

UNIT-2

(Lecture-4)

Convert a Transfer Function to State Space

$$\frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1 \frac{dy}{dt} + a_0 y = b_0 u$$

choose

$$x_1 = y \quad x_2 = \frac{dy}{dt} \quad x_3 = \frac{d^2 y}{dt^2} \quad \cdots \quad x_n = \frac{d^{n-1} y}{dt^{n-1}}$$



diferensiasikan

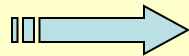
$$\dot{x}_1 = \frac{dy}{dt}$$

$$\dot{x}_2 = \frac{d^2 y}{dt^2}$$

$$\dot{x}_3 = \frac{d^3 y}{dt^3}$$

$\vdots$

$$\dot{x}_n = \frac{d^n y}{dt^n}$$



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$\vdots$

$$\dot{x}_{n-1} = x_n$$

$$\dot{x}_n = -a_0 x_1 - a_1 x_2 \cdots - a_{n-1} x_n + b_0 u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ \vdots & & & & & & \cdots & \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 & -a_4 & -a_5 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

Example : TF to State Space

1. Inverse Laplace

$$\ddot{c} + 9\ddot{c} + 26\dot{c} + 24c = 24r$$

2 Select state variables

$$\cdot \quad x_1 = c \quad x_2 = \dot{c} \quad x_3 = \ddot{c}$$

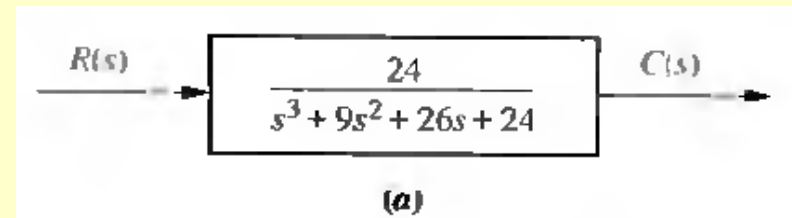
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$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + 24r$$

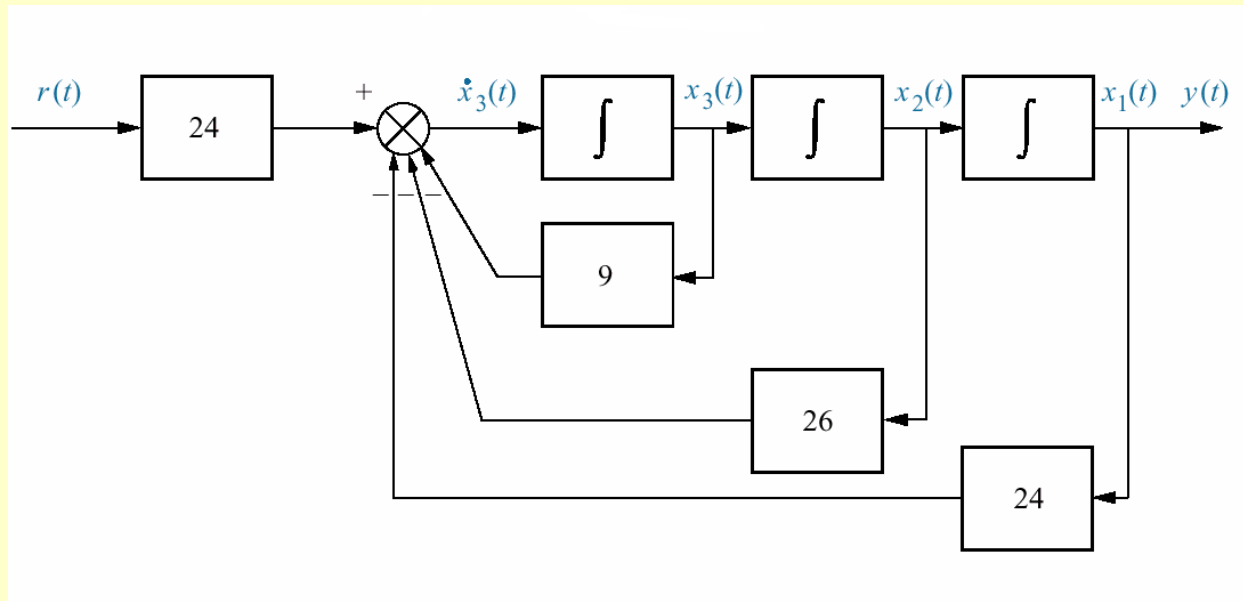
$$y = c = x_1$$



$$G(s) = \frac{N(s)}{D(s)}$$

$N(s)$ **numerator**

$D(s)$ **denominator**



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$