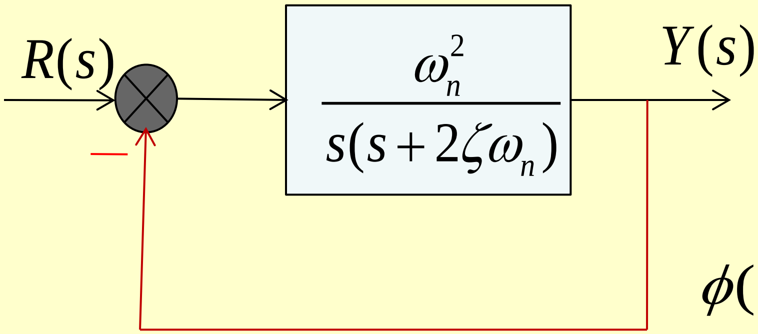
UNIT-3

(Lecture-7)

Effects of Adding Poles and Zeros to Transfer Functions

Effects of Adding Poles



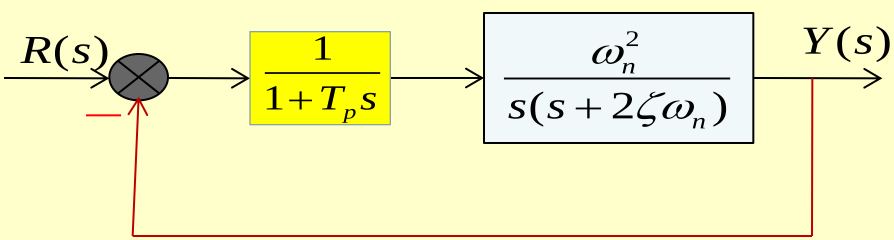
Open-loop TF:

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

Closed-loop TF:

$$\phi(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

1. Adding a pole $s = -1/T_p$ to the open-loop TF



Open-loop TF:

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)(1+T_p s)}$$

Closed-loop TF:

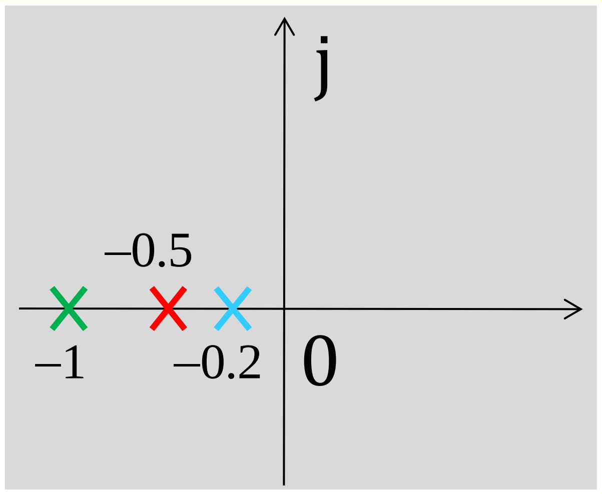
$$\phi(s) = \frac{\omega_n^2}{T_p s^3 + (1+2\zeta\omega_n T_p)s^2 + 2\zeta\omega_n s + \omega_n^2}$$

