

Synchronous Counters, ripple counter & other counters

Lecture 2

Dronacharya Group of Institutions

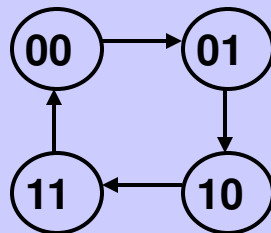
Introduction: Counters

- **Counters** are circuits that cycle through a specified number of states.
- Two types of counters:
 - ❖ synchronous (parallel) counters
 - ❖ asynchronous (ripple) counters
- Ripple counters allow some flip-flop outputs to be used as a source of clock for other flip-flops.
- Synchronous counters apply the same clock to all flip-flops.



Synchronous (Parallel) Counters

- **Synchronous (parallel) counters:** the flip-flops are clocked at the same time by a common clock pulse.
- We can design these counters using the sequential logic design process (covered in Lecture #12).
- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).



Present state		Next state		Flip-flop inputs	
A_1	A_0	A_1^+	A_0^+	TA_1	TA_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

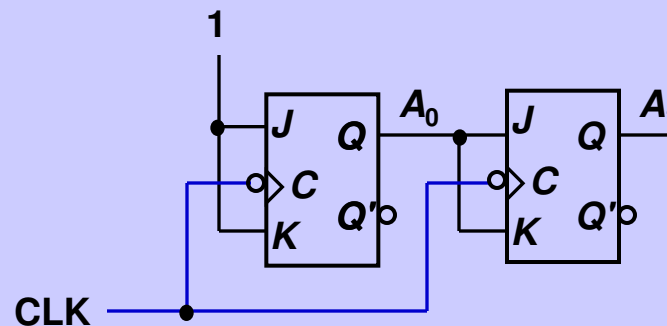
Synchronous (Parallel) Counters

- Example: 2-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J,K inputs).

Present state		Next state		Flip-flop inputs	
A_1	A_0	A_1^+	A_0^+	TA_1	TA_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1

$$TA_1 = A_0$$

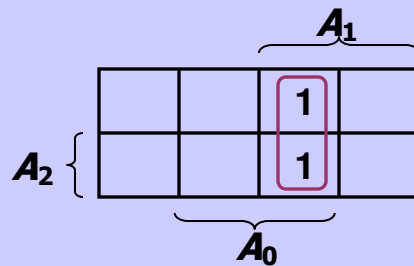
$$TA_0 = 1$$



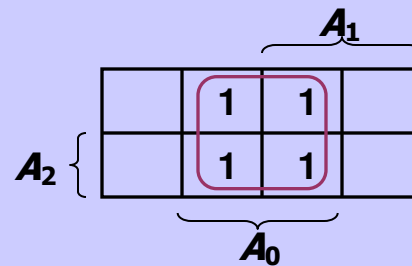
Synchronous (Parallel) Counters

- Example: 3-bit synchronous binary counter (using T flip-flops, or JK flip-flops with identical J, K inputs).

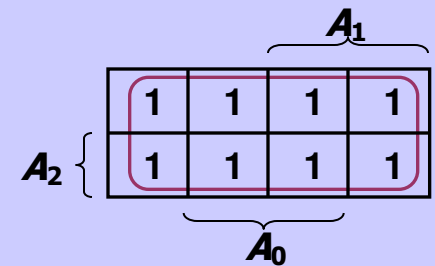
Present state			Next state			Flip-flop inputs		
A_2	A_1	A_0	A_2^+	A_1^+	A_0^+	TA_2	TA_1	TA_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1



