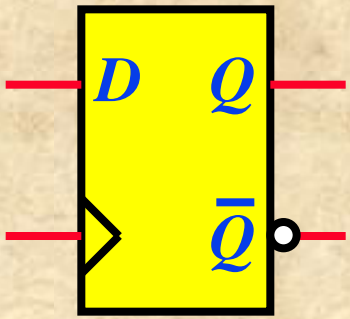


Flip-Flops

★ Master-Slave *D* Flip-Flop



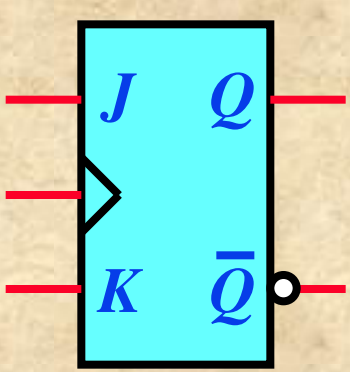
Flip-Flop Characteristic Tables



D	Q(t+1)
0	0
1	1

Reset

Set



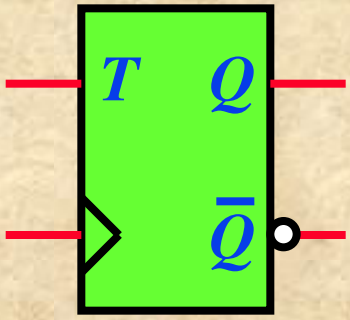
J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	Q'(t)

No change

Reset

Set

Toggle

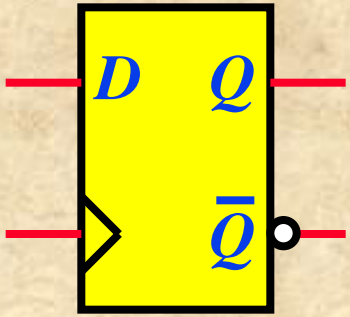


T	Q(t+1)
0	Q(t)
1	Q'(t)

No change

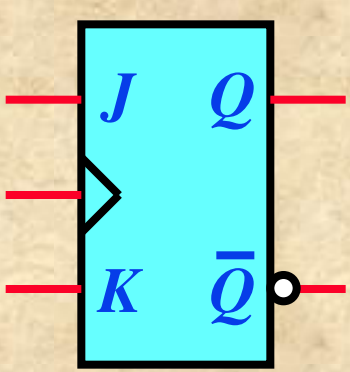
Toggle

Flip-Flop Characteristic Equations



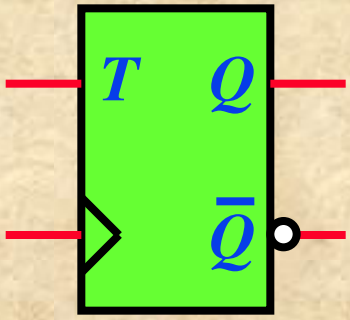
D	Q(t+1)
0	0
1	1

$$Q(t+1) = D$$



J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	Q'(t)

$$Q(t+1) = JQ' + K'Q$$

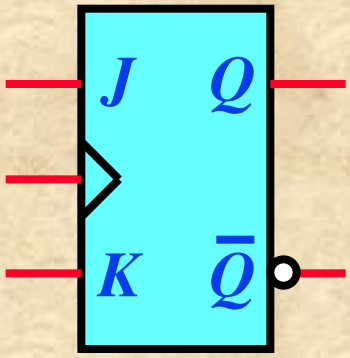


T	Q(t+1)
0	Q(t)
1	Q'(t)

$$Q(t+1) = T \oplus Q$$

Flip-Flop Characteristic Equations

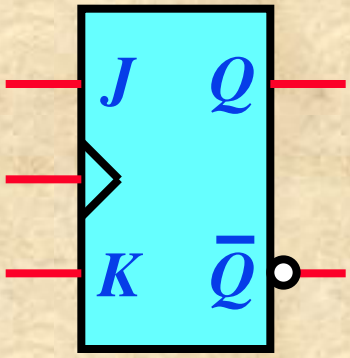
★ **Analysis / Derivation**



J	K	$Q(t)$	$Q(t+1)$	
0	0	0	0	No change
0	0	1	1	
0	1	0		Reset
0	1	1		
1	0	0		Set
1	0	1		
1	1	0		Toggle
1	1	1		

Flip-Flop Characteristic Equations

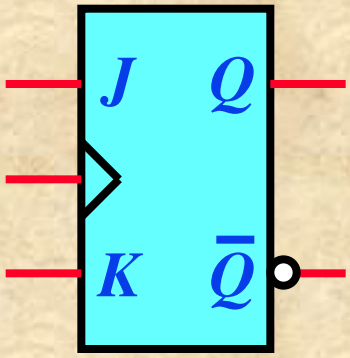
★ **Analysis / Derivation**



J	K	$Q(t)$	$Q(t+1)$	
0	0	0	0	No change
0	0	1	1	
0	1	0	0	Reset
0	1	1	0	
1	0	0		Set
1	0	1		
1	1	0		Toggle
1	1	1		

Flip-Flop Characteristic Equations

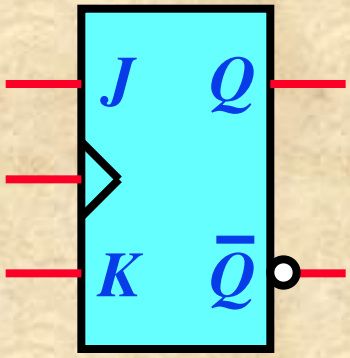
★ Analysis / Derivation



J	K	$Q(t)$	$Q(t+1)$	
0	0	0	0	No change
0	0	1	1	
0	1	0	0	Reset
0	1	1	0	
1	0	0	1	Set
1	0	1	1	
1	1	0		Toggle
1	1	1		

Flip-Flop Characteristic Equations

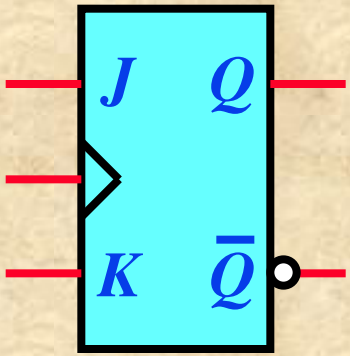
★ **Analysis / Derivation**



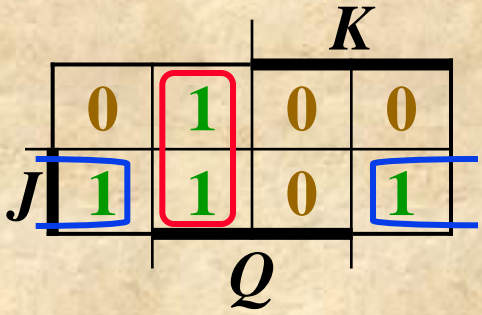
J	K	$Q(t)$	$Q(t+1)$	
0	0	0	0	No change
0	0	1	1	
0	1	0	0	Reset
0	1	1	0	
1	0	0	1	Set
1	0	1	1	
1	1	0	1	Toggle
1	1	1	0	

Flip-Flop Characteristic Equations

★ **Analysis / Derivation**



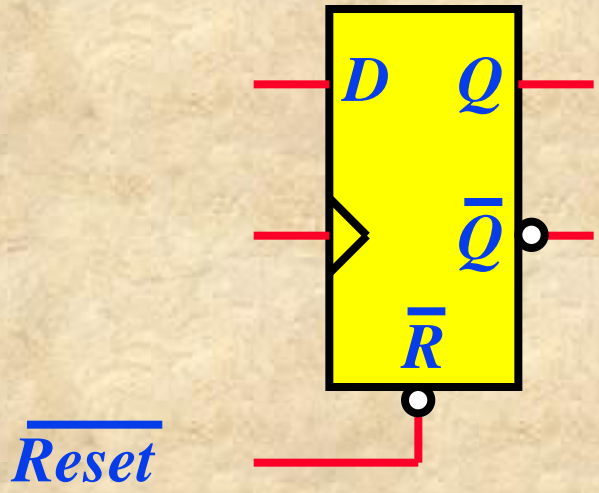
J	K	Q(t)	Q(t+1)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