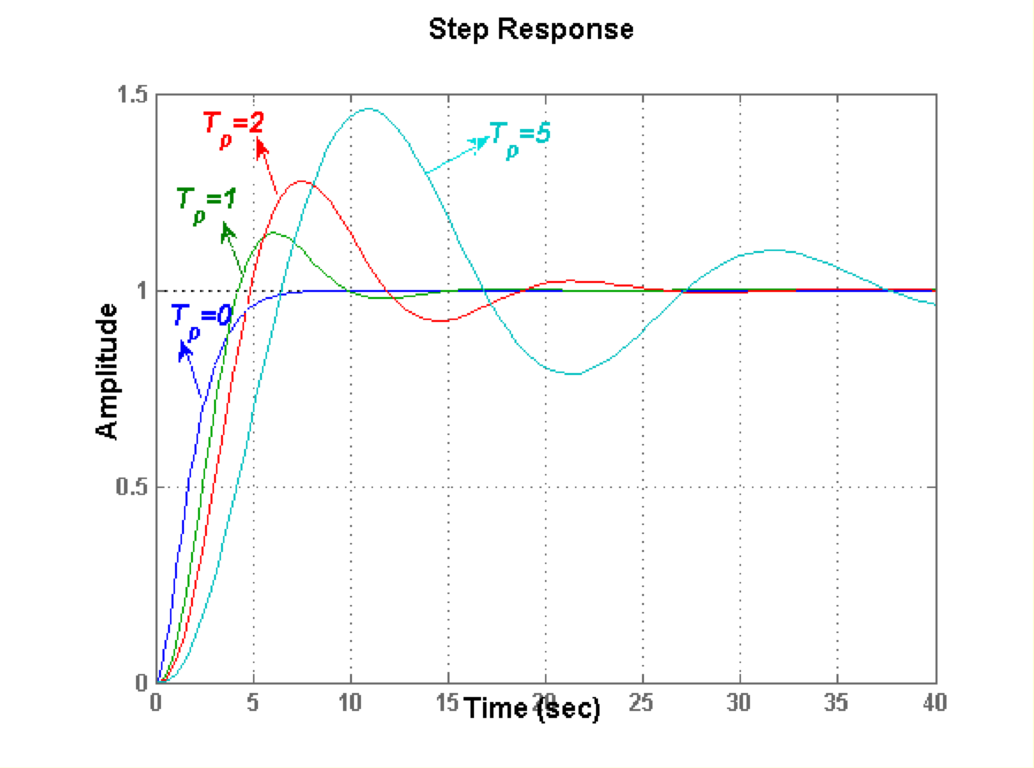
1. Adding a pole at $s = -1/T_p$ to the open-loop TF

How does it affect closed-loop system step-response performance?

$T_p = 0,\ 1,\ 2,\ 5$

$poles:\ s = \infty,\ -1,\ -0.5,\ -0.2$

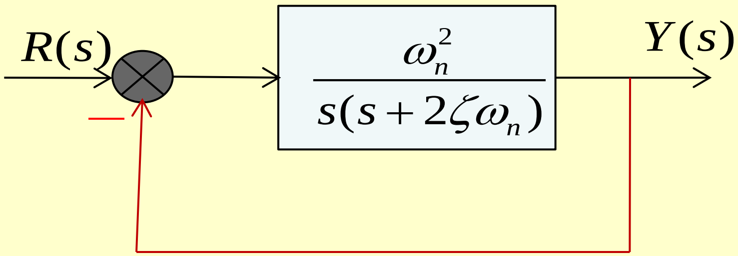




-- Increasing the maximum overshoot of the closed-loop system;

-- Increasing the rise time of the closed-loop system.

Effects of Adding Poles



Open-loop TF: $G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$

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$$\phi(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2. Adding a pole at $s = -1/T_p$ to the closed-loop TF

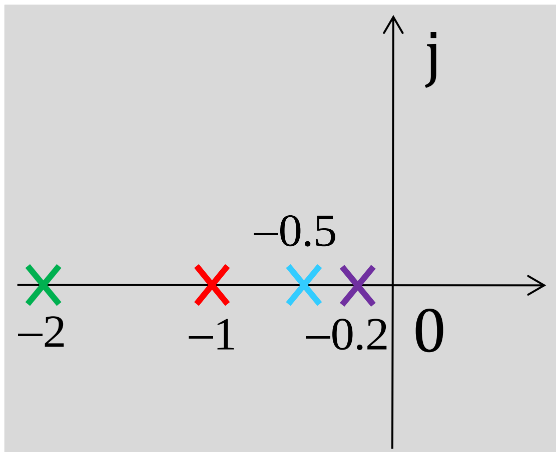
Closed-loop TF: $$\phi(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(1+T_p s)}$$