$$TA_2 = A_1 \cdot A_0$$



$$TA_1 = A_0$$



$$TA_0 = 1$$

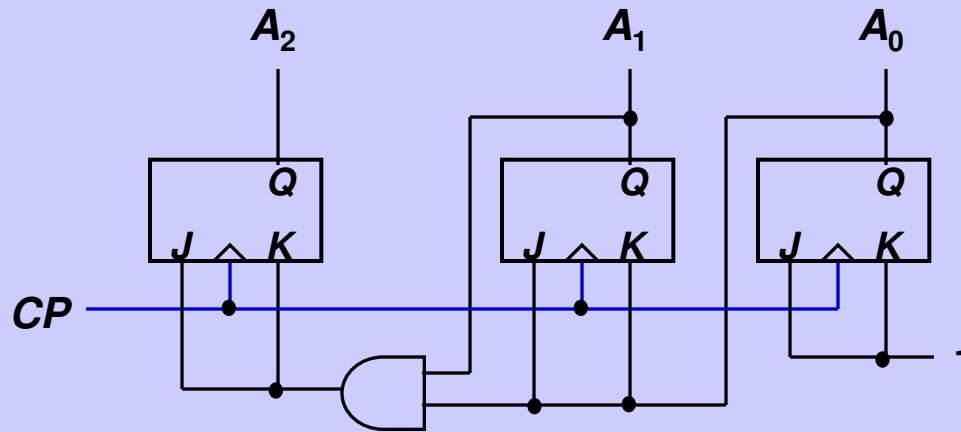
Synchronous (Parallel) Counters

- Example: 3-bit synchronous binary counter (cont'd).

$$TA_2 = A_1.A_0$$

$$TA_1 = A_0$$

$$TA_0 = 1$$



Synchronous (Parallel) Counters

- Note that in a binary counter, the n^{th} bit (shown underlined) is always complemented whenever

$$\underline{0}11\dots11 \rightarrow \underline{1}00\dots00$$

$$\text{or } \underline{1}11\dots11 \rightarrow \underline{0}00\dots00$$

- Hence, X_n is complemented whenever

$$X_{n-1}X_{n-2} \dots X_1X_0 = 11\dots11.$$

- As a result, if T flip-flops are used, then

$$TX_n = X_{n-1} \cdot X_{n-2} \cdot \dots \cdot X_1 \cdot X_0$$



Synchronous (Parallel) Counters

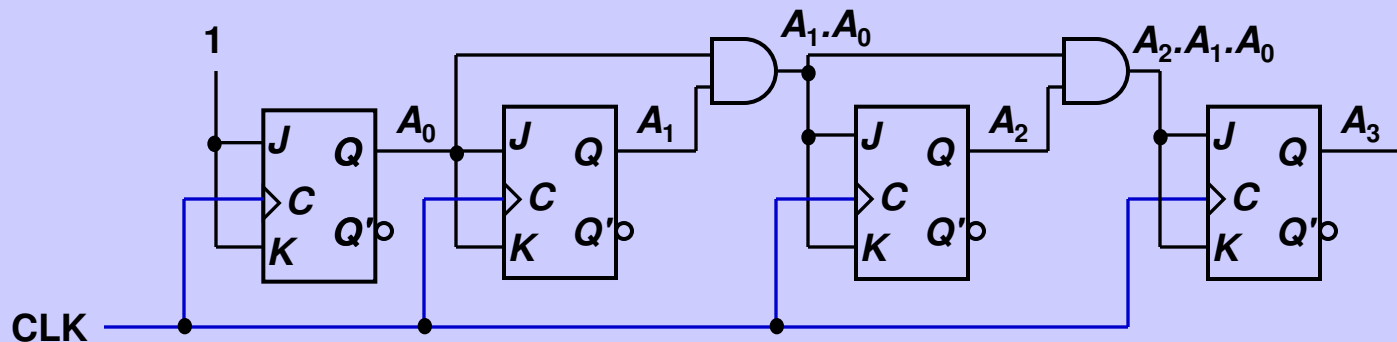
- Example: 4-bit synchronous binary counter.

$$TA_3 = A_2 \cdot A_1 \cdot A_0$$

$$TA_2 = A_1 \cdot A_0$$

$$TA_1 = A_0$$

$$TA_0 = 1$$



Synchronous (Parallel) Counters

- Example: Synchronous decade/BCD counter.