$$Q(t+1) = JQ' + K'Q$$

Flip-Flops with Direct Inputs

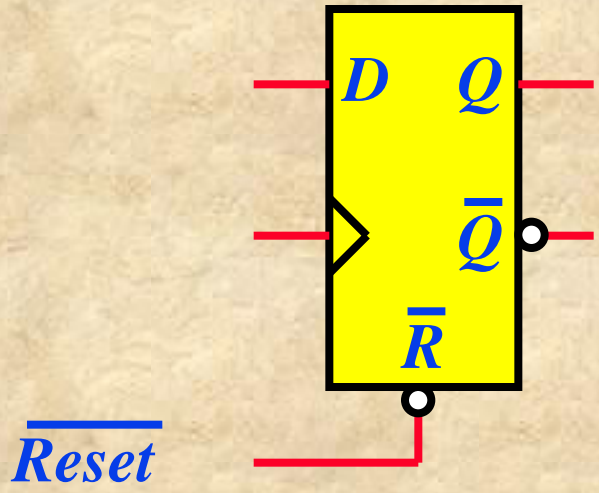
★ Asynchronous Reset



R'	*D*	*CLK*	*Q*(*t*+1)
0	x	x	0

Flip-Flops with Direct Inputs

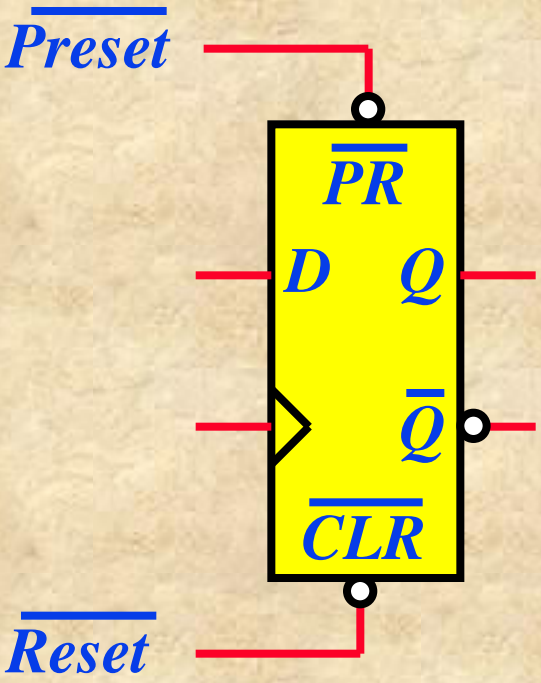
★ Asynchronous Reset



R'	*D*	*CLK*	*Q*(*t*+1)
0	x	x	0
1	0	↑	0
1	1	↑	1

Flip-Flops with Direct Inputs

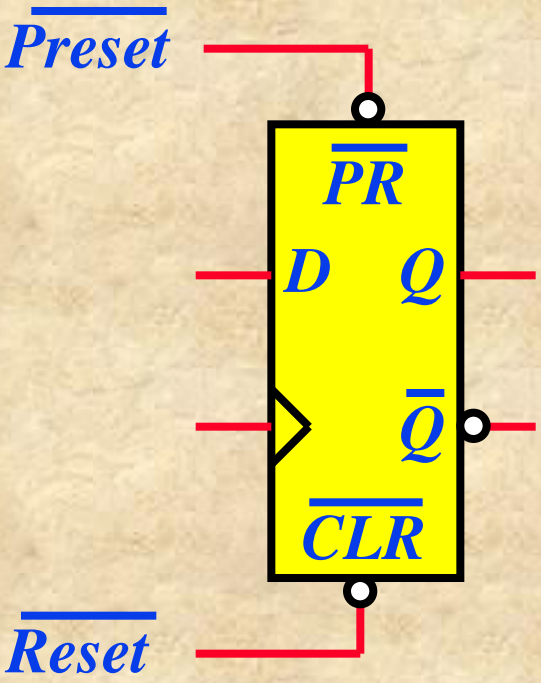
★ **Asynchronous Preset and Clear**



PR'	CLR'	D	CLK	Q(t+1)
1	0	x	x	0

Flip-Flops with Direct Inputs

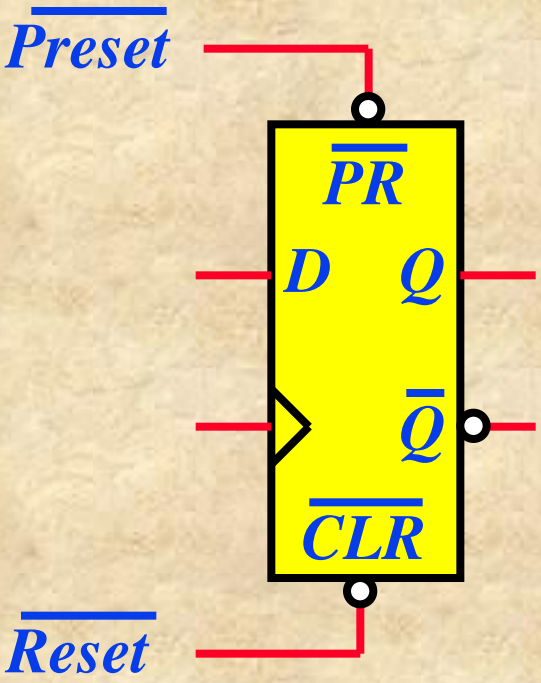
★ **Asynchronous Preset and Clear**



PR'	CLR'	D	CLK	Q(t+1)
1	0	x	x	0
0	1	x	x	1

Flip-Flops with Direct Inputs

★ Asynchronous Preset and Clear



PR'	CLR'	D	CLK	Q(t+1)
1	0	x	x	0
0	1	x	x	1
1	1	0	↑	0
1	1	1	↑	1

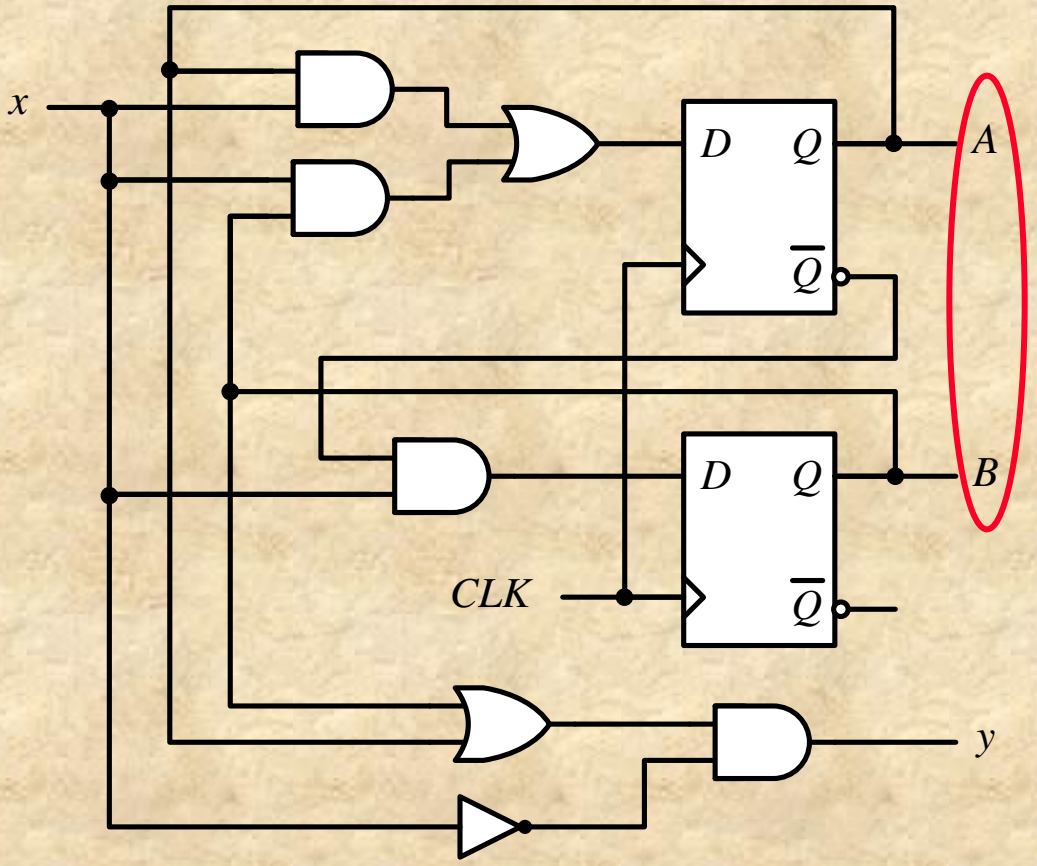
Analysis of Clocked Sequential Circuits

★ The State

- **State = Values of all Flip-Flops**

Example

$A\ B = 0\ 0$



Analysis of Clocked Sequential Circuits

★ State Equations

$$A(t+1) = D_A$$

$$= A(t)\,x(t) + B(t)\,x(t)$$

$$= A\,x + B\,x$$