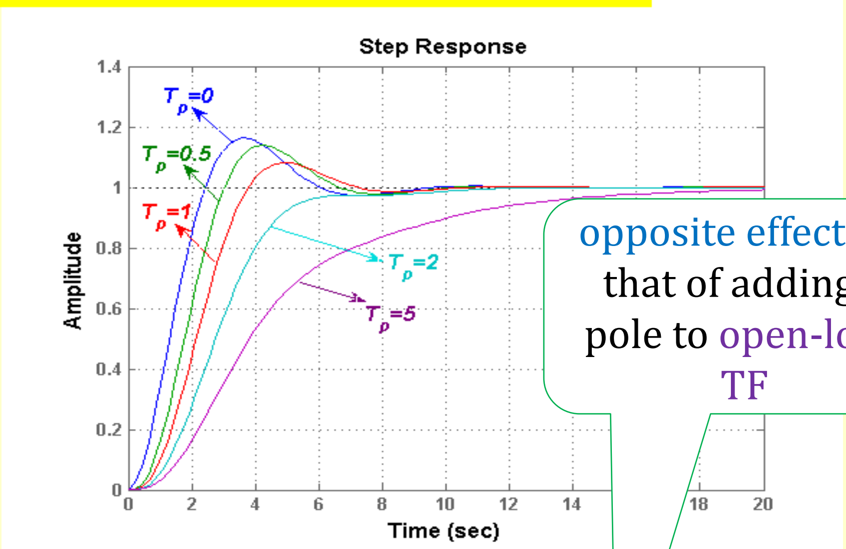
$$= \frac{\omega_n^2}{T_p s^3 + (1+2\zeta\omega_n T_p)s^2 + (2\zeta\omega_n + \omega_n^2 T_p)s + \omega_n^2}$$

2. Adding a pole at $s=-1/T_p$ to the closed-loop TF

$T_p = 0,\ 0.5,\ 1,\ 2,\ 5$

$s = \infty,\ -2,\ -1,\ -0.5,\ -0.2$





opposite effects to that of adding a pole to open-loop TF

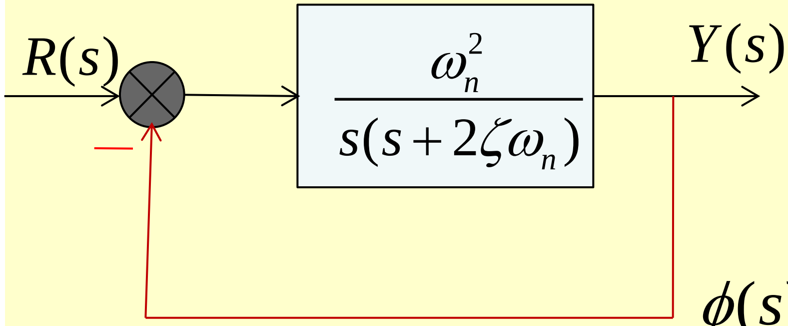
As the pole at $s=-1/T_p$ is moved toward the origin in the s-plane

-- the maximum overshoot of the closed-loop system decreases;

-- the rise time of the closed-loop system increases.

1. Adding a pole to the closed-loop system has the effect as increasing the damping ratio;
2. An originally underdamped system can be made into overdamped by adding a closed-loop pole.

Effects of Adding Zeros



Open-loop TF: $G(s) = \dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}$

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1. Adding a zero at $s = -1/T_z$ the closed-loop TF

Closed-loop TF:

$$\phi(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2(1+T_z s)}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \frac{\omega_n^2 T_z s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

1. Adding a zero at $s=-1/T_z$ to the closed-loop TF

Closed-loop TF:

$$\phi(s)=\frac{Y(s)}{R(s)}=\frac{\omega_n^2(1+T_z s)}{s^2+2\zeta\omega_n s+\omega_n^2}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}+\frac{\omega_n^2 T_z s}{s^2+2\zeta\omega_n s+\omega_n^2}$$

