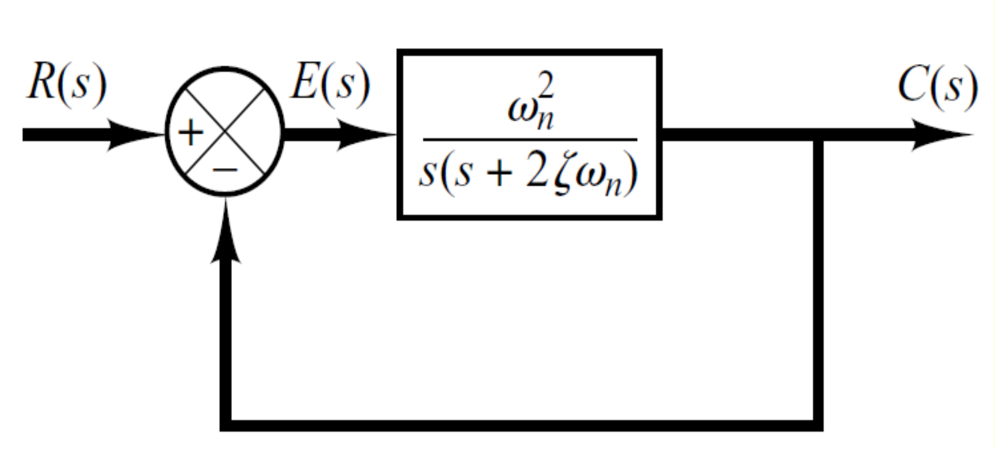
UNIT-3

(Lecture-5)

Examples Based on Time Domain Specification

Example#1

- Consider the system shown in following figure, where damping ratio is 0.6 and natural undamped frequency is 5 rad/sec. Obtain the rise time t_r, peak time t_p, maximum overshoot M_p, and settling time 2% and 5% criterion t_s when the system is subjected to a unit-step input.



Example#1

Rise Time

$$t_r = \frac{\pi - \theta}{\omega_d}$$

Peak Time

$$t_p = \frac{\pi}{\omega_d}$$

Settling Time (2%)

$$t_s = 4T = \frac{4}{\zeta\omega_n}$$

Maximum Overshoot

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

$$t_s = 3T = \frac{3}{\zeta\omega_n}$$

Settling Time (4%)

Example#1

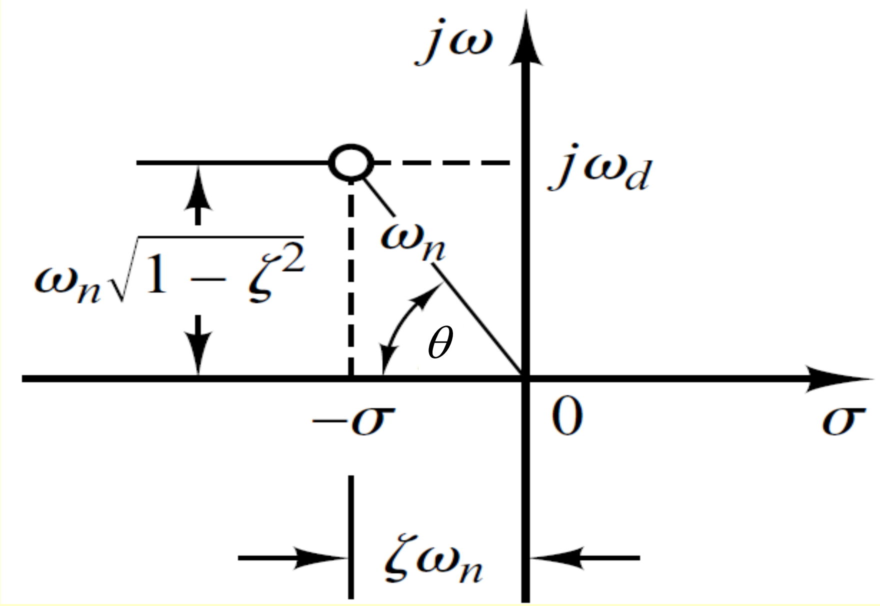
Rise Time

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$t_r = \frac{3.141 - \theta}{\omega_n \sqrt{1-\zeta^2}}$$