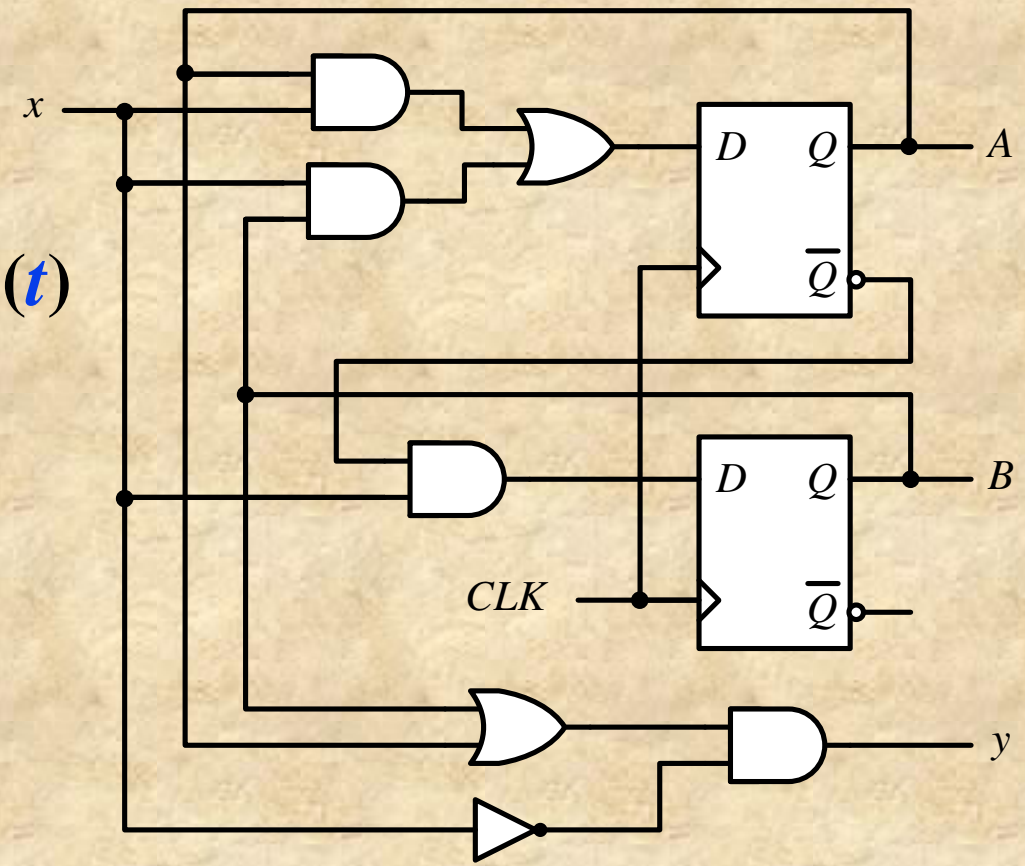
$$B(t+1) = D_B$$

$$= A'(t)\,x(t)$$

$$= A'\,x$$

$$y(t) = [A(t) + B(t)]\,x'(t)$$

$$= (A + B)\,x'$$

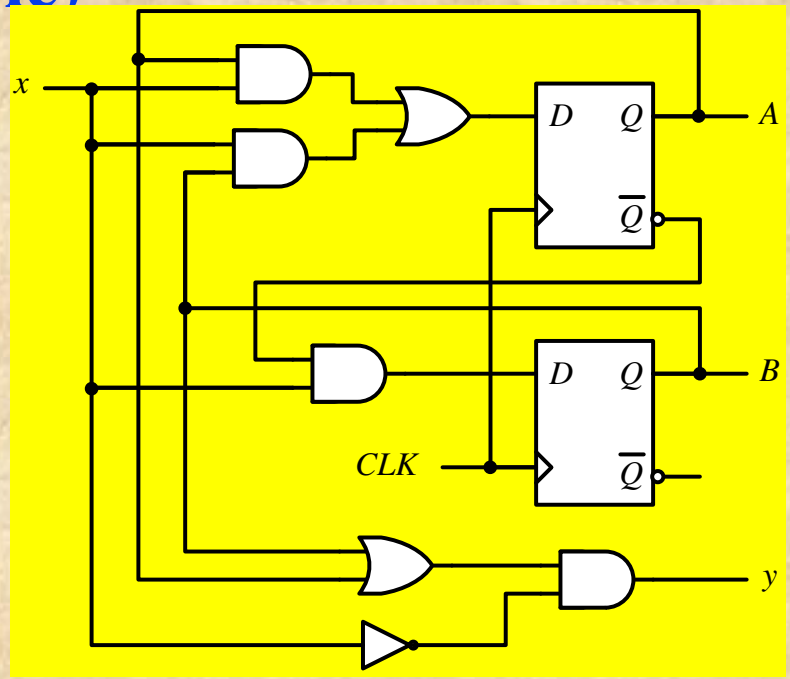


Analysis of Clocked Sequential Circuits

★ State Table (Transition Table)

Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

t *t*+1 *t*



$$A(t+1) = A\,x + B\,x$$

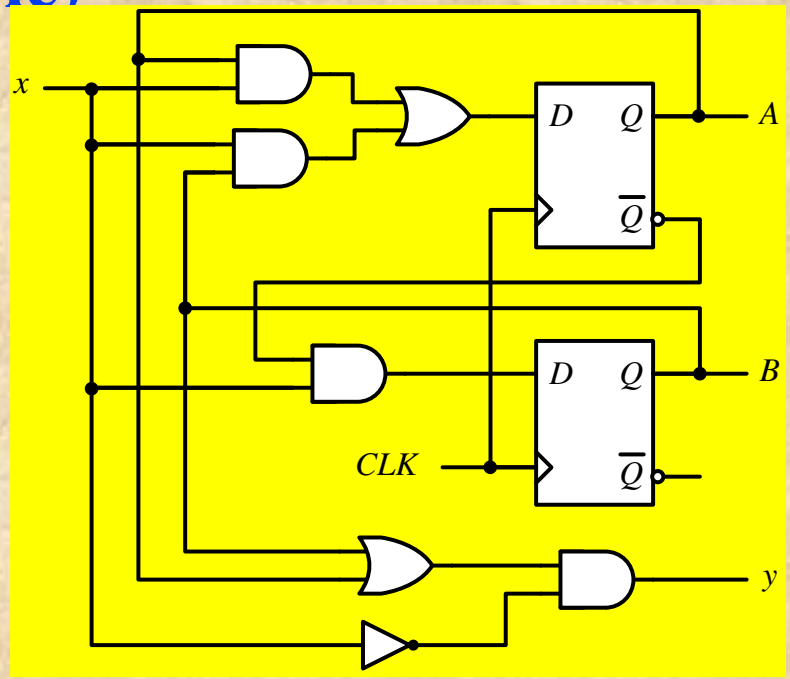
$$B(t+1) = A'\,x$$

$$y(t) = (A + B)\,x'$$

Analysis of Clocked Sequential Circuits

★ State Table (Transition Table)

Present State	Next State		Output	
	$x = 0$	$x = 1$	$x = 0$	$x = 1$
$A\ B$	$A\ B$	$A\ B$	y	y
0 0	0 0	0 1	0	0
0 1	0 0	1 1	1	0
1 0	0 0	1 0	1	0
1 1	0 0	1 0	1	0
t	$t+1$		t	



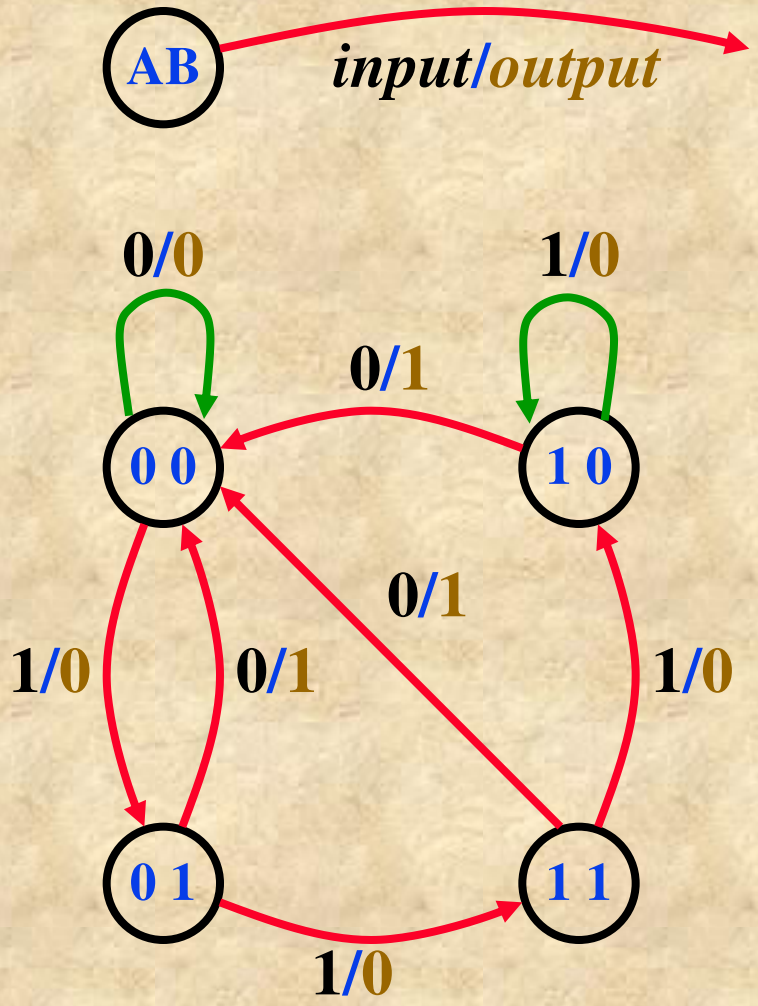
$$A(t+1) = A\,x + B\,x$$

$$B(t+1) = A'\,x$$

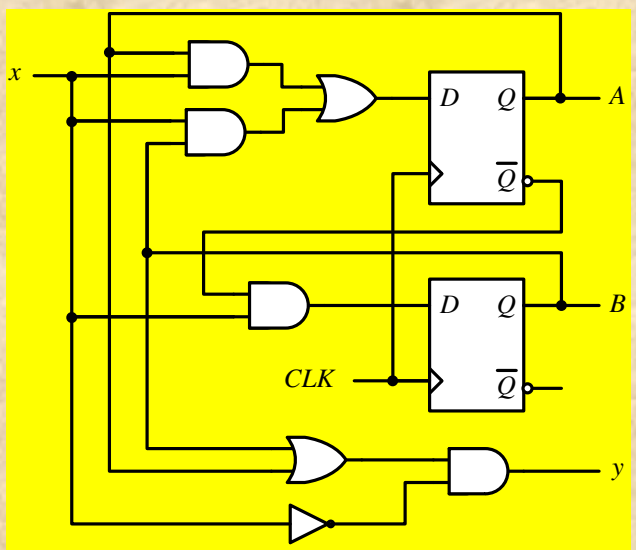
$$y(t) = (A + B)\,x'$$

Analysis of Clocked Sequential Circuits

★ State Diagram



Present State	Next State				Output	
	$x = 0$		$x = 1$		$x = 0$	$x = 1$
A B	A	B	A	B	y	y
0 0	0	0	0	1	0	0
0 1	0	0	1	1	1	0
1 0	0	0	1	0	1	0
1 1	0	0	1	0	1	0