For a unit-step input $1(t)$, $R(s)=\frac{1}{s}$

The step response of the closed-loop system

$$Y(s)=\phi(s)R(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s}+T_z s\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s}$$

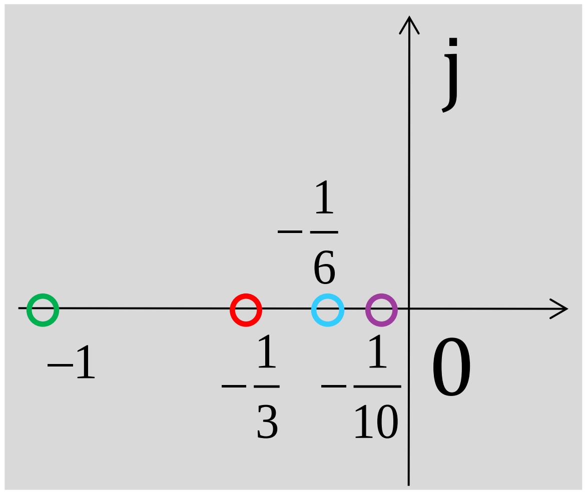
$$y(t)=L^{-1}[Y(s)]=y_1(t)+T_z\frac{dy_1(t)}{dt}$$

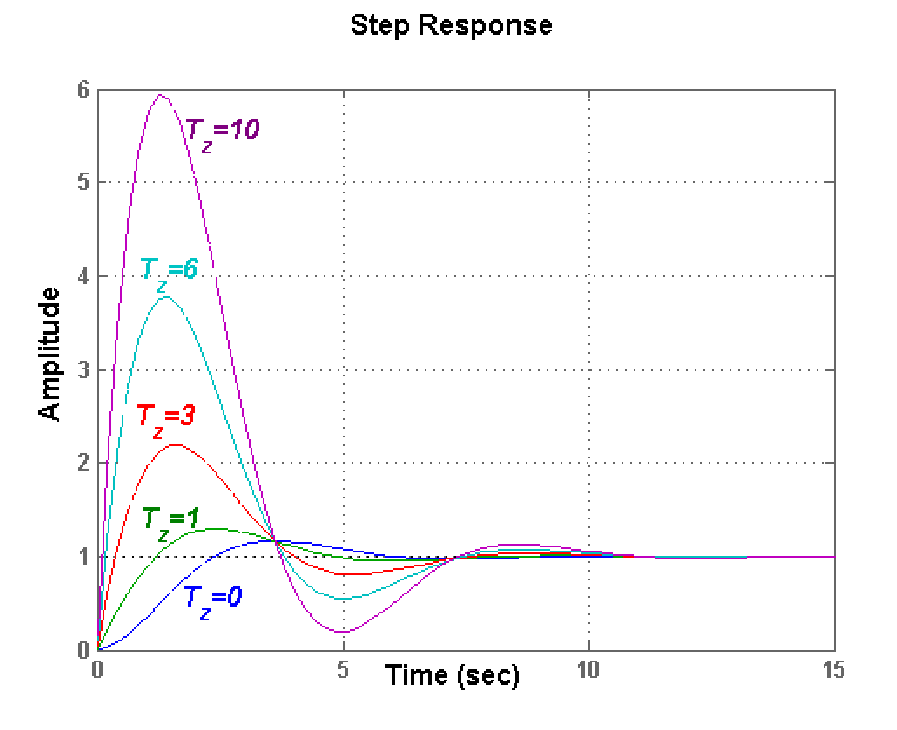
1. Adding a zero at $s=-1/T_z$ to the closed-loop TF

a) Its effects on an underdamped $(0<\zeta<1)$ system

$T_Z = 0,\ 1,\ 3,\ 6,\ 10$

$s = \infty,\ -1,\ -\frac{1}{3},\ -\frac{1}{6},\ -\frac{1}{10}$





As the zero at $s=-1/T_z$ is moved toward the origin in the s-plane

-- the maximum overshoot of the closed-loop system increases;

-- the rise time of the closed-loop system decreases.

