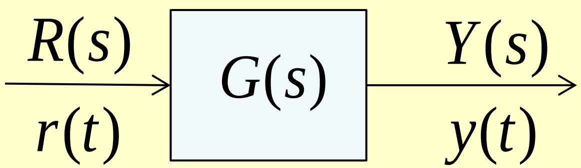
UNIT-3

(Lecture-6)

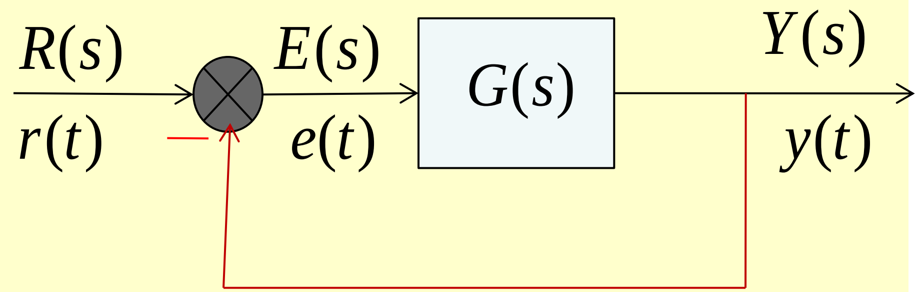
Steady-state error

Error and steady-state error

Open-loop control system



Closed-loop control system



Error: $e(t) = r(t) - y(t)$

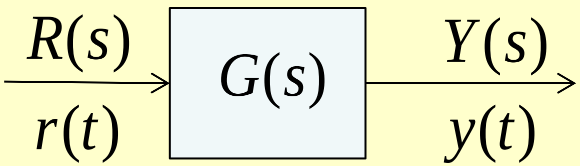
Steady-state error: $e_{ss} = \lim_{t \to \infty} e(t)$

Utilizing the final value theorem: $\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

Assuming r(t)=1(t) is a unit-step input, according to the above definition, could you calculate the steady-state error of the open-loop and closed-loop control systems?

Open-loop control system

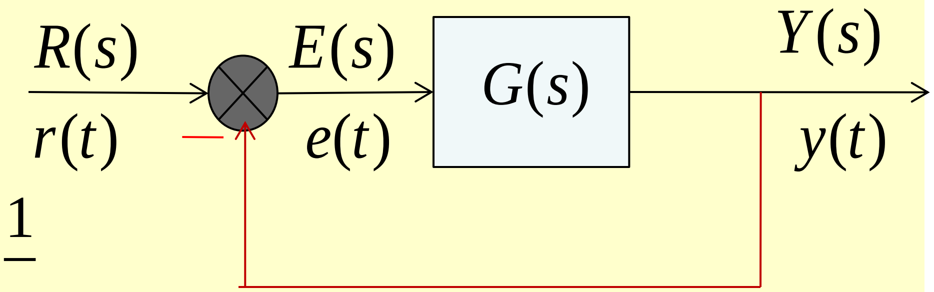


Unit-step input r(t)=1(t), $R(s) = \frac{1}{s}$

$$E(s) = R(s) - Y(s)$$
$$= R(s) - G(s)R(s)$$
$$= [1 - G(s)]R(s)$$

$$e_{ss} = \lim_{s \to 0} sE(s)$$
$$= \lim_{s \to 0} s[1 - G(s)]\frac{1}{s}$$
$$= \lim_{s \to 0}[1 - G(s)]$$
$$= 1 - G(0)$$

Closed-loop control system



$$E(s) = R(s) - Y(s)$$
$$= R(s) - \frac{G(s)}{1 + G(s)}R(s)$$
$$= \frac{1}{1 + G(s)}R(s)$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s\frac{1}{1 + G(s)}\frac{1}{s}$$
$$= \lim_{s \to 0}\frac{1}{1 + G(s)} = \frac{1}{1 + G(0)}$$

The forward-path transfer function G(s) can be formulated as

$$G(s) = \frac{k(\tau_1 s+1)\cdots(\tau_2^2 s^2 + 2\xi_1\tau_2 s+1)}{s^\nu (T_1 s+1)\cdots(T_2^2 s^2 + 2\xi_2 T_2 s+1)}$$

$$= \frac{k}{s^\nu} G_0(s) \qquad \text{when } s \to 0, \quad G_0(s) \to 1$$

ν is the order of the pole of G(s) at s=0

System Type: the order of the pole of G(s) at s=0.

When ν=0,1,2, the system is called type 0, type 1, type 2; k is called open-loop gain.

$$E(s) = \frac{1}{1+G(s)} R(s)$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{1+\frac{k}{s^\nu} G_0(s)} R(s)$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{1}{1 + \frac{k}{s^0} G_0(s)} R(s)$$

ν=0, type 0 system

Step input:

$r(t) = 1(t) \quad R(s) = \frac{1}{s}$

$$e_{ss} = \frac{1}{1+k}$$

Steady-state error exists and is finite.

Ramp input:

$r(t) = t \quad R(s) = \frac{1}{s^2}$

$$e_{ss} = \infty$$

Unstable

Parabolic input:

$r(t) = \frac{1}{2} t^2 \quad R(s) = \frac{1}{s^3}$

$$e_{ss} = \infty$$

Unstable

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{1+\frac{k}{s^{1}}G_0(s)} R(s)$$

ν=1, type 1 system

Step input:

$r(t) = 1(t) \quad R(s) = \frac{1}{s}$

$$e_{ss} = \frac{1}{1+\infty} = 0$$

No steady-state error

Ramp input:

$r(t) = t \quad R(s) = \frac{1}{s^2}$

$$e_{ss} = \frac{1}{k}$$

Steady-state error exists

Parabolic input:

$r(t) = \frac{1}{2}t^2 \quad R(s) = \frac{1}{s^3}$

$$e_{ss} = \infty$$

Unstable

Type-1 system can track step signal accurately.

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{1+\frac{k}{s^2}G_0(s)} R(s)$$

ν=2, type 2 system

Step input:

$r(t) = 1(t) \quad R(s) = \frac{1}{s}$

$$e_{ss} = \frac{1}{1+\infty} = 0$$

No steady-state error

Ramp input:

$r(t) = t \quad R(s) = \frac{1}{s^2}$

$$e_{ss} = 0$$

No steady-state error

Parabolic input:

$r(t) = \frac{1}{2}t^2 \quad R(s) = \frac{1}{s^3}$

$$e_{ss} = \frac{1}{k}$$

Steady-state error exists

Type-2 system can track step and ramp signals accurately.

Steady-state error constants

with step input

$$k_p = \lim_{s\to 0} G(s)$$ - step-error constant

with ramp input

$$k_v = \lim_{s\to 0} sG(s)$$ - ramp-error constant

with parabolic input

$$k_a = \lim_{s\to 0} s^2 G(s)$$ - parabolic-error constant

Type of System	Error constants			Steady-state error e_{ss}		
j	k_p	k_v	k_a	$r(t)=R_0\cdot 1(t)$	$r(t)=V_0 t$	$r(t)=A_0 t^2/2$
0	k	0	0	$\frac{R_0}{1+k}$	∞	∞
I	∞	k	0	0	$\frac{V_0}{k}$	∞
II	∞	∞	k	0	0	$\frac{A_0}{k}$

Summary of steady-state error and error constants for unit-feedback systems (H(s)=1)