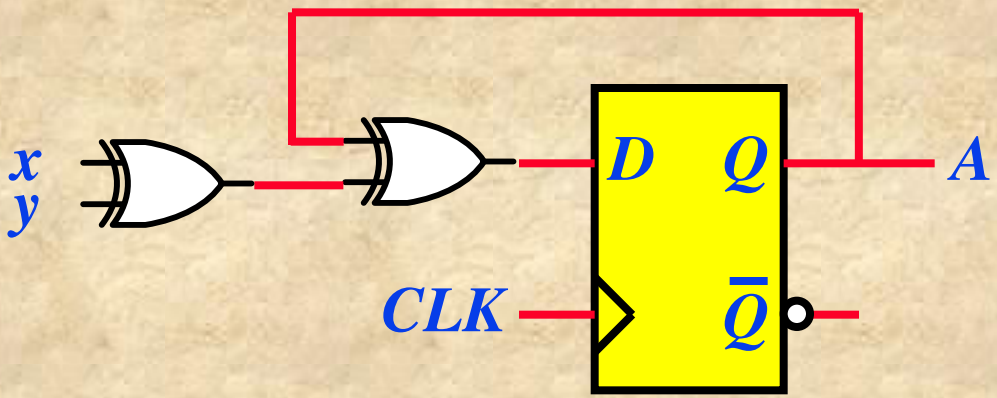


Analysis of Clocked Sequential Circuits

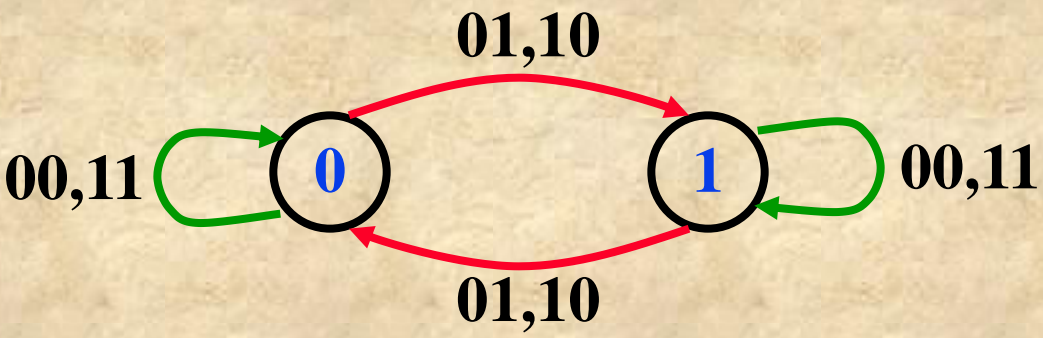
★ *D* Flip-Flops

Example:

Present State	Input		Next State
A	*x*	*y*	*A*
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



$$A(t+1) = D_A = A \oplus x \oplus y$$

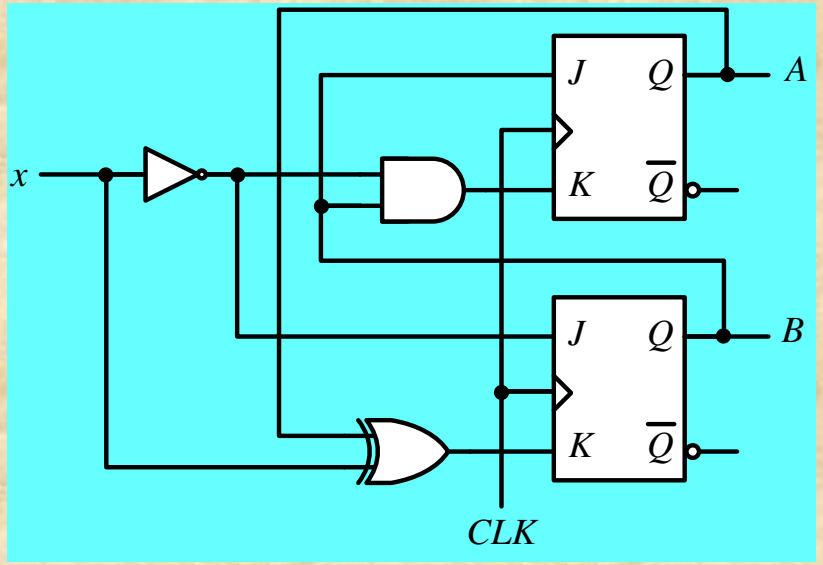


Analysis of Clocked Sequential Circuits

★ *JK* Flip-Flops

***Example*:**

Present State		I/P	Next State		Flip-Flop Inputs			
A	*B*	*x*	*A*	*B*	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0



$J_A = B$ $\quad K_A = B\,x'$

$J_B = x'$ $\quad K_B = A \oplus x$

$$A(t+1) = J_A Q'_A + K'_A Q_A$$
$$= A'B + AB' + Ax$$

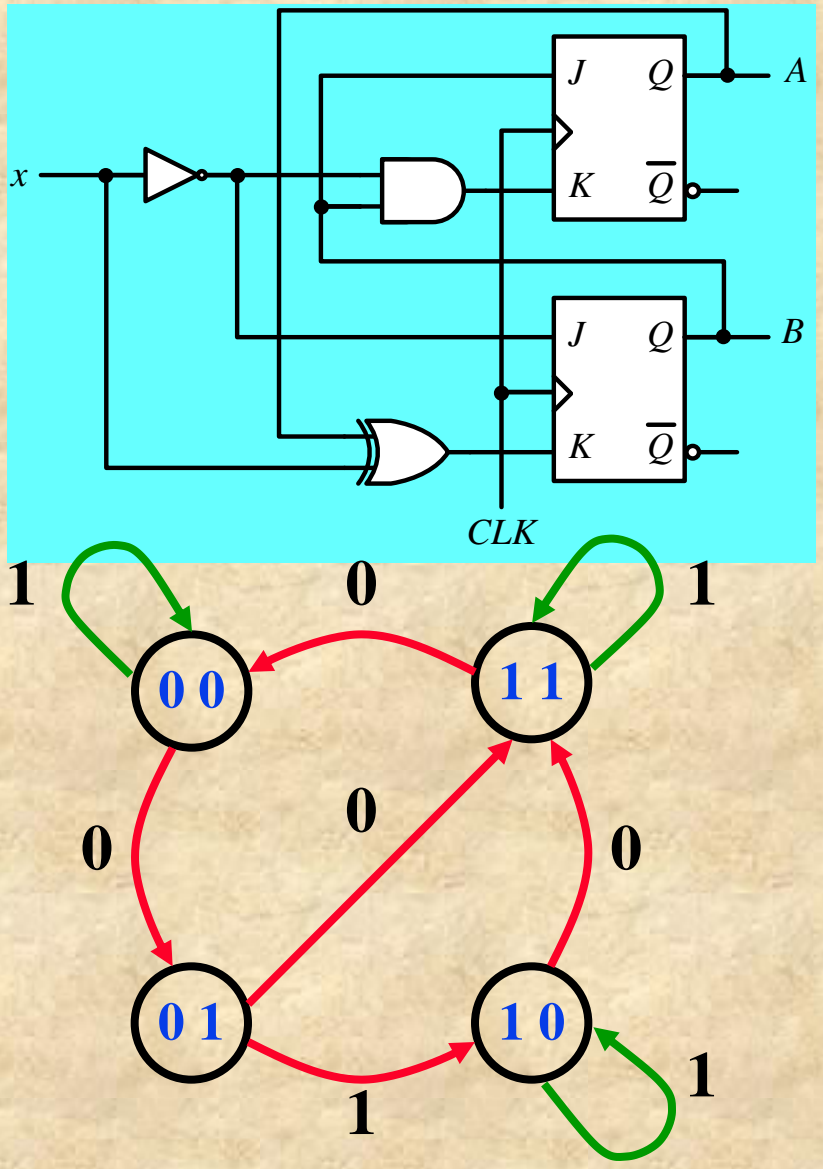
$$B(t+1) = J_B Q'_B + K'_B Q_B$$
$$= B'x' + ABx + A'Bx'$$

Analysis of Clocked Sequential Circuits

★ *JK* Flip-Flops

***Example*:**

Present State		I/P	Next State		Flip-Flop Inputs			
A	*B*	*x*	*A*	*B*	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

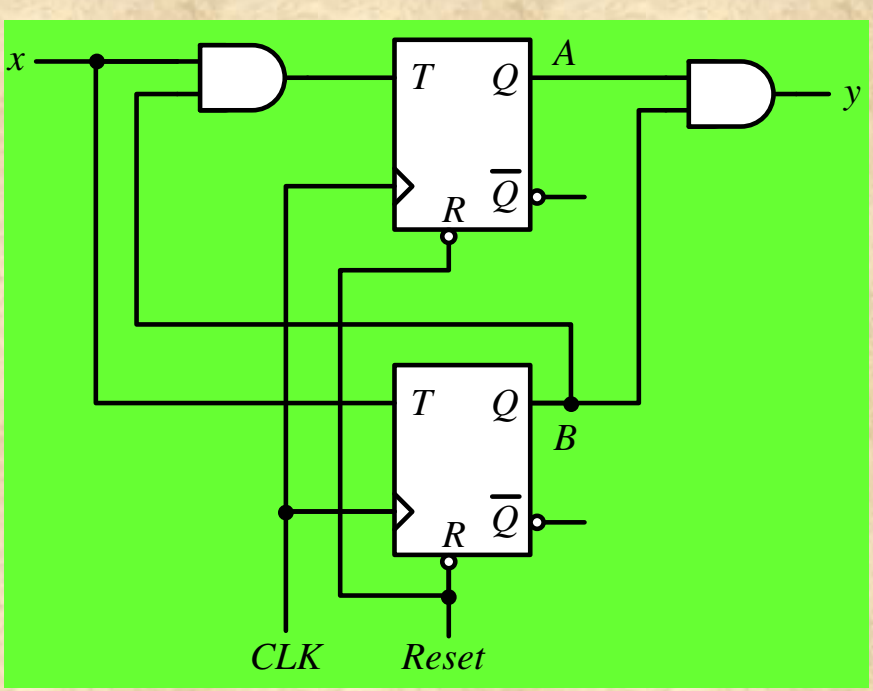


Analysis of Clocked Sequential Circuits

★ *T* Flip-Flops

Example:

Present State		I/P	Next State		F.F Inputs		O/P
A	*B*	*x*	*A*	*B*	T_A	T_B	*y*
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	0	1	0	0	0
0	1	1	1	0	1	1	0
1	0	0	1	0	0	0	0
1	0	1	1	1	0	1	0
1	1	0	1	1	0	0	1
1	1	1	0	0	1	1	1



$$T_A = B\,x \qquad T_B = x$$

$$y = A\,B$$

$$A(t+1) = T_A Q'_A + T'_A Q_A$$
$$= AB' + Ax' + A'Bx$$

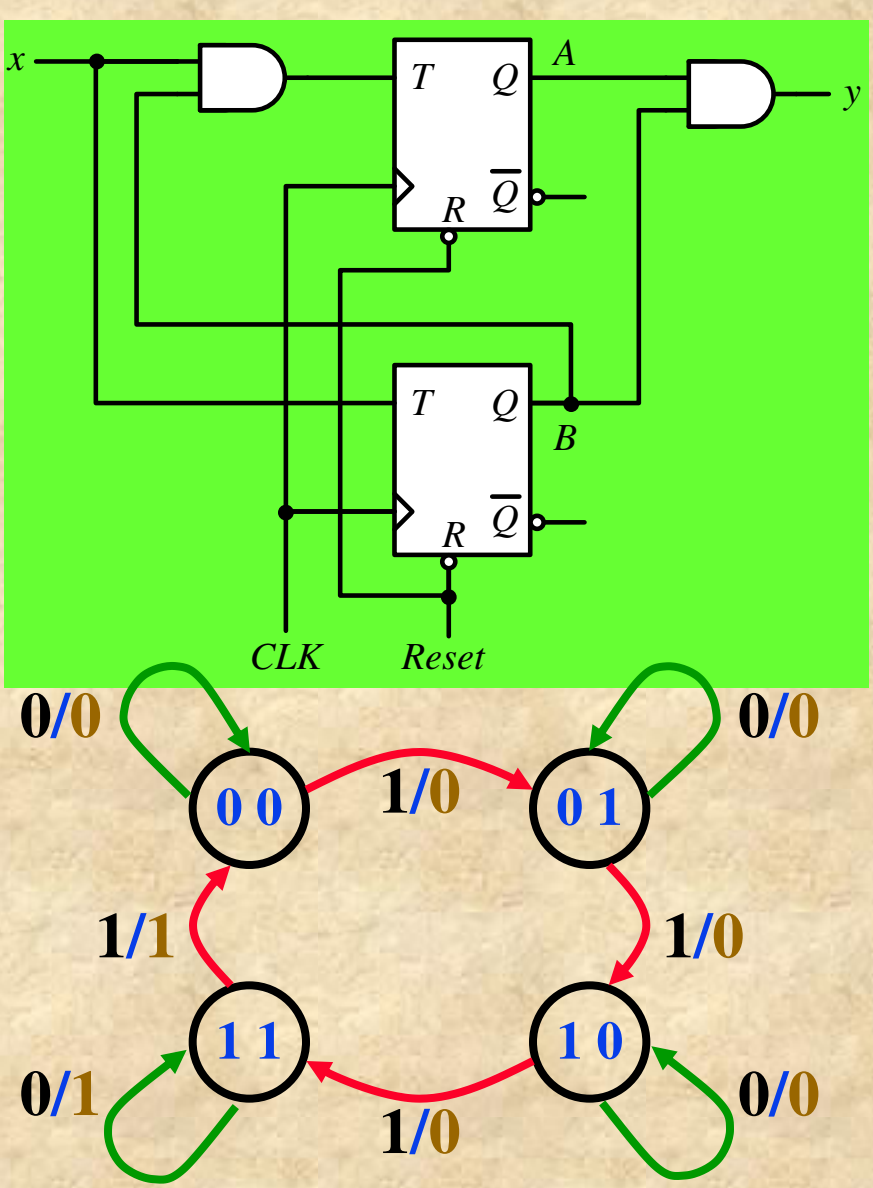
$$B(t+1) = T_B Q'_B + T'_B Q_B$$
$$= x \oplus B$$

Analysis of Clocked Sequential Circuits

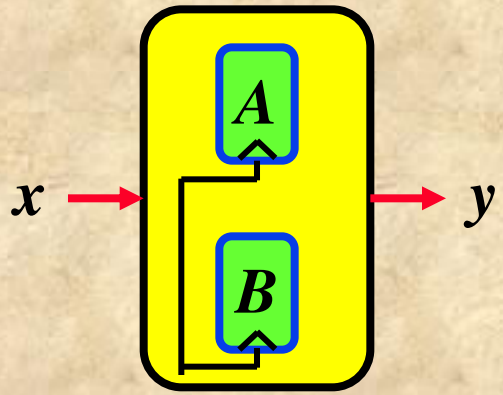
★ *T* Flip-Flops

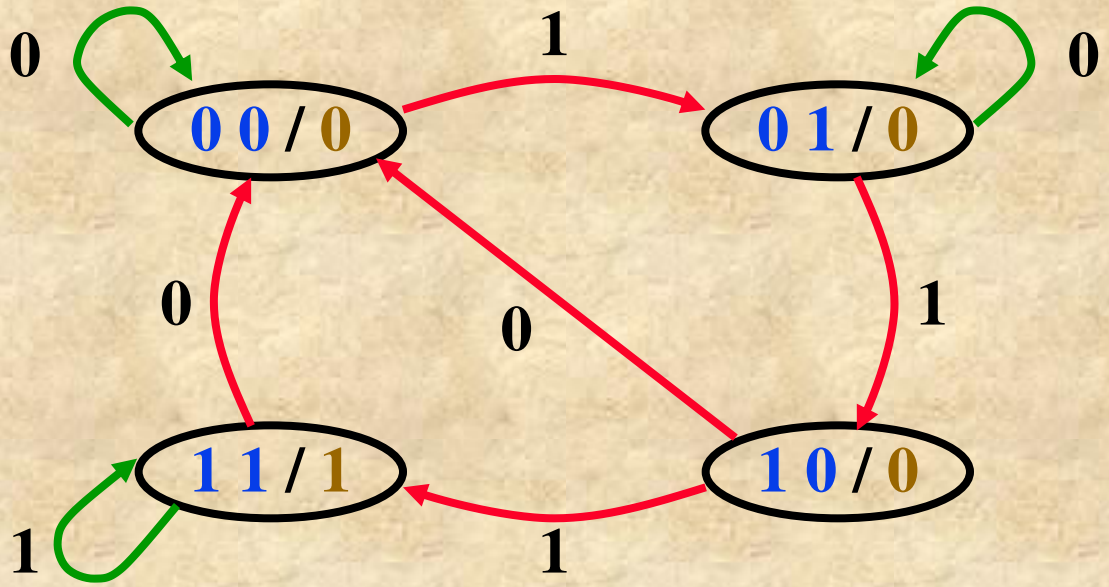
Example:

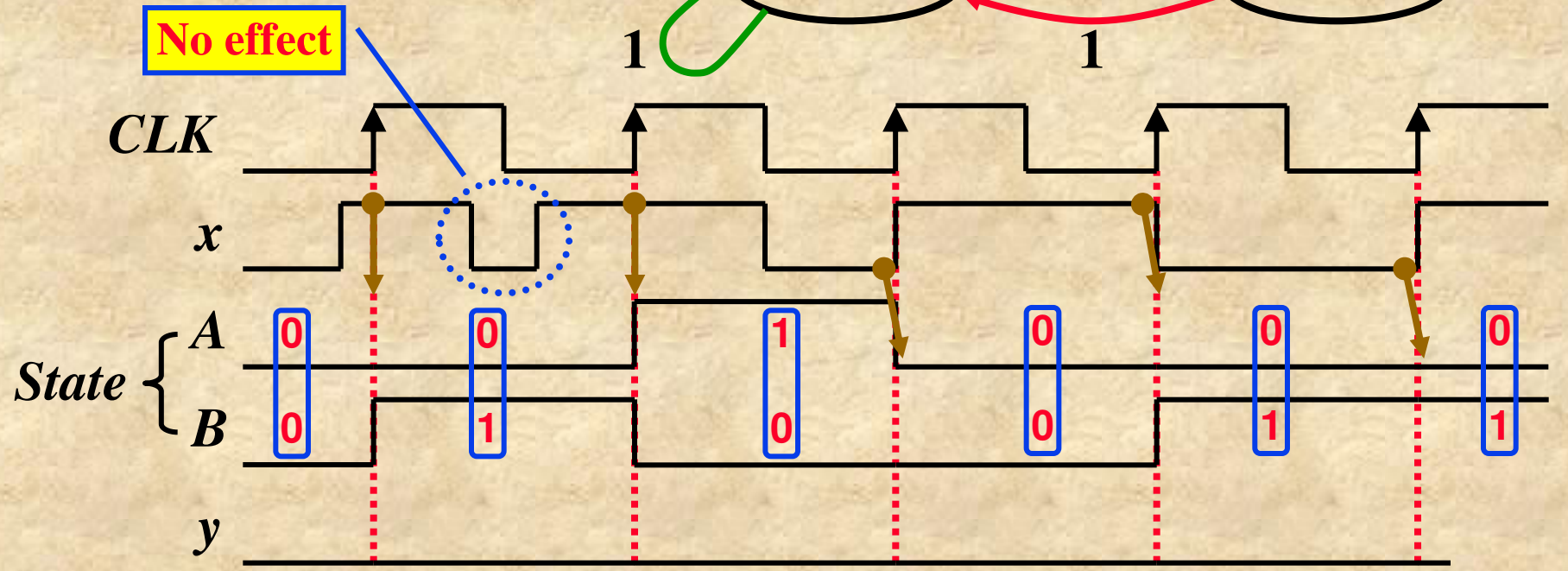
Present State		I/P	Next State		F.F Inputs		O/P
A	*B*	*x*	*A*	*B*	T_A	T_B	*y*
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	0	1	0	0	0
0	1	1	1	0	1	1	0
1	0	0	1	0	0	0	0
1	0	1	1	1	0	1	0
1	1	0	1	1	0	0	1
1	1	1	0	0	1	1	1