$$\theta = \tan^{-1}\left(\frac{\omega_n \sqrt{1-\zeta^2}}{\zeta\omega_n}\right) = 0.93 \text{ rad}$$

$$t_r = \frac{3.141 - 0.93}{5\sqrt{1-0.6^2}} = 0.55s$$



Example#1

Peak Time

$$t_p = \frac{\pi}{\omega_d}$$

$$t_p = \frac{3.141}{4} = 0.785s$$

Settling Time (2%)

$$t_s = \frac{4}{\zeta\omega_n}$$

$$t_s = \frac{4}{0.6 \times 5} = 1.33s$$

Settling Time (4%)

$$t_s = \frac{3}{\zeta\omega_n}$$

$$t_s = \frac{3}{0.6 \times 5} = 1s$$

Example#1

Maximum Overshoot

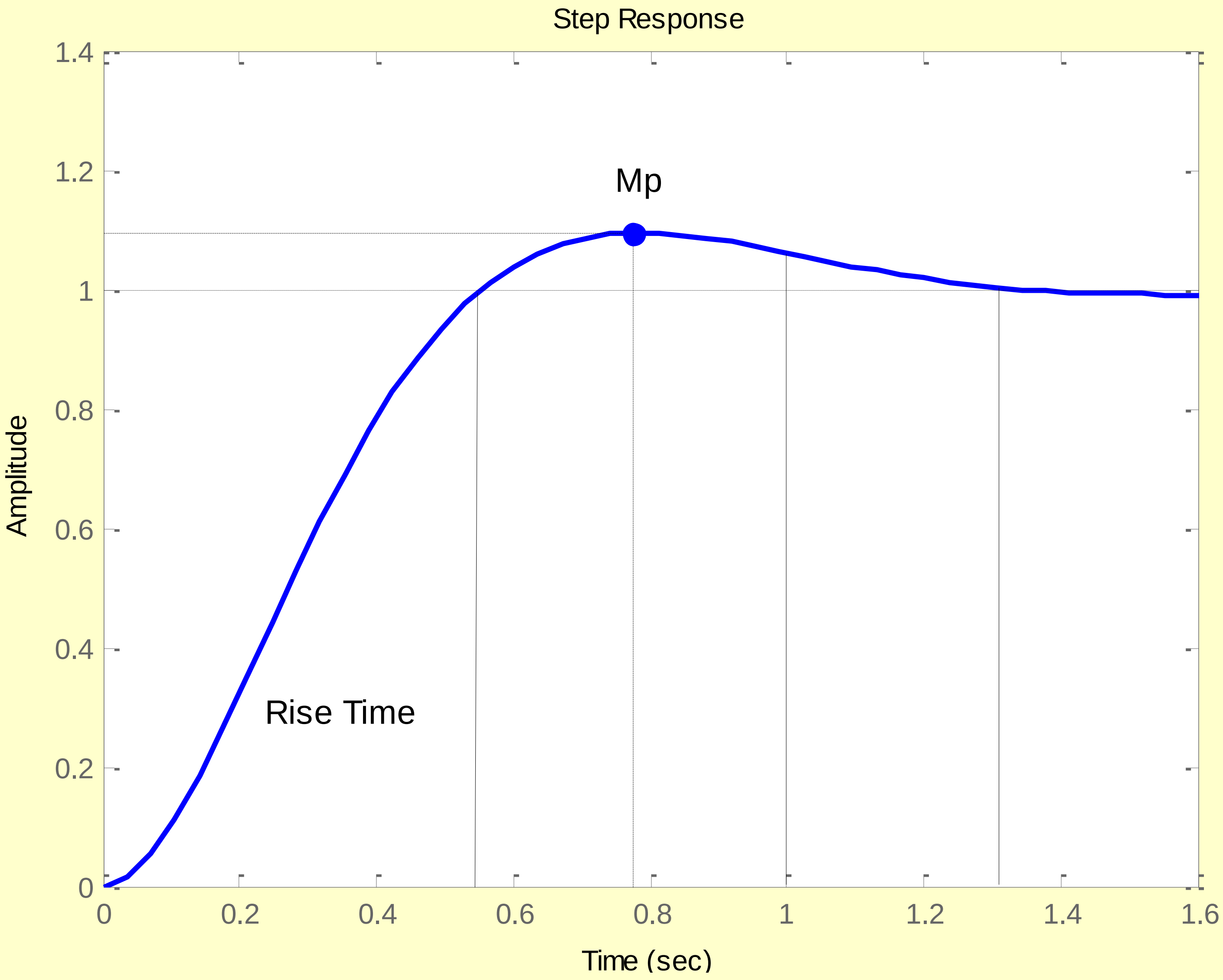
$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

