UNIT-3

(Lecture-1)

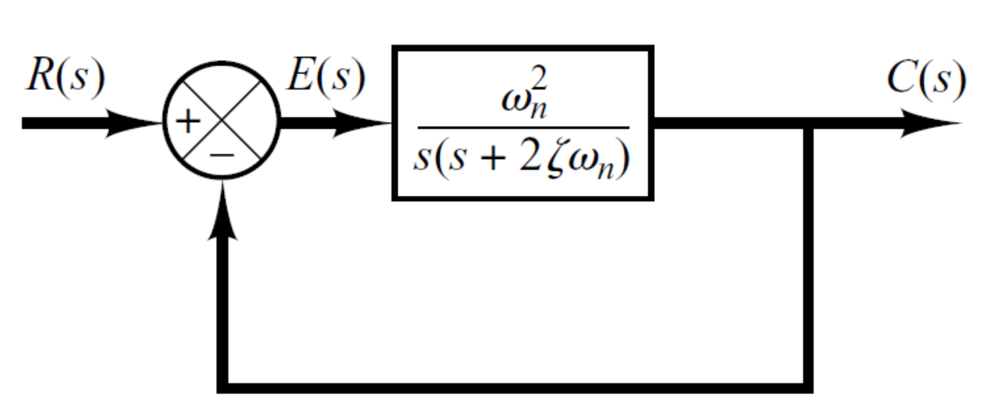
Time Domain Analysis of Control Systems

Introduction

- We have already discussed the affect of location of poles and zeros on the transient response of 1st order systems.
- Compared to the simplicity of a first-order system, a second-order system exhibits a wide range of responses that must be analysed and described.
- Varying a first-order system's parameter (T, K) simply changes the speed and offset of the response
- Whereas, changes in the parameters of a second-order system can change the *form of* the response.
- A second-order system can display characteristics much like a first-order system or, depending on component values, display damped or *pure oscillations* for its *transient response*.

Introduction

- A general second-order system is characterized by the following transfer function.



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Introduction

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n \longrightarrow$ un-damped natural frequency of the second order system, which is the frequency of oscillation of the system without damping.

$\zeta \longrightarrow$ damping ratio of the second order system, which is a measure of the degree of resistance to change in the system output.

Example#1

- Determine the un-damped natural frequency and damping ratio of the following second order system.

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 2s + 4}$$

- Compare the numerator and denominator of the given transfer function with the general 2nd order transfer function.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = 4 \Rightarrow \omega_n = 2\ rad/\mathrm{sec}$$

$$\cancel{s^2} + 2\zeta\omega_n s + \cancel{\omega_n^2} = \cancel{s^2} + 2s + \cancel{4}$$

$$\Rightarrow 2\zeta\omega_n s = 2s$$

$$\Rightarrow \zeta\omega_n = 1$$

$$\Rightarrow \zeta = 0.5$$

Introduction

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- **Two poles of the system are**

