UNIT-3

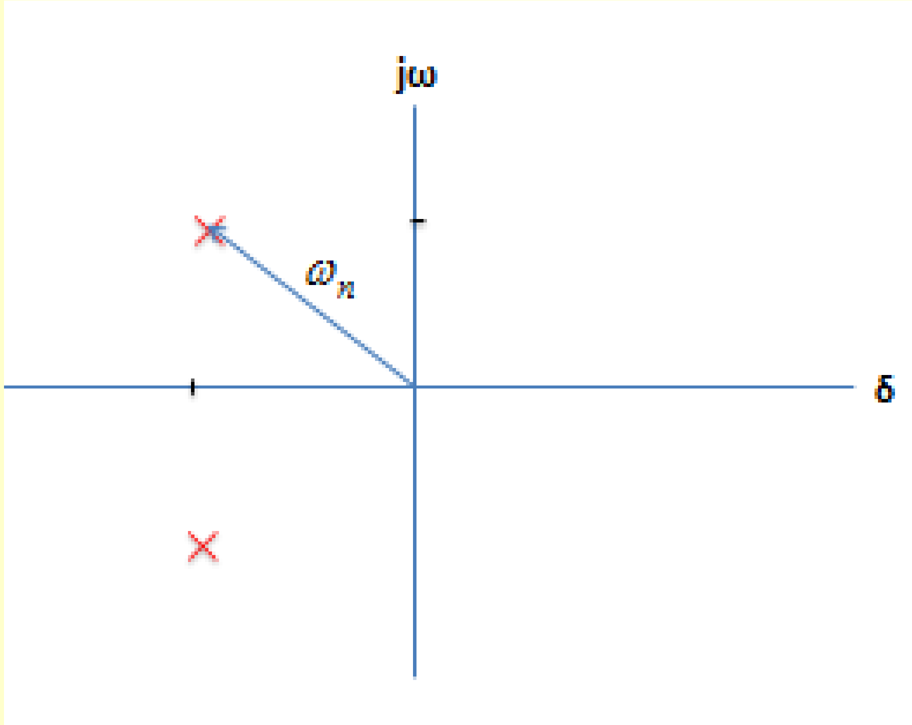
(Lecture-8)

S-Plane

S-Plane

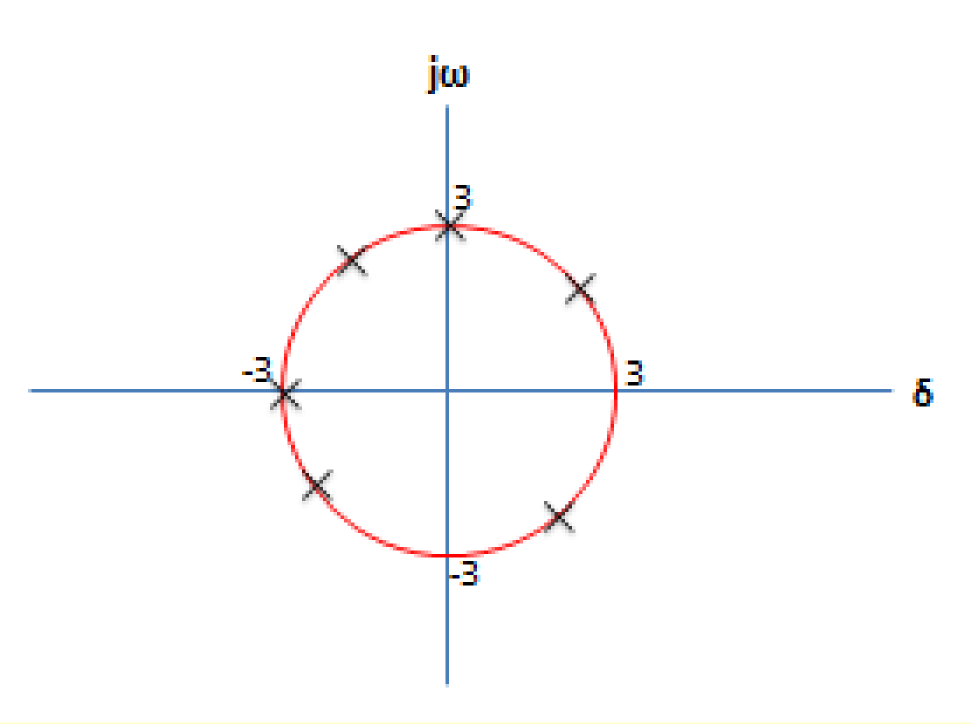
- **Natural Undamped Frequency.**

- Distance from the origin of s-plane to pole is natural undamped frequency in rad/sec.



