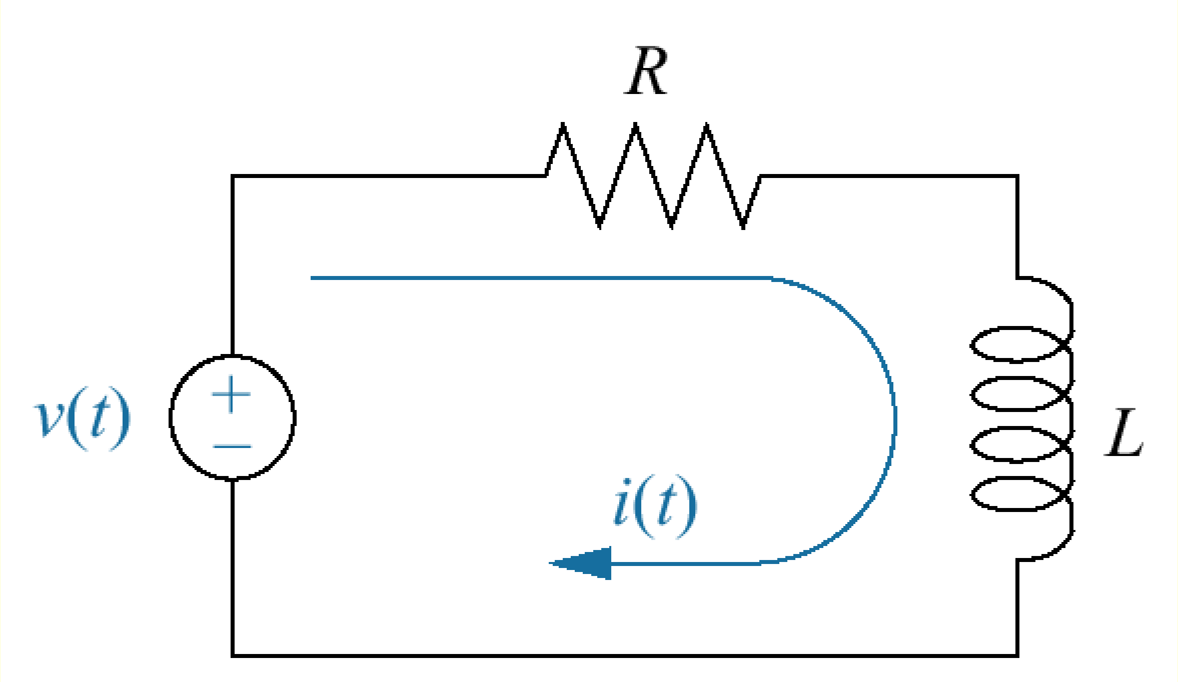
UNIT-2

(Lecture-7)

Application to Network Circuits

RL network



1. Select $i(t)$ As state variables

2. Write loop equation $$L\frac{di}{dt}+Ri=v(t)$$ **(1)**

3. Solve the equation using Laplace transform

$$L[sI(s)-i(0)]+RI(s)=V(s)$$

(1) Not unique

Assumption $v(t)$: Unit step

$$I(s)=\frac{1}{R}\left(\frac{1}{s}-\frac{1}{s+R/L}\right)+\frac{i(0)}{s+R/L}$$

↓

$$i(t)=\frac{1}{R}\left(1-e^{-(R/L)t}\right)+i(0)e^{-(R/L)t}$$

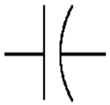
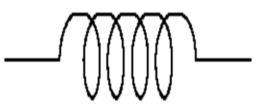
4. We can solve all other network variables