Clock pulse	Q_3	Q_2	Q_1	Q_0
Initially	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10 (recycle)	0	0	0	0

$$T_0 = 1$$

$$T_1 = Q_3' \cdot Q_0$$

$$T_2 = Q_1 \cdot Q_0$$

$$T_3 = Q_2 \cdot Q_1 \cdot Q_0 + Q_3 \cdot Q_0$$

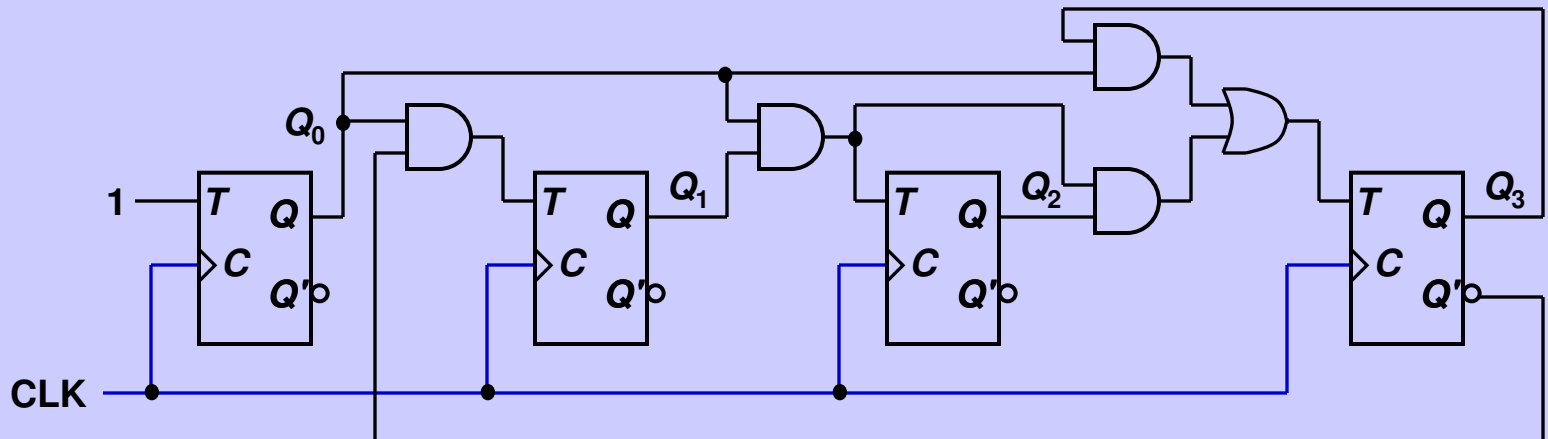
- Example: Synchronous decade/BCD counter (cont'd).

$$T_0 = 1$$

$$T_1 = Q_3' \cdot Q_0$$

$$T_2 = Q_1 \cdot Q_0$$

$$T_3 = Q_2 \cdot Q_1 \cdot Q_0 + Q_3 \cdot Q_0$$



Up/Down Synchronous Counters

- Up/down synchronous counter: a *bidirectional* counter that is capable of counting either up or down.
- An input (control) line Up/\overline{Down} (or simply Up) specifies the direction of counting.
 - ❖ $Up/\overline{Down} = 1 \rightarrow$ Count upward
 - ❖ $Up/\overline{Down} = 0 \rightarrow$ Count downward



Up/Down Synchronous Counters

- Example: A 3-bit up/down synchronous binary counter.

Clock pulse	<i>Up</i>	Q_2	Q_1	Q_0	<i>Down</i>
0		0	0	0	
1		0	0	1	
2		0	1	0	
3		0	1	1	
4		1	0	0	
5		1	0	1	
6		1	1	0	
7		1	1	1	