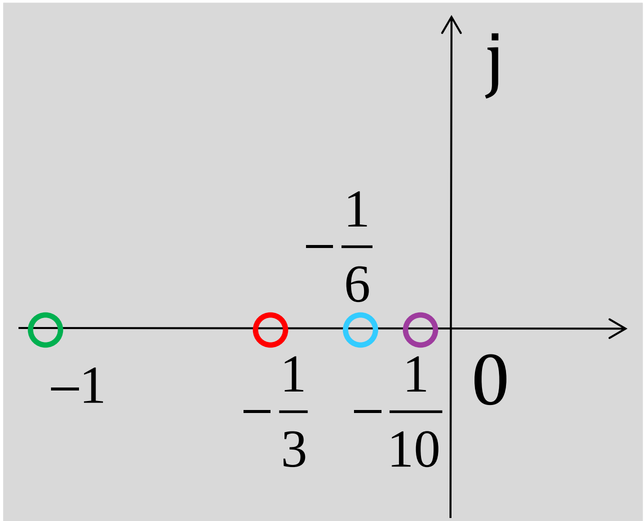
The additional zero has the effect as reducing the damping ratio

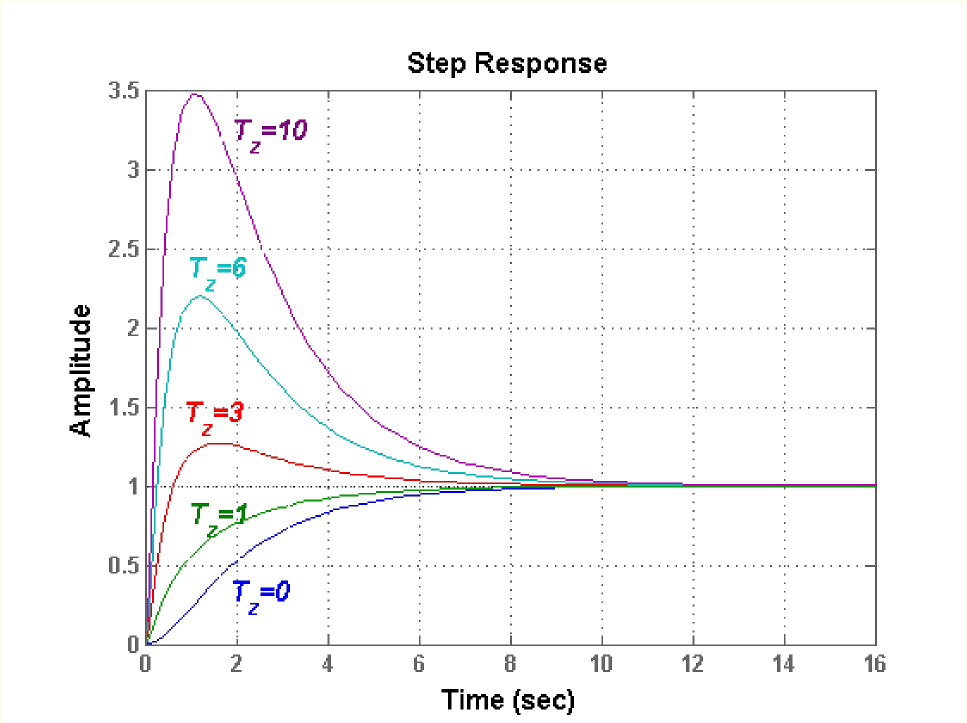
1. Adding a zero at $s=-1/T_z$ to the closed-loop TF

