

UNIT-2

(Lecture-6)

State Space to Transfer Function

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

→ **Laplace Transform**

$$sX(s) = AX(s) + BU(s)$$

$$Y(s) = CX(s) + DU(s)$$

$$\longrightarrow X(s) = (sI - A)^{-1}BU(s)$$

$$\longrightarrow Y(s) = [C(sI - A)^{-1}B + D]U(s)$$

$$T(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0]x$$

$$\longrightarrow (sI - A) = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{\text{adj}(sI - A)}{\det(sI - A)}$$

$$= \frac{\begin{bmatrix} (s^2 + 3s + 2) & s + 3 & 1 \\ -1 & s(s + 3) & s \\ -s & -(2s + 1) & s^2 \end{bmatrix}}{s^3 + 3s^2 + 2s + 1}$$

$$T(s) = C(sI - A)^{-1}B + D$$

$$\longrightarrow T(s) = \frac{10(s^2 + 3s + 2)}{s^3 + 3s^2 + 2s + 1}$$

Appendix

$$x(s) = \frac{1}{(s^2 + 3s + 2)s} = \frac{1}{(s+1)(s+2)s} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$A = \lim_{s \rightarrow 0} \left[s \left(\frac{1}{(s+1)(s+2)s} \right) \right] = \frac{1}{2}$$

$$B = \lim_{s \rightarrow -1} \left[(s+1) \left(\frac{1}{(s+1)(s+2)s} \right) \right] = -1$$

$$C = \lim_{s \rightarrow -2} \left[(s+2) \left(\frac{1}{(s+1)(s+2)s} \right) \right] = \frac{1}{2}$$