$$TQ_0 = 1$$

$$TQ_1 = (Q_0 \cdot Up) + (Q_0' \cdot Up')$$

$$TQ_2 = (Q_0 \cdot Q_1 \cdot Up) + (Q_0' \cdot Q_1' \cdot Up')$$

Up counter

$$TQ_0 = 1$$

$$TQ_1 = Q_0$$

$$TQ_2 = Q_0 \cdot Q_1$$

Down counter

$$TQ_0 = 1$$

$$TQ_1 = Q_0'$$

$$TQ_2 = Q_0' \cdot Q_1'$$

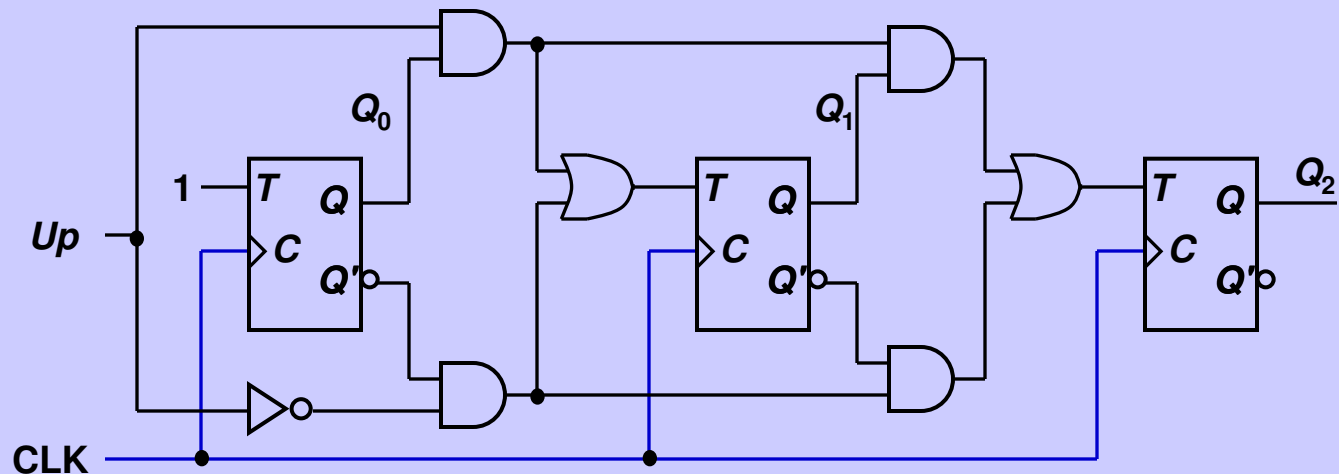
Up/Down Synchronous Counters

- Example: A 3-bit up/down synchronous binary counter (cont'd).

$$TQ_0 = 1$$

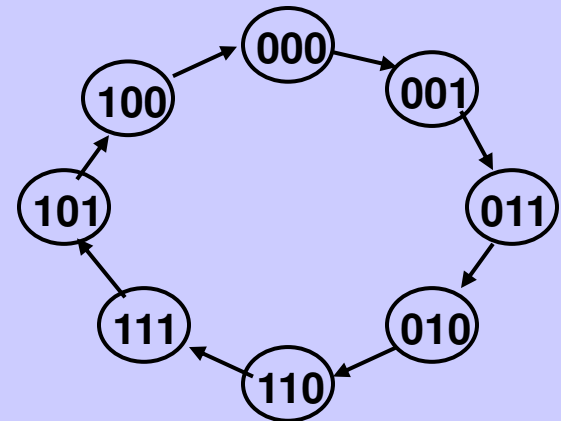
$$TQ_1 = (Q_0 \cdot Up) + (Q_0' \cdot Up')$$

$$TQ_2 = (Q_0 \cdot Q_1 \cdot Up) + (Q_0' \cdot Q_1' \cdot Up')$$



Designing Synchronous Counters

- Covered in Lecture #12.
- Example: A 3-bit Gray code counter (using JK flip-flops).



Present state			Next state			Flip-flop inputs					
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	JQ_2	KQ_2	JQ_1	KQ_1	JQ_0	KQ_0
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	1	0	X	1	X	X	0
0	1	0	1	1	0	1	X	X	0	0	X
0	1	1	0	1	0	0	X	X	0	X	1
1	0	0	0	0	0	X	1	0	X	0	X
1	0	1	1	0	0	X	0	0	X	X	1
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	1	0	1	X	0	X	1	X	0

Designing Synchronous Counters

- 3-bit Gray code counter: flip-flop inputs.