Output equations

$$v_R(t)=Ri(t) \quad (2)$$

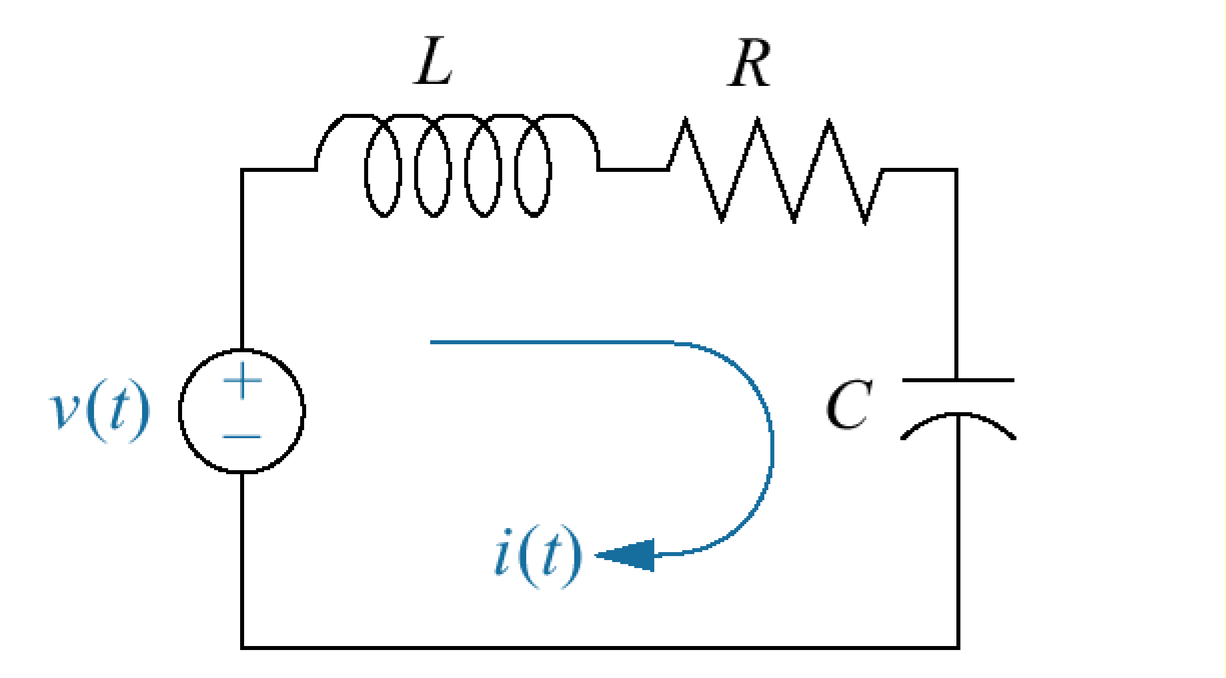
$$v_L(t)=v(t)-Ri(t) \quad (3)$$

(1), (2),(3) : state-space representation

 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

RLC network



1. State variables $i(t)$ $q(t)$

2. $$L\frac{di}{dt}+Ri+\frac{1}{C}\int idt=v(t)$$

Using $i(t)=dq/dt$

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=v(t)$$

$$\frac{dq}{dt}=i$$

$$\frac{di}{dt}=-\frac{1}{LC}q-\frac{R}{L}i+\frac{1}{L}v(t) \qquad \textbf{(1)}$$

3. $q(t)\ i(t)$ **Can be solved using Laplace Transform**

4. Other network variables can be obtained

$$v_L(t)=-\frac{1}{C}q(t)-Ri(t)+v(t) \qquad \textbf{(2)}$$

5. (1),(2) : state-space representation

Other variables $v_R(t)$ $v_C(t)$

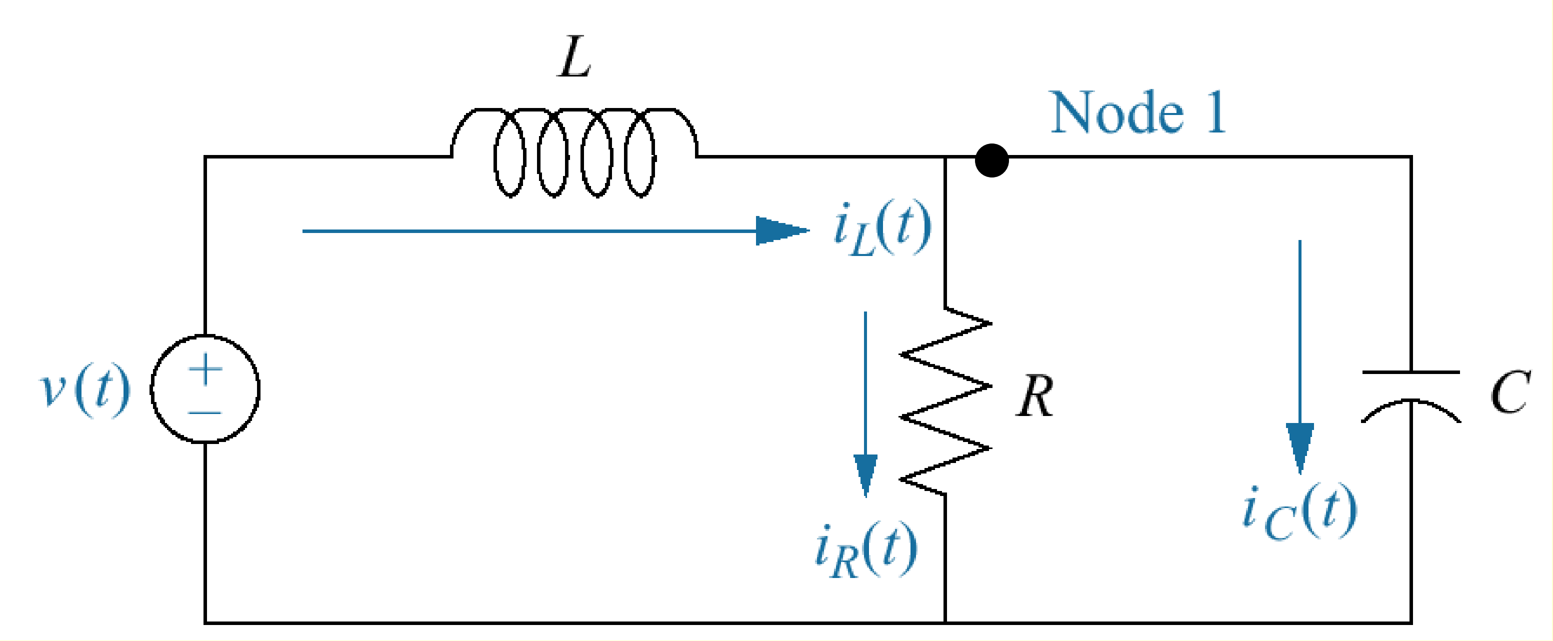
$$\frac{dv_R}{dt} = -\frac{R}{L}v_R - \frac{R}{L}v_C + \frac{R}{L}v(t)$$