$$M_p = e^{-\frac{3.141\times 0.6}{\sqrt{1-0.6^2}}} \times 100$$

$$M_p = 0.095 \times 100$$

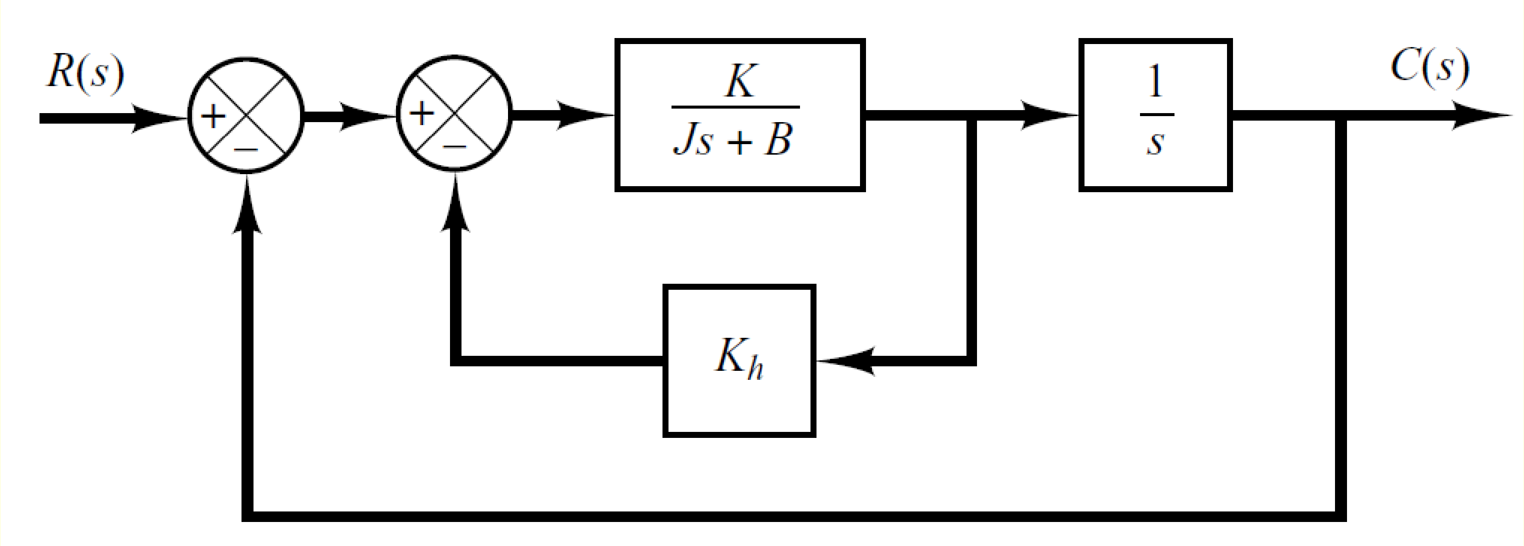
$$M_p = 9.5\%$$

Example#1

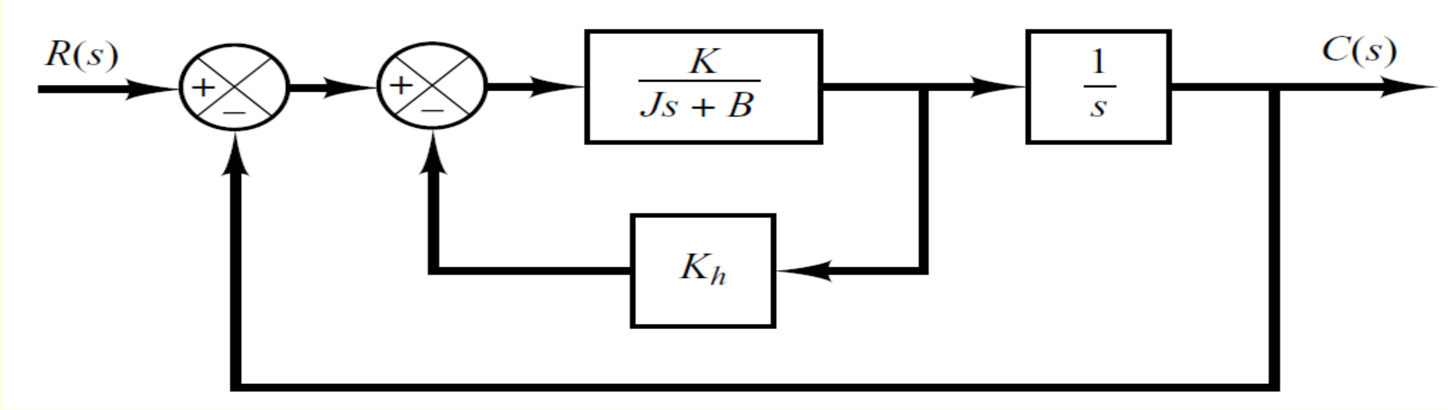

Step Response
Mp
Rise Time
Amplitude
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

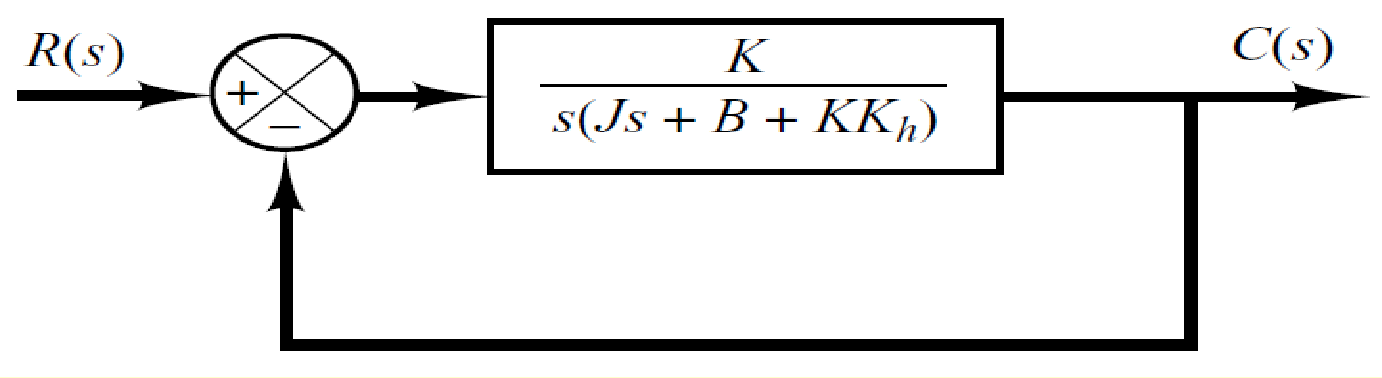
Example#2

- For the system shown in Figure-(a), determine the values of gain K and velocity-feedback constant K_h so that the maximum overshoot in the unit-step response is 0.2 and the peak time is 1 sec. With these values of K and K_h, obtain the rise time and settling time. Assume that J=1 kg-m^2 and B=1 N-m/rad/sec.