		$Q_1 Q_0$			
		00	01	11	10
Q_2	0				1
	1	X	X	X	X

$JQ_2 = Q_1 \cdot Q_0'$

		$Q_1 Q_0$			
		00	01	11	10
Q_2	0		1	X	X
	1			X	X

$JQ_1 = Q_2' \cdot Q_0$

		$Q_1 Q_0$			
		00	01	11	10
Q_2	0	1	X	X	
	1		X	X	1

$JQ_0 = Q_2 \cdot Q_1 + Q_2' \cdot Q_1'$
 $= (Q_2 \oplus Q_1)'$

		$Q_1 Q_0$			
		00	01	11	10
Q_2	0	X	X	X	X
	1	1			

$KQ_2 = Q_1' \cdot Q_0'$

		$Q_1 Q_0$			
		00	01	11	10
Q_2	0	X	X		
	1	X	X	1	

$KQ_1 = Q_2 \cdot Q_0$

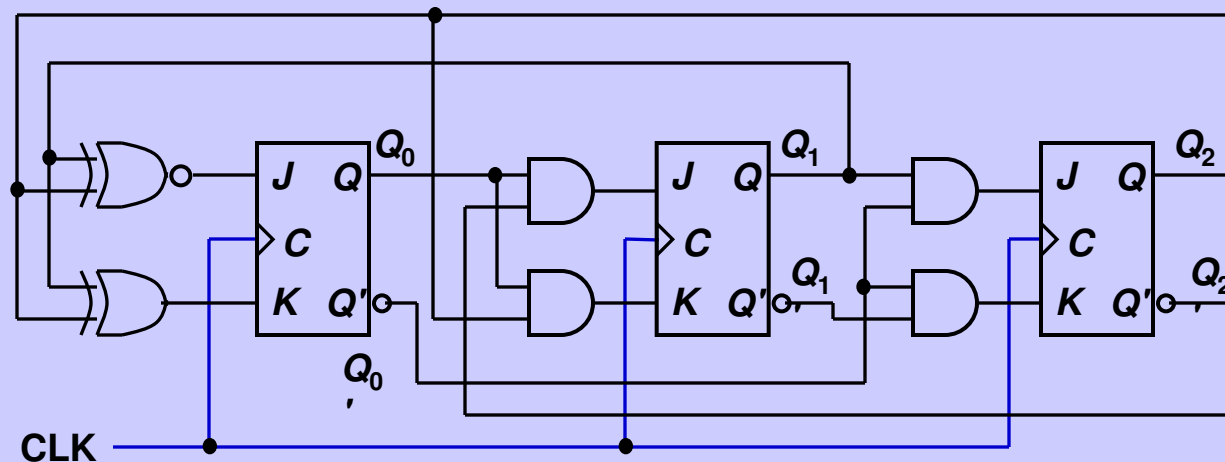
		$Q_1 Q_0$			
		00	01	11	10
Q_2	0	X		1	X
	1	X	1		X

$KQ_0 = Q_2 \cdot Q_1' + Q_2' \cdot Q_1$
 $= Q_2 \oplus Q_1$

Designing Synchronous Counters

- 3-bit Gray code counter: logic diagram.

$$\begin{array}{lll} JQ_2 = Q_1 \cdot Q_0' & JQ_1 = Q_2' \cdot Q_0 & JQ_0 = (Q_2 \oplus Q_1)' \\ KQ_2 = Q_1' \cdot Q_0' & KQ_1 = Q_2 \cdot Q_0 & KQ_0 = Q_2 \oplus Q_1 \end{array}$$