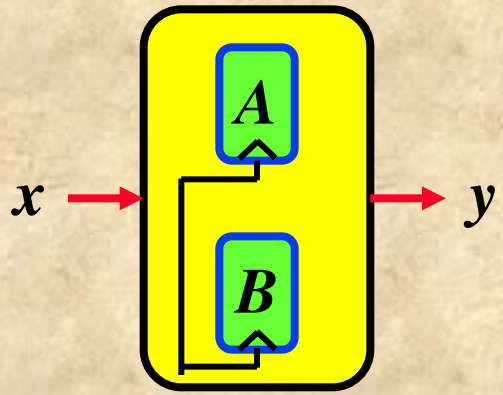
Timing Diagram

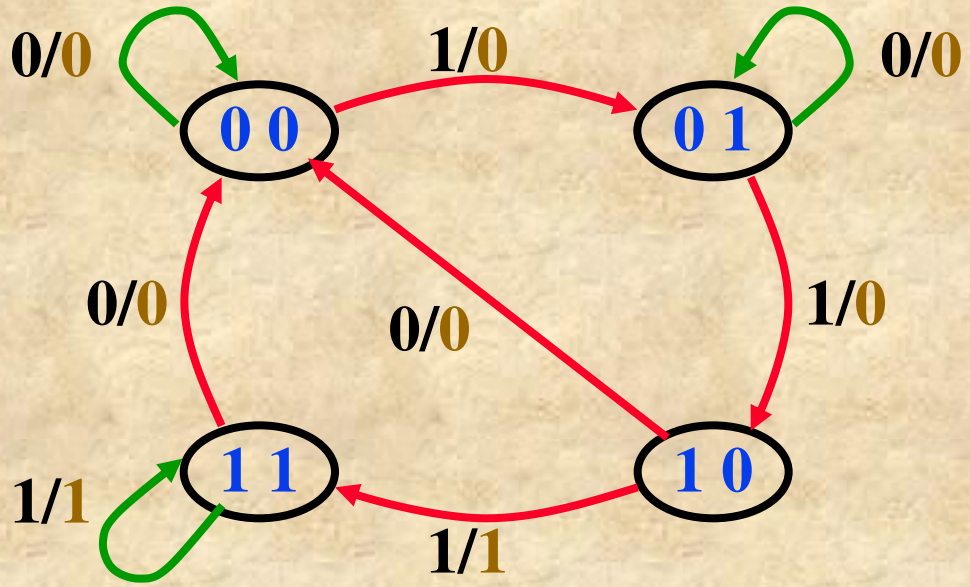


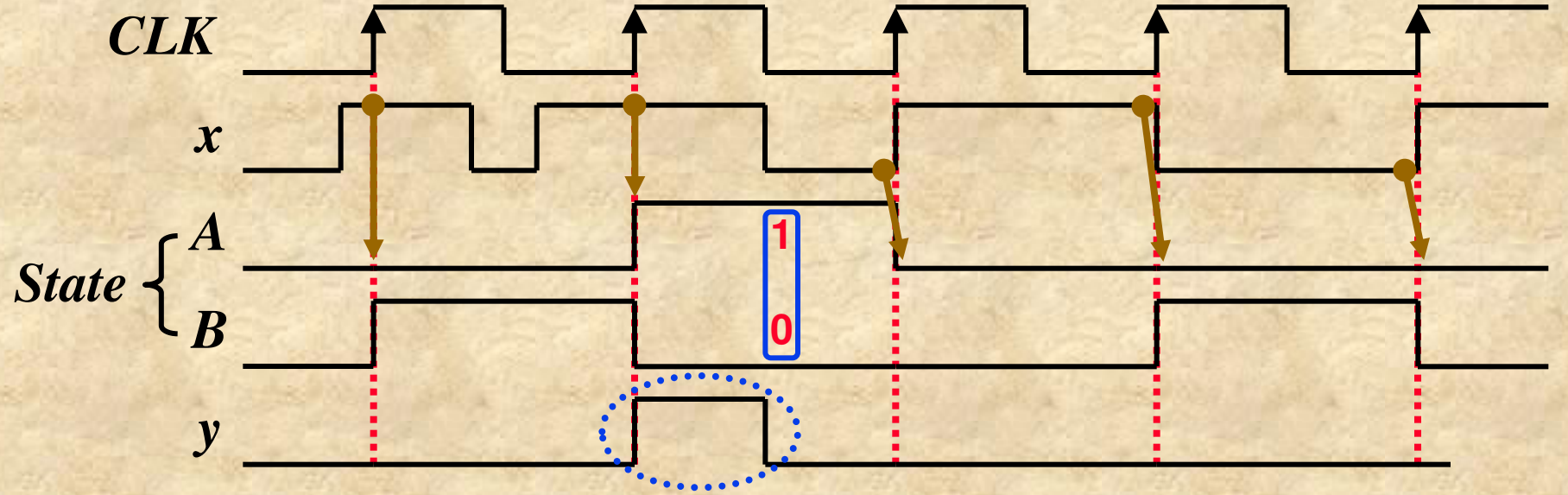




Timing Diagram



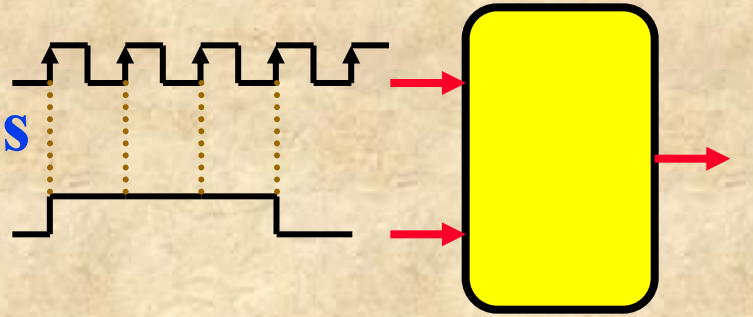


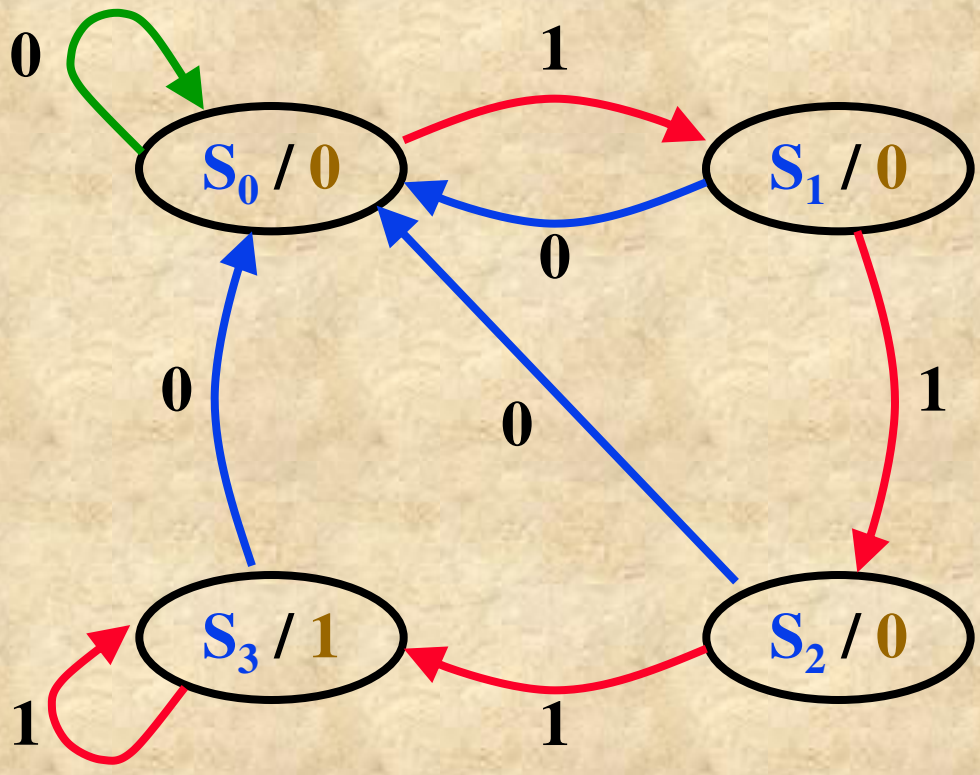


Design of Clocked Sequential Circuits

★ *Example*:

Detect 3 or more consecutive 1's



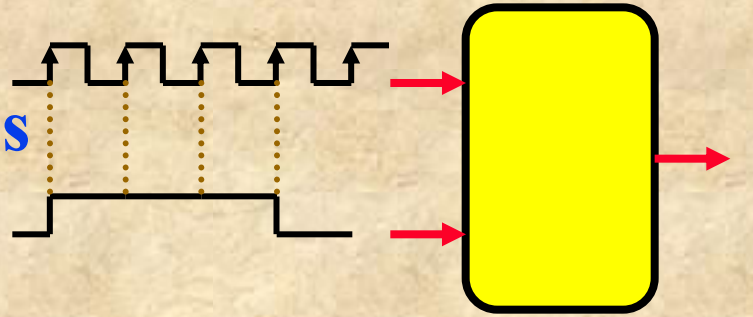


State	*A* *B*
S_0	0 0
S_1	0 1
S_2	1 0
S_3	1 1

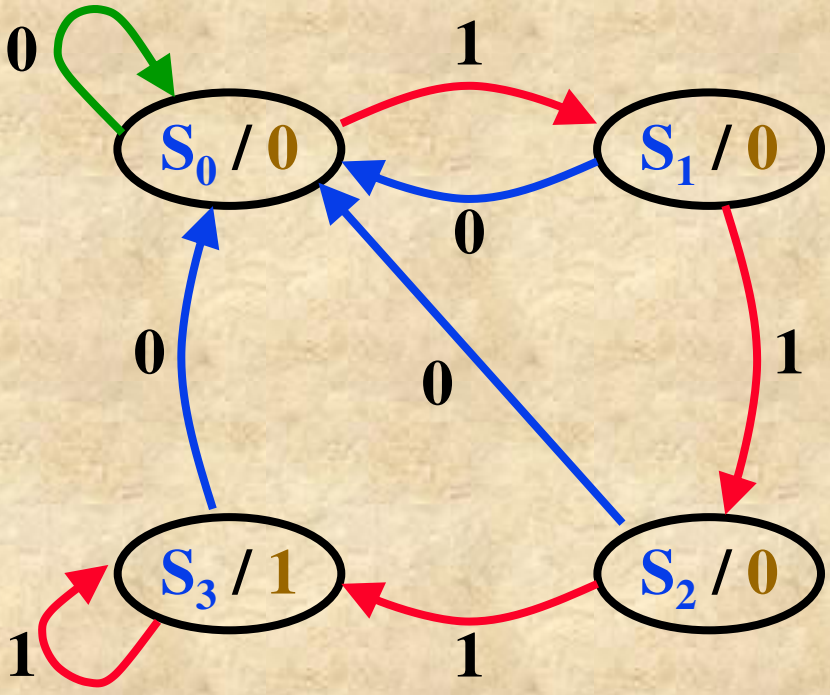
Design of Clocked Sequential Circuits

★ *Example*:

Detect 3 or more consecutive 1's



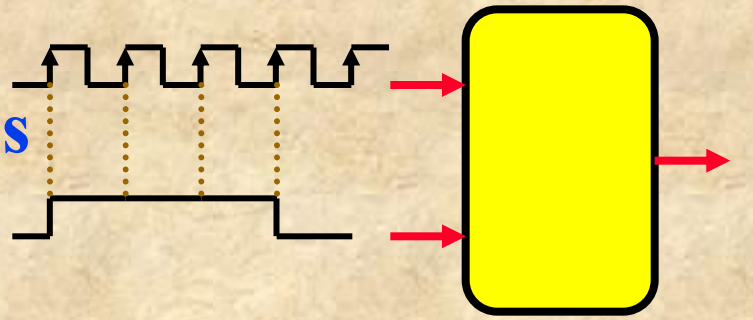
Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	1	1	1	1



Design of Clocked Sequential Circuits

★ *Example*:

Detect 3 or more consecutive 1's



Present State		Input	Next State		Output
A	*B*	*x*	*A*	*B*	*y*
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	1	1	1	1

Synthesis using *D* Flip-Flops

$$A(t+1) = D_A(A, B, x)$$

$$= \sum(3, 5, 7)$$

$$B(t+1) = D_B(A, B, x)$$

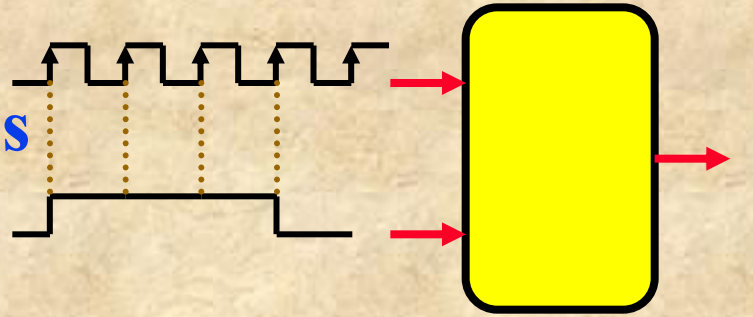
$$= \sum(1, 5, 7)$$

$$y(A, B, x) = \sum(6, 7)$$

Design of Clocked Sequential Circuits with *D* F.F.

★ *Example*:

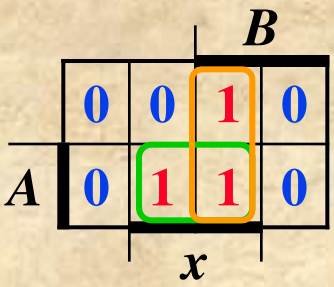
Detect 3 or more consecutive 1's



Synthesis using *D* Flip-Flops

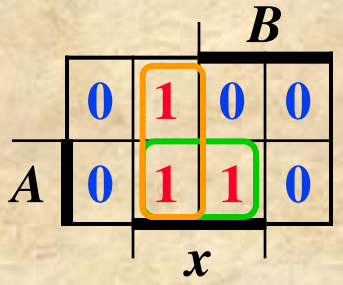
$$D_A(A, B, x) = \sum(3, 5, 7)$$

$$= A\,x + B\,x$$



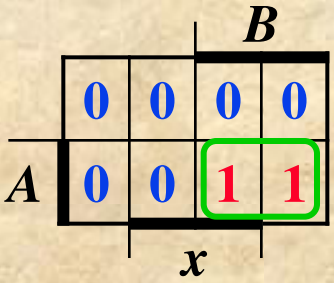
$$D_B(A, B, x) = \sum(1, 5, 7)$$

$$= A\,x + B'x$$



$$y(A, B, x) = \sum(6, 7)$$

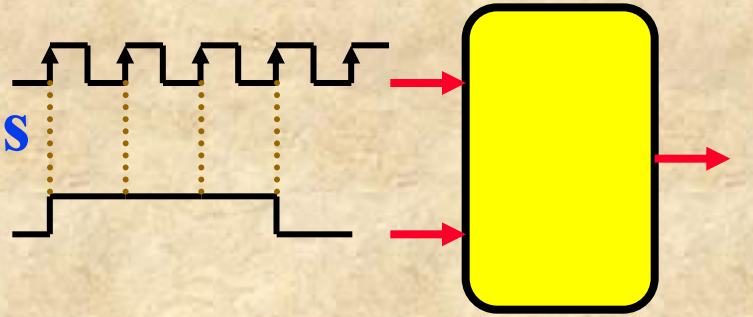
$$= A\,B$$



Design of Clocked Sequential Circuits with *D* F.F.

★ *Example*:

Detect 3 or more consecutive 1's

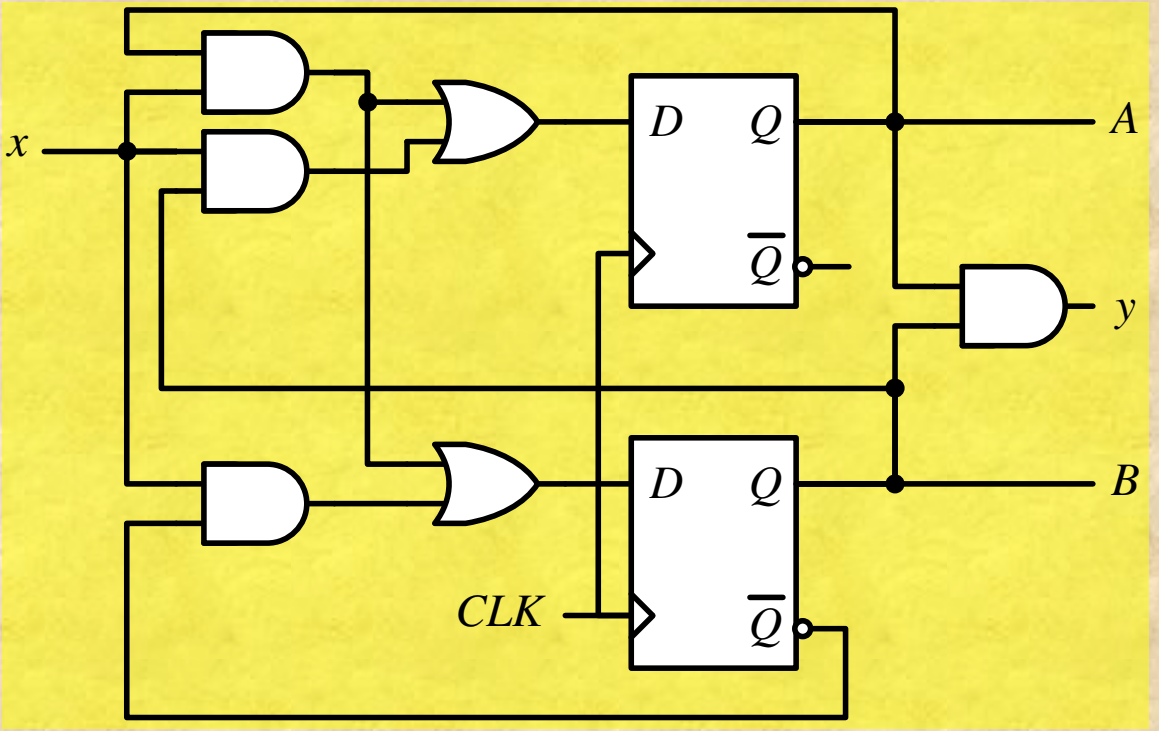


Synthesis using *D* Flip-Flops

$$D_A = A\,x + B\,x$$

$$D_B = A\,x + B'\,x$$

$$y = A\,B$$



Flip-Flop Excitation Tables

Present State	Next State	F.F. Input
$Q(t)$	$Q(t+1)$	D
0	0	0
0	1	1
1	0	0
1	1	1

Present State	Next State	F.F. Input	
$Q(t)$	$Q(t+1)$	J	K
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