b) Its effects on an overdamped $(\zeta > 1)$ system

$T_Z = 0,\ 1,\ 3,\ 6,\ 10$

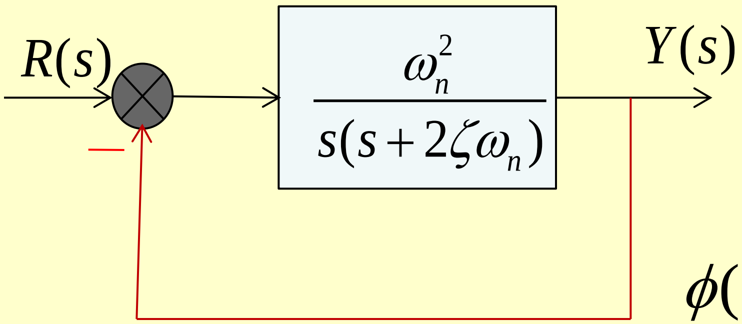
$s = \infty,\ -1,\ -\frac{1}{3},\ -\frac{1}{6},\ -\frac{1}{10}$





Adding a zero to an overdamped system can change it into an underdamped system by putting the zero at a proper position.

Effects of Adding Zeros



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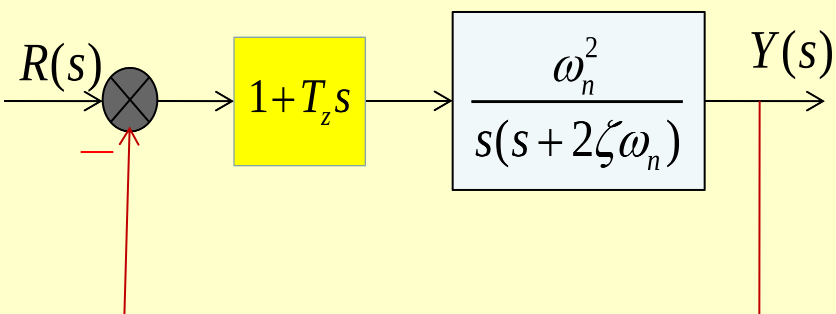
1. Adding a zero at $s=-1/T_z$ to the open-loop TF



Open-loop TF:

$$G(s) = \frac{(1+T_z s)\omega_n^2}{s(s+2\zeta\omega_n)}$$

Closed-loop TF: $\phi(s) = \dfrac{\omega_n^2(1+T_z s)}{s^2 + (2\zeta\omega_n + T_z\omega_n^2)s + \omega_n^2}$

The additional zero changes both numerator and denominator.

1. Adding a zero at $s = -1/T_z$ to the open-loop TF

The additional zero changes both numerator and denominator.

Closed-loop TF:

$$\phi(s) = \frac{\omega_n^2(1+T_z s)}{s^2 + (2\zeta\omega_n + T_z\omega_n^2)s + \omega_n^2} = \frac{\omega_n^2(1+T_z s)}{s^2 + 2\zeta_z\omega_n s + \omega_n^2}$$

An additional zero $s = -1/T_z$ will increase overshoot

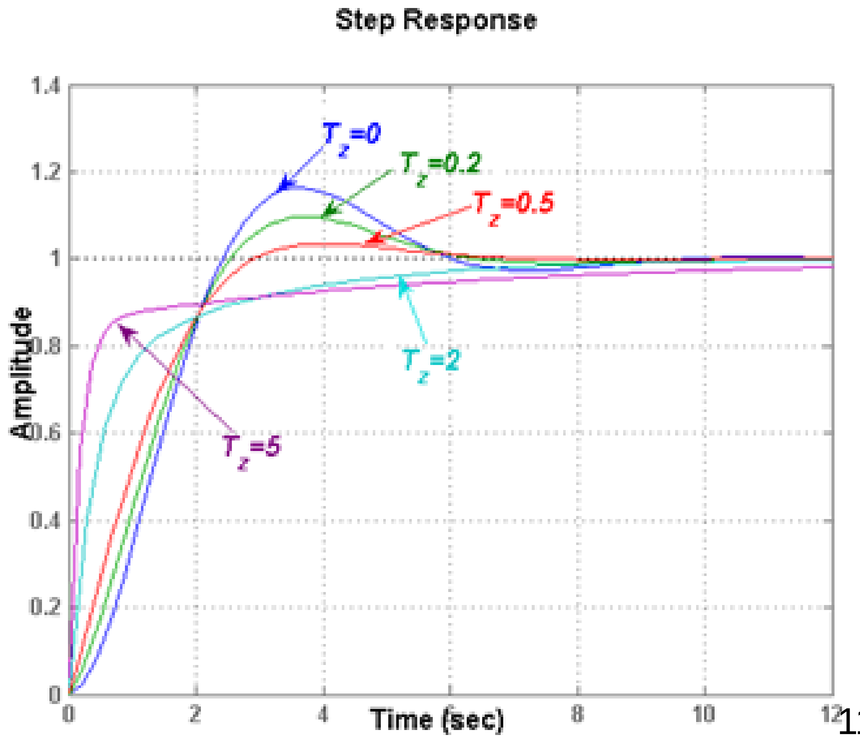
$T_z \uparrow$ overshoot $\sigma\% \uparrow$