Example#2





$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

Example#2

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

Since $J = 1\ kgm^2$ and $B = 1$ Nm/rad/se

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + (1 + KK_h)s + K}$$

- Comparing above T.F with general 2nd order T.F

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{K} \qquad \zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

Example#2

$$\omega_n = \sqrt{K}$$

$$\zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

- **Maximum overshoot is 0.2.**

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$$

$$e^{-(\zeta/\sqrt{1-\zeta^2})\pi} = 0.2$$

$$\ln(e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}) = \ln(0.2)$$

$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.61$$

$$\zeta = 0.456$$

- **The peak time is 1 sec**

$$t_p = \frac{\pi}{\omega_d}$$

$$1 = \frac{3.141}{\omega_n\sqrt{1-\zeta^2}}$$

$$\omega_n = \frac{3.141}{\sqrt{1-0.456^2}}$$

$$\omega_n = 3.53$$

Example#2

$$\zeta = 0.456$$

$$\omega_n = 3.96$$

$$\omega_n = \sqrt{K}$$

$$\zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

$$3.53 = \sqrt{K}$$

$$0.456 \times 2\sqrt{12.5} = (1 + 12.5K_h)$$

$$3.53^2 = K$$

$$K_h = 0.178$$

$$K = 12.5$$

Example#2

$$\zeta = 0.456$$

$$\omega_n = 3.96$$

$$t_r = \frac{\pi - \theta}{\omega_n \sqrt{1 - \zeta^2}}$$

$$t_s = \frac{4}{\zeta \omega_n}$$

$$t_r = 0.65s$$

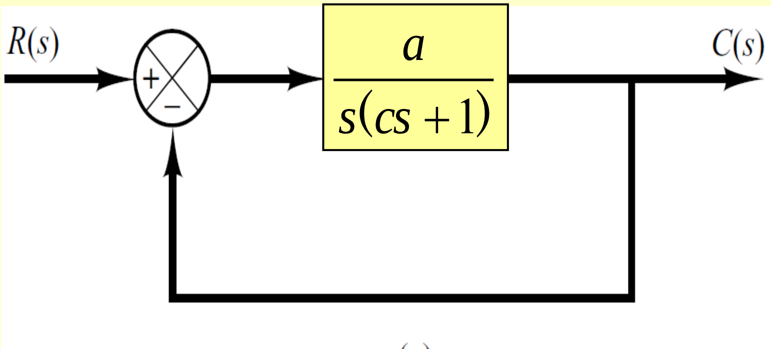
$$t_s = 2.48s$$

$$t_s = \frac{3}{\zeta \omega_n}$$

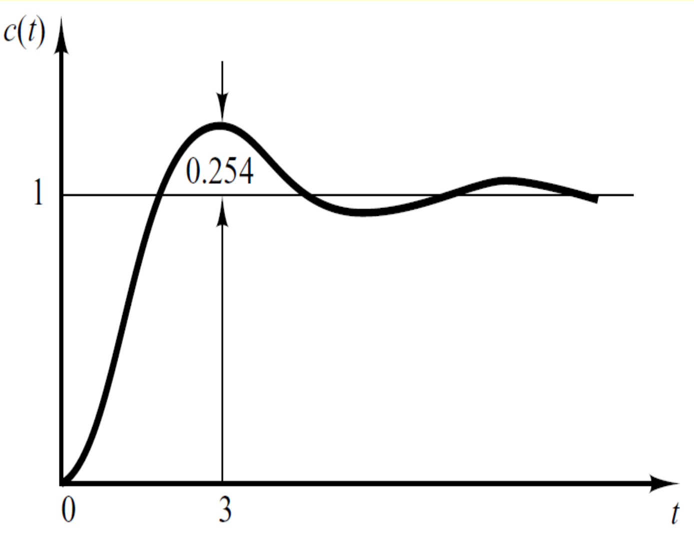
$$t_s = 1.86s$$

Example#3

When the system shown in Figure(a) is subjected to a unit-step input, the system output responds as shown in Figure(b). Determine the values of *a* and *c* from the response curve.