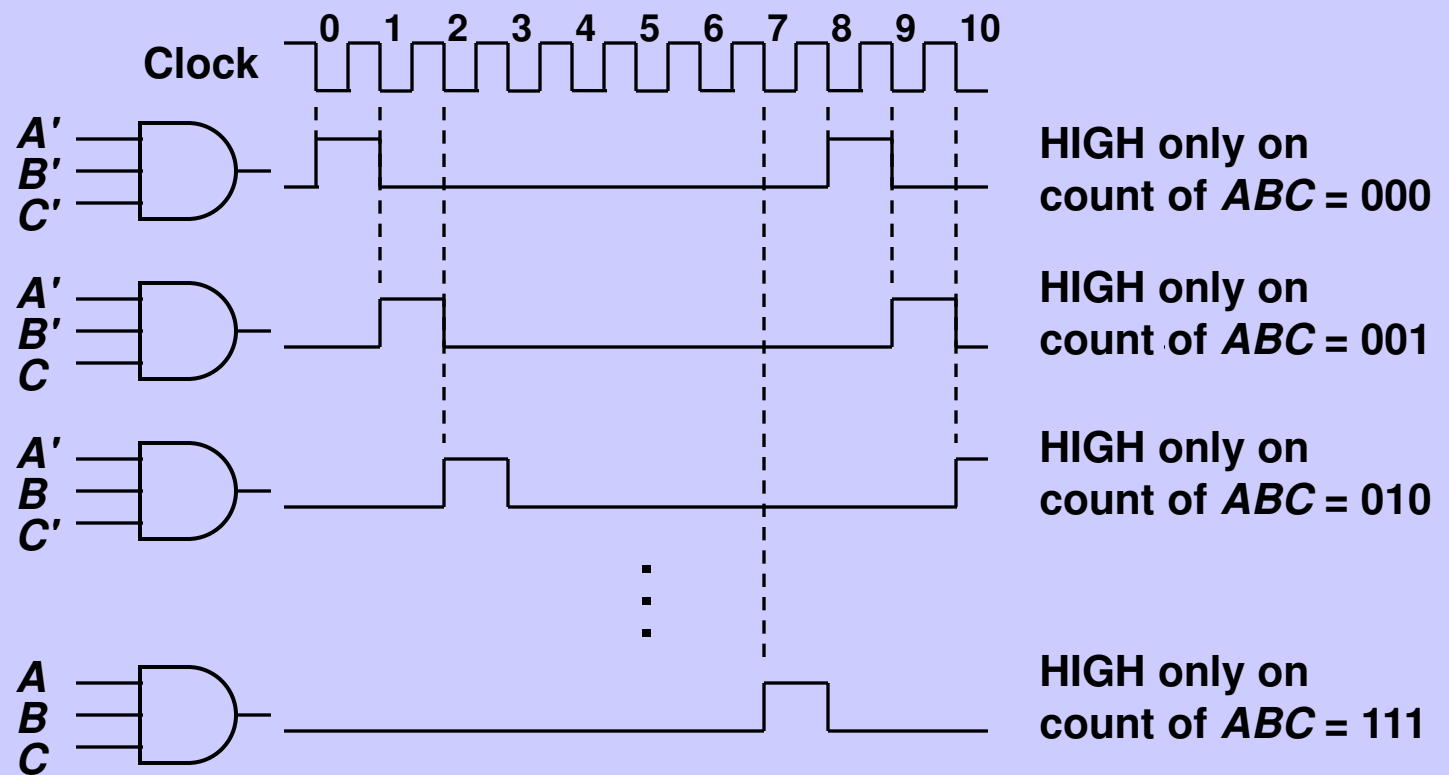
Decoding A Counter

- **Decoding a counter** involves determining which state in the sequence the counter is in.
- Differentiate between *active-HIGH* and *active-LOW* decoding.
- Active-HIGH decoding: output HIGH if the counter is in the state concerned.
- Active-LOW decoding: output LOW if the counter is in the state concerned.



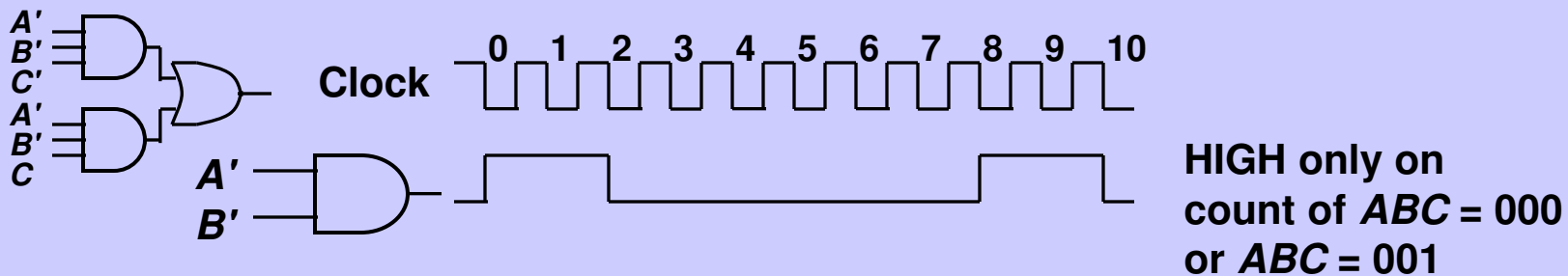
Decoding A Counter

- Example: MOD-8 ripple counter (active-HIGH decoding).