$$\frac{dv_C}{dt} = \frac{1}{RC}v_R$$

Each variables : linearly independent

Example 1

Electrical network



1. Select state variables $v_C \quad i_L$

$$C\frac{dv_C}{dt} = i_C \qquad L\frac{di_L}{dt} = v_L \qquad (1)$$

Express (1) using $v_C \quad i_L \quad v(t)$

2. $i_C = -i_R + i_L \qquad v_L = -v_C + v(t) \qquad$ **(2)**

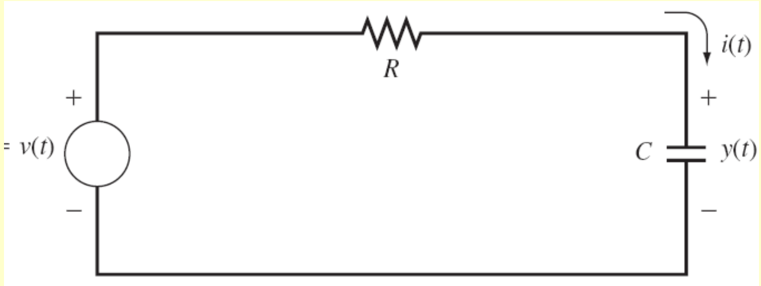
$$= -\frac{1}{R}v_C + i_L$$

3. $\frac{dv_C}{dt} = -\frac{1}{RC}v_c + \frac{1}{C}i_L$

$$\frac{di_L}{dt} = -\frac{1}{L}v_C + \frac{1}{L}v(t) \qquad (3)$$

4. Output equation $i_R = \frac{1}{R}v_C$

Example 2 & 3



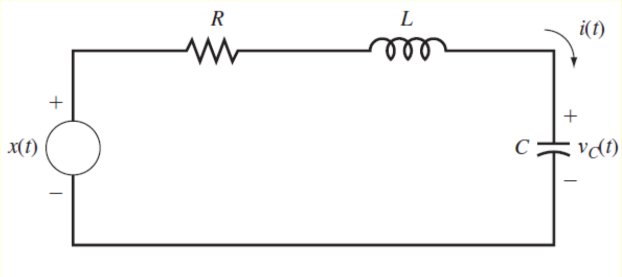
$$\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} v(t)$$

$$\mathbf{A} = [-a_0] = \left[-\frac{1}{RC}\right]$$

$$\mathbf{B} = [b_0] = \left[\frac{1}{RC}\right] \quad \mathbf{C} = [1] \quad \mathbf{D} = 0$$

$$[\dot{x}_1(t)] = \left[-\frac{1}{RC}\right][x_1(t)] + \left[\frac{1}{RC}\right] v(t)$$

$$y(t) = [1][x_1(t)]$$



$$\frac{d^2 y(t)}{dt^2} + \left(\frac{R}{L}\right)\frac{dy(t)}{dt} + \frac{1}{LC} y(t) = \frac{1}{LC} v(t)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1/LC & -R/L \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ b_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/LC \end{bmatrix}$$

$$\mathbf{C} = [1 \quad 0] \quad \mathbf{D} = 0$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1/LC & -R/L \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/LC \end{bmatrix} v(t)$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$