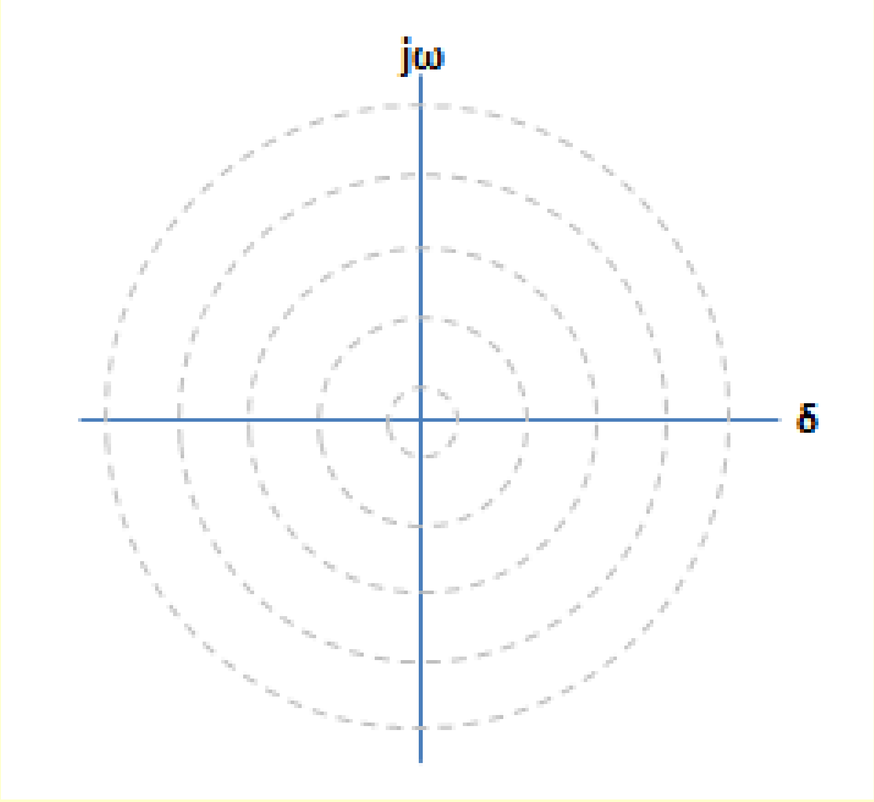
S-Plane

- Let us draw a circle of radius 3 in s-plane.
- If a pole is located anywhere on the circumference of the circle the natural undamped frequency would be *3 rad/sec*.



S-Plane

- Therefore the s-plane is divided into Constant Natural Undamped Frequency (ω_n) Circles.