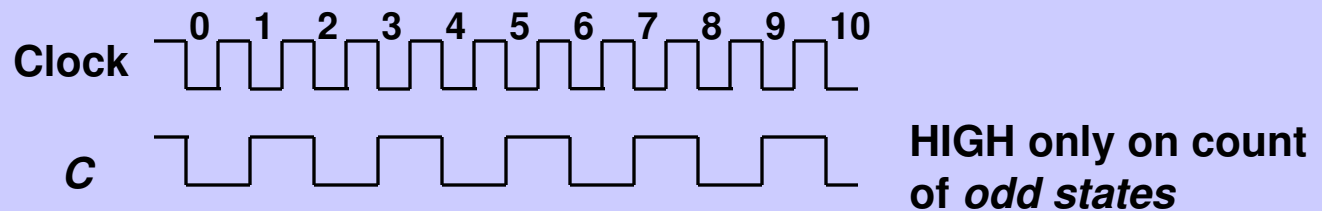


Decoding A Counter

- Example: To detect that a MOD-8 counter is in state 0 (000) or state 1 (001).



- Example: To detect that a MOD-8 counter is in the odd states (states 1, 3, 5 or 7), simply use C .



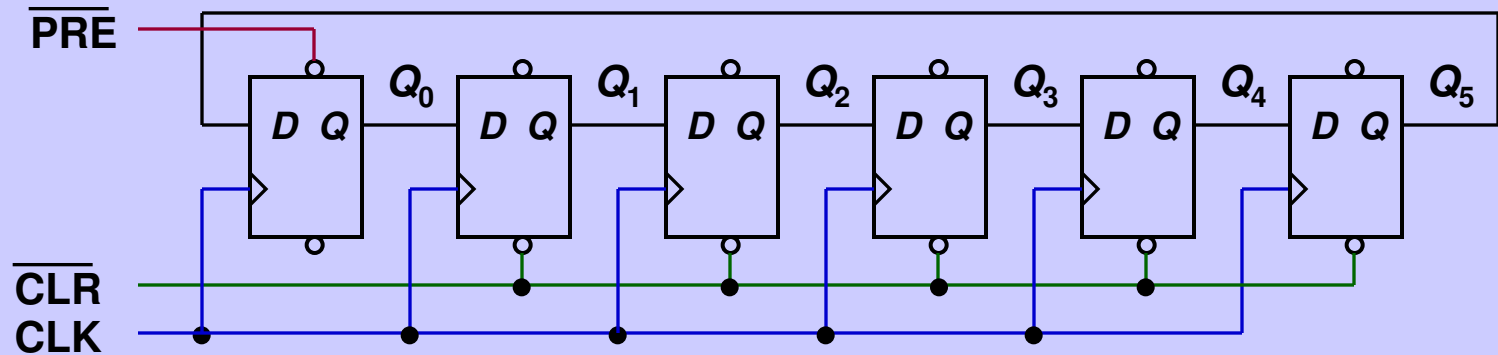
Ring Counters

- One flip-flop (stage) for each state in the sequence.
- The output of the last stage is connected to the D input of the first stage.
- An n -bit ring counter cycles through n states.
- No decoding gates are required, as there is an output that corresponds to every state the counter is in.