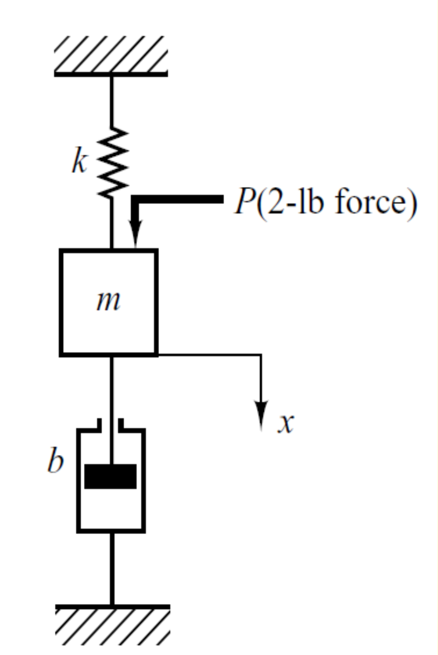
(a)



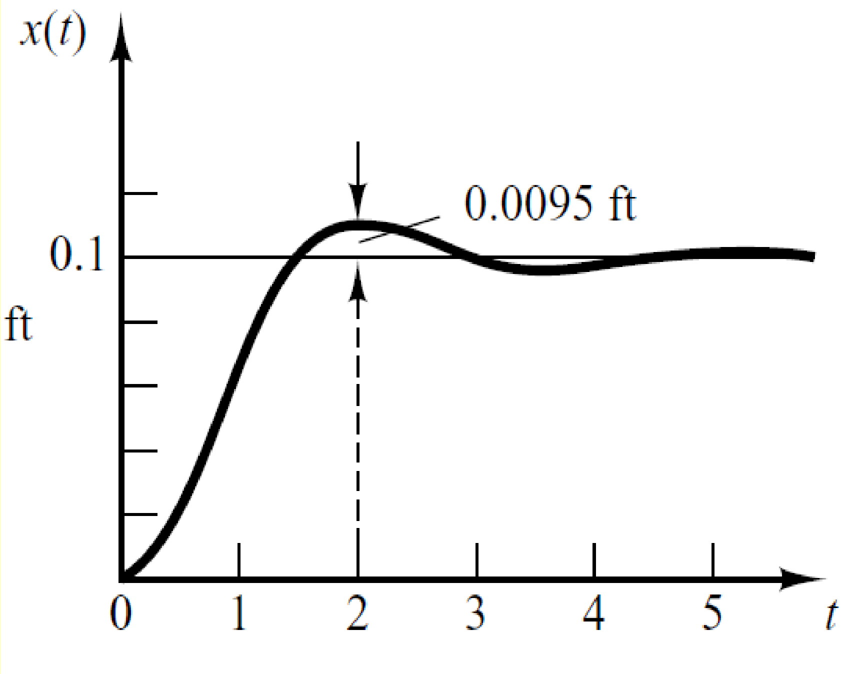
(b)

Example#4

Figure (a) shows a mechanical vibratory system. When 2 lb of force (step input) is applied to the system, the mass oscillates, as shown in Figure (b). Determine m, b, and k of the system from this response curve.



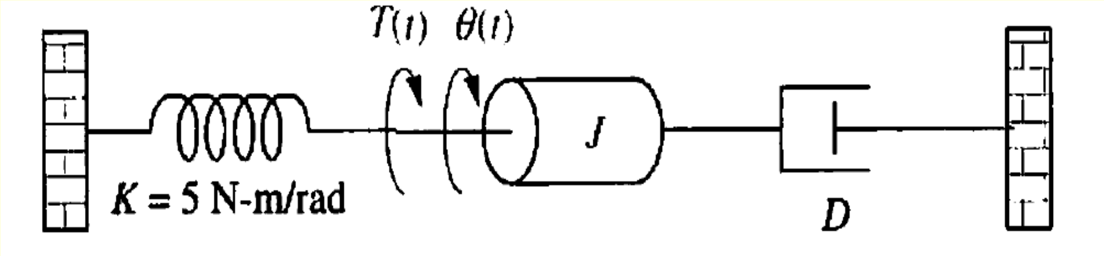
(a)



(b)