0 0 (No change)
0 1 (Reset)

1 0 (Set)
1 1 (Toggle)

0 1 (Reset)
1 1 (Toggle)

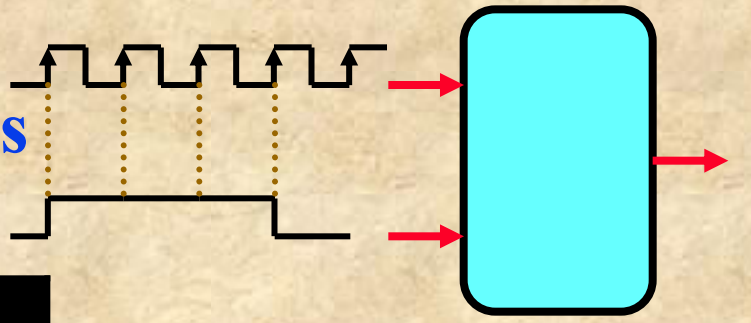
0 0 (No change)
1 0 (Set)

$Q(t)$	$Q(t+1)$	T
0	0	0
0	1	1
1	0	1
1	1	0

Design of Clocked Sequential Circuits with *JK* F.F.

★ *Example*:

Detect 3 or more consecutive 1's



Present State		Input	Next State		Flip-Flop Inputs			
A	*B*	*x*	*A*	*B*	J_A	K_A	J_B	K_B
0	0	0	0	0	0	x	0	x
0	0	1	0	1	0	x	1	x
0	1	0	0	0	0	x	x	1
0	1	1	1	0	1	x	x	1
1	0	0	0	0	x	1	0	x
1	0	1	1	1	x	0	1	x
1	1	0	0	0	x	1	x	1
1	1	1	1	1	x	0	x	0

Synthesis using *JK* F.F.

$J_A(A, B, x) = \sum(3)$

$d_{JA}(A, B, x) = \sum(4,5,6,7)$

$K_A(A, B, x) = \sum(4, 6)$

$d_{KA}(A, B, x) = \sum(0,1,2,3)$

$J_B(A, B, x) = \sum(1, 5)$

$d_{JB}(A, B, x) = \sum(2,3,6,7)$

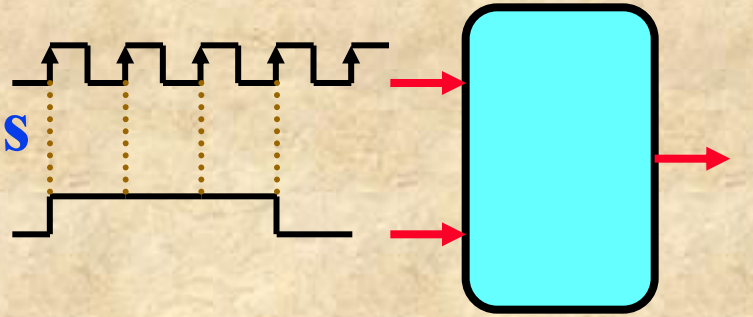
$K_B(A, B, x) = \sum(2, 3, 6)$

$d_{KB}(A, B, x) = \sum(0,1,4,5)$

Design of Clocked Sequential Circuits with *JK* F.F.

★ *Example*:

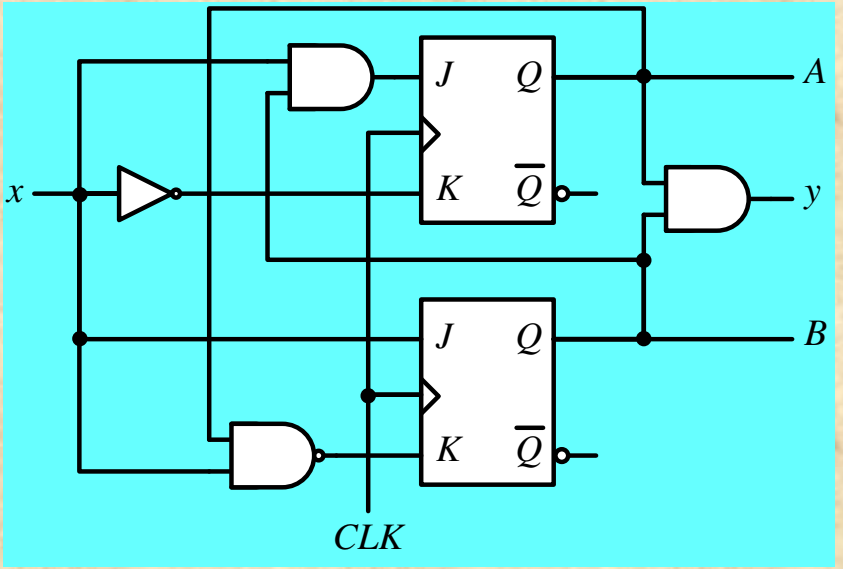
Detect 3 or more consecutive 1's

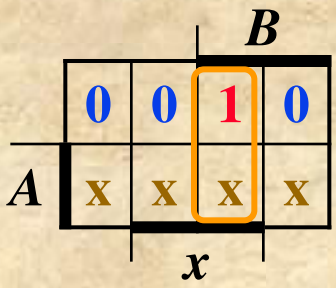


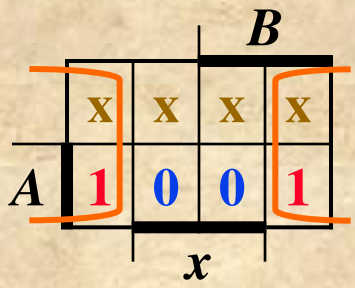
Synthesis using *JK* Flip-Flops

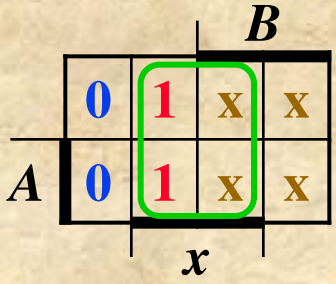
$J_A = B\,x$ $\quad K_A = x'$

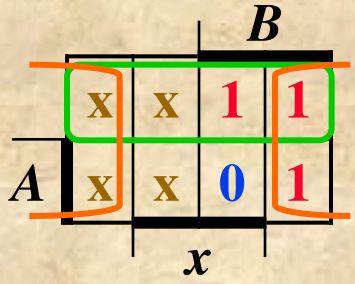
$J_B = x$ $\quad K_B = A' + x'$







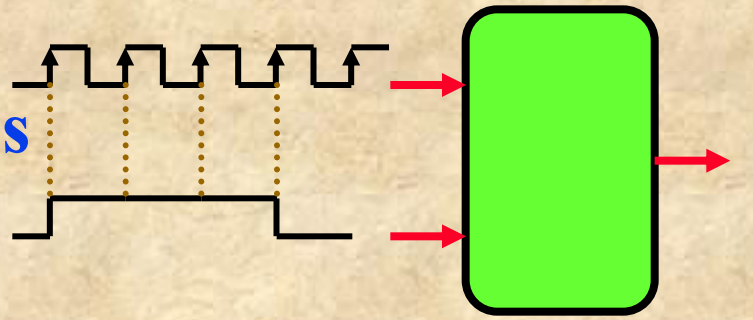




Design of Clocked Sequential Circuits with *T* F.F.

★ *Example*:

Detect 3 or more consecutive 1's



Present State		Input	Next State		F.F. Input	
A	*B*	*x*	*A*	*B*	T_A	T_B
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	0	0	0	1
0	1	1	1	0	1	1
1	0	0	0	0	1	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	1	1	1	0	0

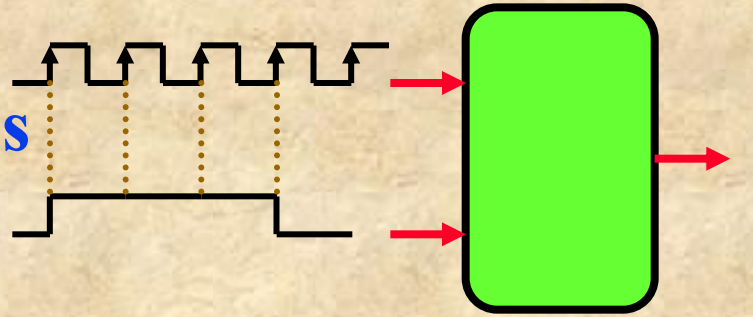
Synthesis using *T* Flip-Flops

$$T_A(A, B, x) = \sum(3, 4, 6)$$
$$T_B(A, B, x) = \sum(1, 2, 3, 5, 6)$$

Design of Clocked Sequential Circuits with *T* F.F.

★ *Example*:

Detect 3 or more consecutive 1's



Synthesis using *T* Flip-Flops

$T_A = A\,x' + A'\,B\,x$

$T_B = A'\,B + B \oplus x$