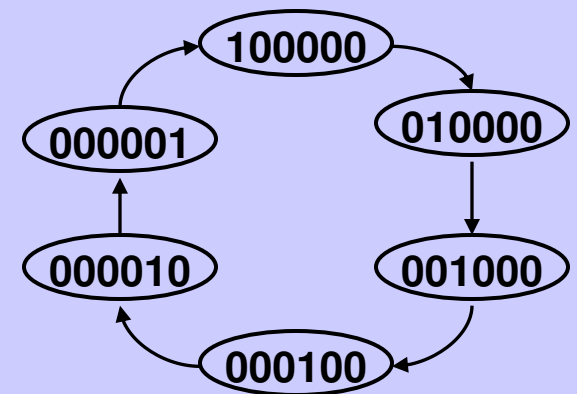


Ring Counters

- Example: A 6-bit (MOD-6) ring counter.



Clock	Q_0	Q_1	Q_2	Q_3	Q_4	Q_5
0	1	0	0	0	0	0
1	0	1	0	0	0	0
2	0	0	1	0	0	0
3	0	0	0	1	0	0
4	0	0	0	0	1	0
5	0	0	0	0	0	1



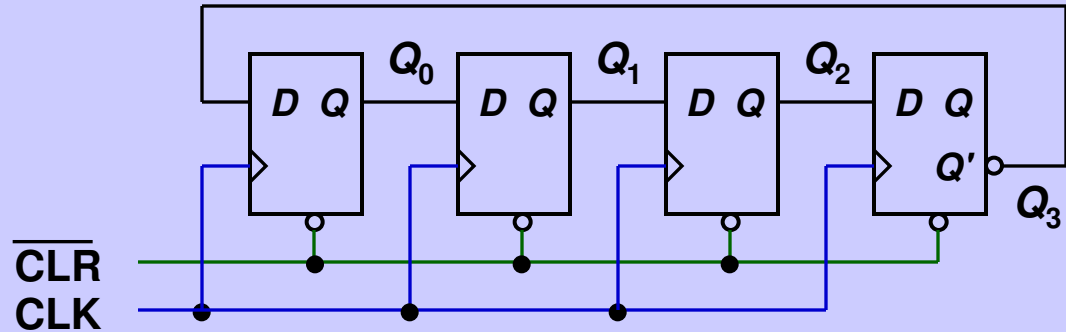
Johnson Counters

- The complement of the output of the last stage is connected back to the D input of the first stage.
- Also called the *twisted-ring counter*.
- Require fewer flip-flops than ring counters but more flip-flops than binary counters.
- An n -bit Johnson counter cycles through $2n$ states.
- Require more decoding circuitry than ring counter but less than binary counters.

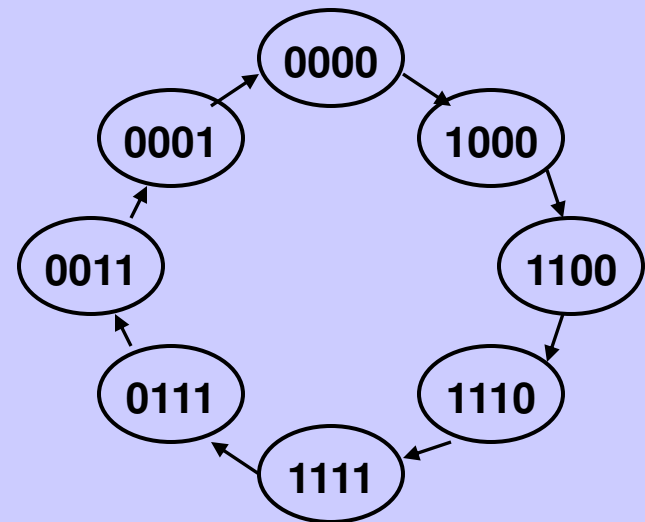


Johnson Counters

- Example: A 4-bit (MOD-8) Johnson counter.



Clock	Q_0	Q_1	Q_2	Q_3
0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1
5	0	1	1	1
6	0	0	1	1
7	0	0	0	1



Johnson Counters

- Decoding logic for a 4-bit Johnson counter.

Clock	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	Decoding
0	0	0	0	0	$A'.D'$
1	1	0	0	0	$A.B'$
2	1	1	0	0	$B.C'$
3	1	1	1	0	$C.D'$
4	1	1	1	1	$A.D$
5	0	1	1	1	$A'.B$
6	0	0	1	1	$B'.C$
7	0	0	0	1	$C'.D$