$$-\omega_n\zeta + \omega_n\sqrt{\zeta^2 - 1}$$

$$-\omega_n\zeta - \omega_n\sqrt{\zeta^2 - 1}$$

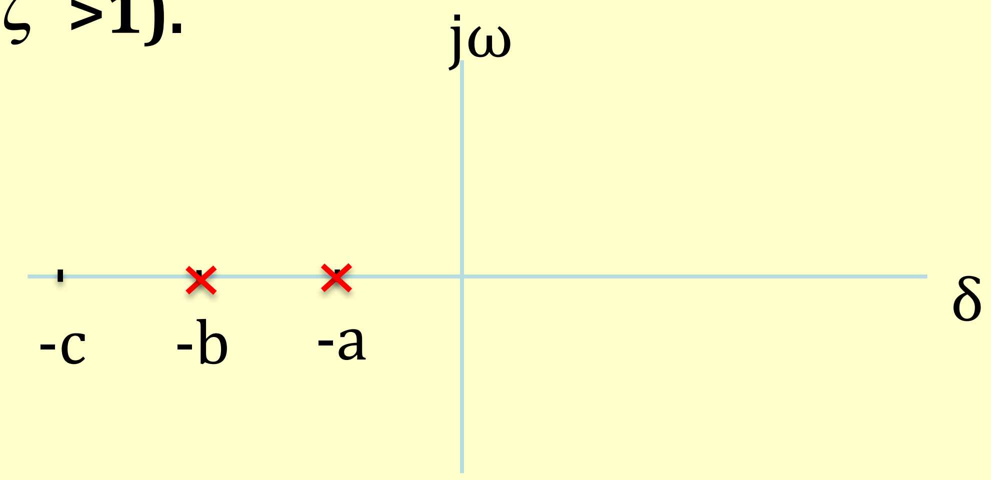
Introduction

$$-\omega_n\zeta + \omega_n\sqrt{\zeta^2 - 1}$$

$$-\omega_n\zeta - \omega_n\sqrt{\zeta^2 - 1}$$

- **According the value of ζ , a second-order system can be set into one of the four categories:**

1. *Overdamped* - when the system has two real distinct poles (ζ >1).



Introduction

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$$-\omega_n\zeta - \omega_n\sqrt{\zeta^2 - 1}$$

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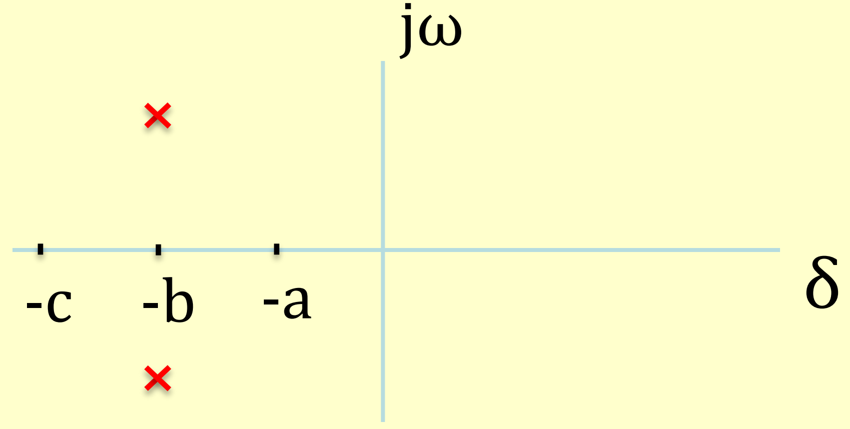
2. *Underdamped* - when the system has two complex conjugate poles (0 < ζ <1)



Introduction

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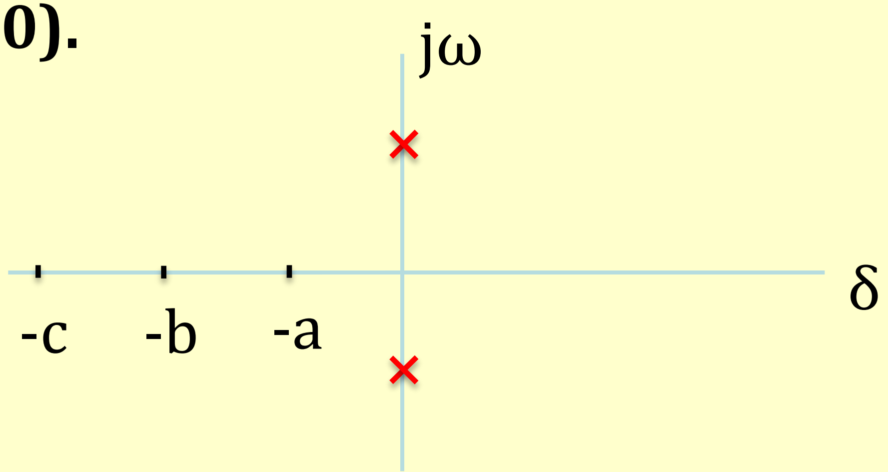
- **According the value of ζ , a second-order system can be set into one of the four categories:**

3. *Undamped* - when the system has two imaginary poles (ζ = 0).



Introduction

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$$-\omega_n\zeta - \omega_n\sqrt{\zeta^2 - 1}$$

- **According the value of ζ , a second-order system can be set into one of the four categories:**

4. *Critically damped* - when the system has two real but equal poles (ζ = 1).