The equivalent damping ratio:

$$\zeta_z = \zeta + \frac{T_z\omega_n}{2} > \zeta$$

$T_z \uparrow$ $\zeta_z \uparrow$ overshoot $\sigma\% \downarrow$

when $\zeta_z > 1$, the closed-loop system becomes overdamped (no overshoot no matter how large T_z is)



Dominant Poles of Transfer Function

Dominant poles: those poses that have a dominant effect on the transient response.

By identifying dominant poles, high-order systems can be approximated by lower ones as the transient response is concerned.

e.g.

$$Y(s) = \frac{1}{(s+p_1)(s+p_2)(s+p_3)} = \frac{c_1}{s+p_1} + \frac{c_2}{s+p_3} + \frac{c_3}{s+p_3}$$

$$y(t) = c_1 e^{-p_1 t} + c_2 e^{-p_2 t} + c_3 e^{-p_3 t}$$

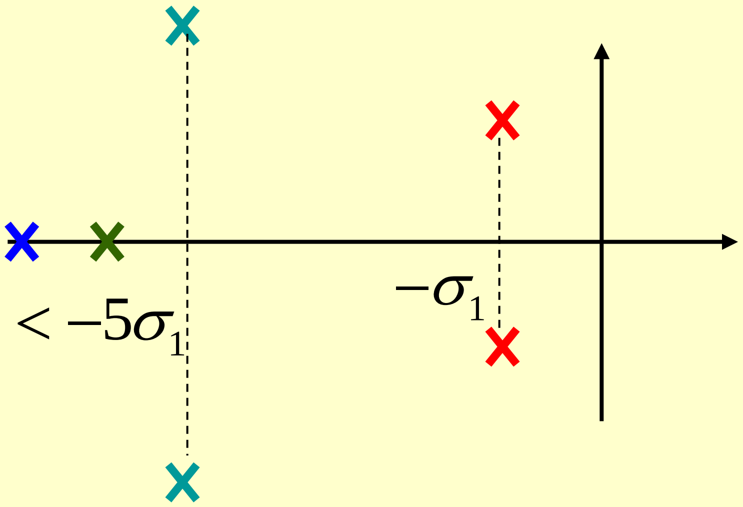
poles: $-p_1, -p_2, -p_3$

If $p_1 > p_2 > p_3$, $c_1 e^{-p_1 t}$ decays fastest, $c_3 e^{-p_3 t}$ decays slowest.

Position of Poles in the left-half s-plane	Their effects on transient response
close to the imaginary axis	decaying relatively slowly
far away from the imaginary axis	decaying fast

Dominant Poles of Transfer Function

If the ratio of real parts exceed 5 and no zeros nearby, the closed-loop poles nearest the imaginary-axis will dominate in the transient response behavior.



The dominant poles can be a real pole, but a pair of complex conjugate poles are more preferable in control engineering（why?）.

In order to apply second-order system in approximating the dynamic performance of higher-order system