Example#5

Given the system shown in following figure, find *J and D to yield 20%* overshoot and a settling time of 2 seconds for a step input of torque *T(t).*



$$G(s) = \frac{1/J}{s^2 + \frac{D}{J}s + \frac{K}{J}}$$

$$2\zeta\omega_n = 4$$

$$\omega_n = \sqrt{\frac{K}{J}}$$

$$\zeta = \frac{4}{2\omega_n} = 2\sqrt{\frac{J}{K}}$$

$$T_s = 2 = \frac{4}{\zeta\omega_n}$$

Example#5

$$\omega_n = \sqrt{\frac{K}{J}}$$

$$\zeta = 2\sqrt{\frac{J}{K}}$$

20% overshoot implies $\zeta = 0.456$. Therefore,

$$\zeta = 2\sqrt{\frac{J}{K}} = 0.456$$

Hence,

$$\frac{J}{K} = 0.052$$

From the problem statement, $K = 5$ N-m/rad.

$$J = 0.26 \text{ kg-m}^2$$

Example#5

$$G(s) = \frac{1/J}{s^2 + \frac{D}{J}s + \frac{K}{J}}$$

$$2\zeta\omega_n = \frac{D}{J}$$

$$D = 1.04 \text{ N-m-s/rad}$$

Second – Order System

Example 6: Describe the nature of the second-order system response via the value of the damping ratio for the systems with transfer function

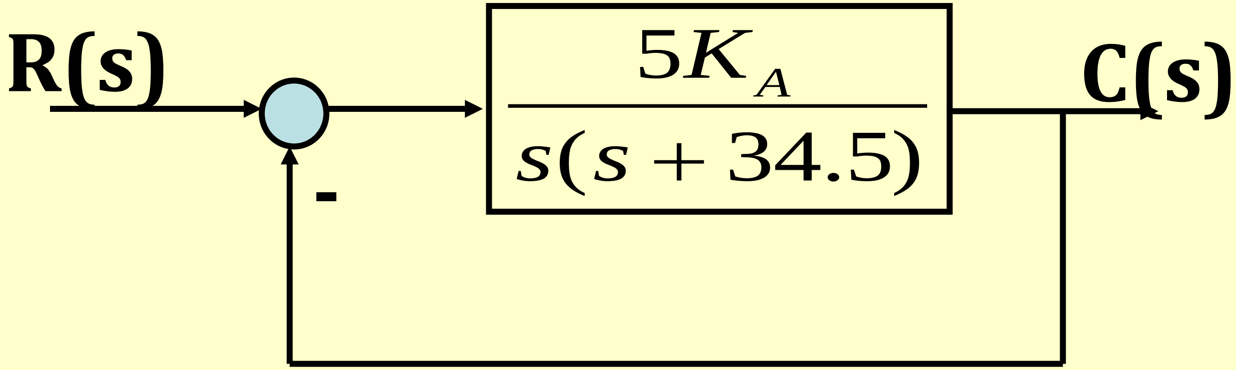
1. $G(s) = \dfrac{12}{s^2+8s+12}$

2. $G(s) = \dfrac{16}{s^2+8s+16}$

3. $G(s) = \dfrac{20}{s^2+8s+20}$

Do them as your own revision

Example7: Consider the following unit-feedback system



System input is the unit-step function, When the amplifier gains are KA=200, KA=1500, KA=13.5 respectively, can you calculate the time-domain specifications of the unit-step response ?

Investigate the effect of the amplifier gain KA on the system response

Solution: The closed-loop transfer function is

$$\phi(s) = \frac{G(s)}{1+G(s)} = \frac{5K_A}{s^2 + 34.5s + 5K_A}$$

$$K_A = 200, \therefore \phi(s) = \frac{1000}{s^2 + 34.5s + 1000}$$

$$\therefore \omega_n^2 = 1000, \quad 2\zeta\omega_n = 34.5$$

$$\therefore \omega_n = 31.6(rad/s), \zeta = \frac{34.5}{2\omega_n} = 0.545$$

According to the formula to calculate the performance indices, it follows that

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 0.12(\text{sec})$$

$$t_s \approx \frac{3}{\zeta \omega_n} = 0.174(\text{sec})$$

$$\sigma\% = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\% = 13\%$$

$$K_A = 1500$$

$$\text{If } K_A = 200, \text{ then } \omega_n = 34.5(rad/s); \quad \zeta = 0.545$$
$$\therefore t_p = 0.12(s), \quad t_s = 0.174(s), \quad \sigma\% = 13\%$$

$$\text{If } K_A = 1500, \text{ then } \omega_n = 86.2(rad/s); \quad \zeta = 0.2$$
$$\therefore t_p = 0.037(s), \quad t_s = 0.174(s), \quad \sigma\% = 52.7\%$$

Thus, the greater the *KA*, the less theξ,the greater the *wn*, the less the *tp*, the greater the*б%,* while the settling time *ts* has no change.

$\zeta > 1$
Overdamped

$$K_A = 13.5$$

$$\text{When } K_A = 13.5, \omega_n = 8.22(rad/s), \zeta = 2.1$$

When $K_A = 13.5, \omega_n = 8.22(rad/s), \zeta = 2.1$

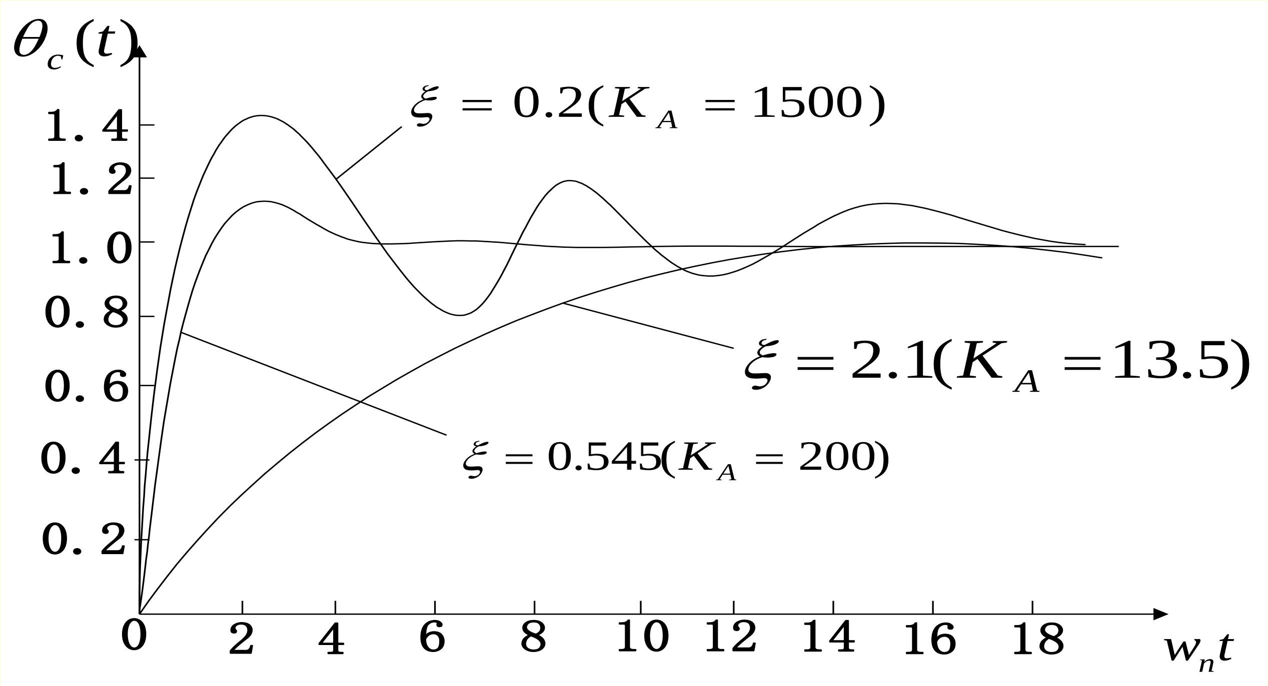
When system is over-damped, there is no peak time, overshoot and oscillation.

The settling time can be calculated approximately:

$$t_s \approx 3T = 1.46(\sec)$$

$$\frac{1}{T} = \omega_n(\zeta - \sqrt{\zeta^2 - 1})$$

The settling time is greater than previous cases, although the response has no overshoot, the transition process is very slow, the curves are as follows:



Note: When K_A increases, tp decreases, tr decreases, the speed of response increases, meanwhile, the overshoot increases. Therefore, to improve the dynamic performance indexes of system, we adopt PD-control or velocity feedback control , namely, PD compensation 。