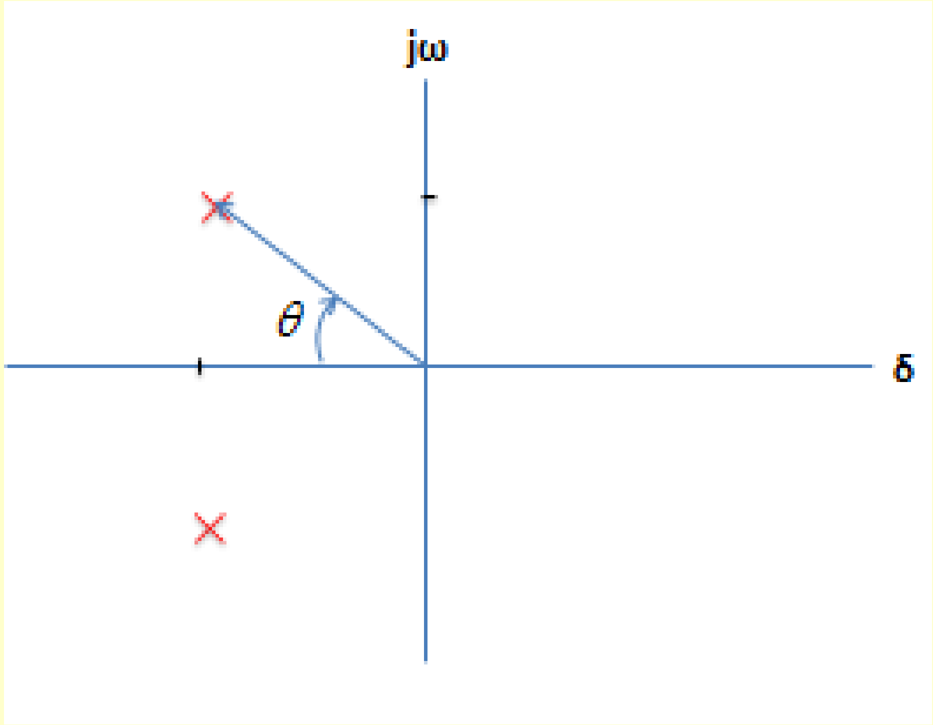


S-Plane

- **Damping ratio.**

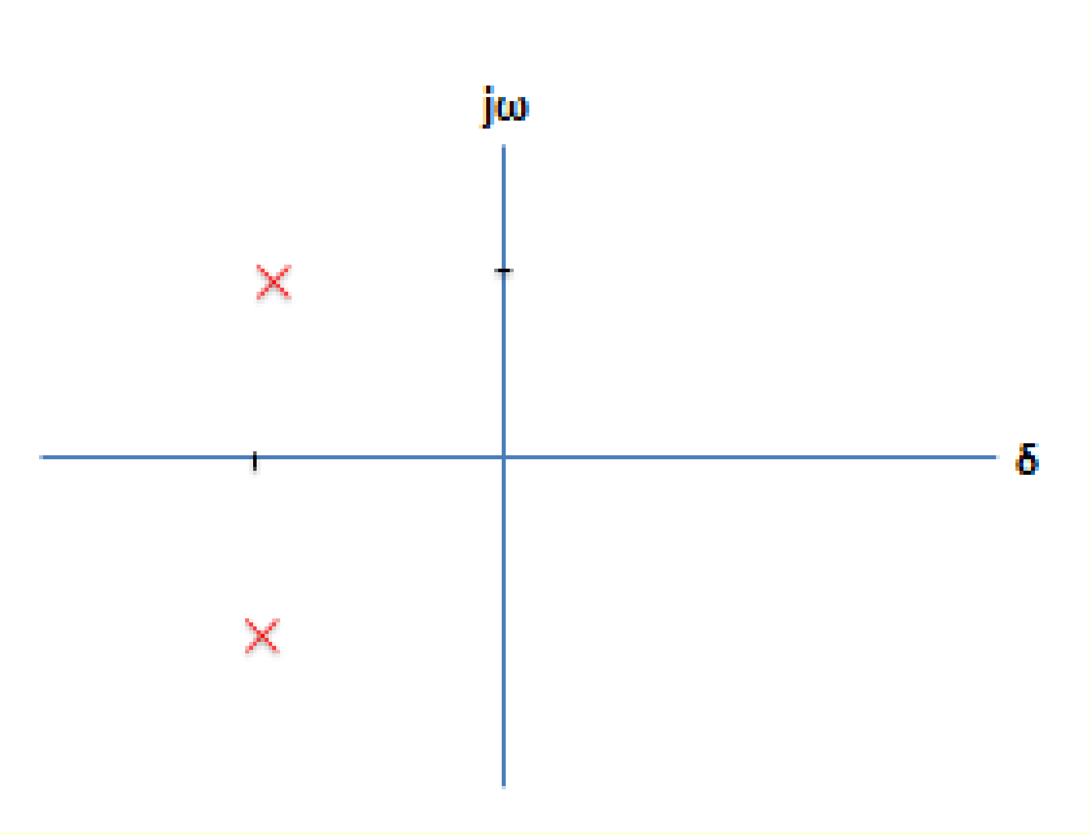
  - Cosine of the angle between vector connecting origin and pole and –ve real axis yields damping ratio.

$$\zeta = \cos\theta$$



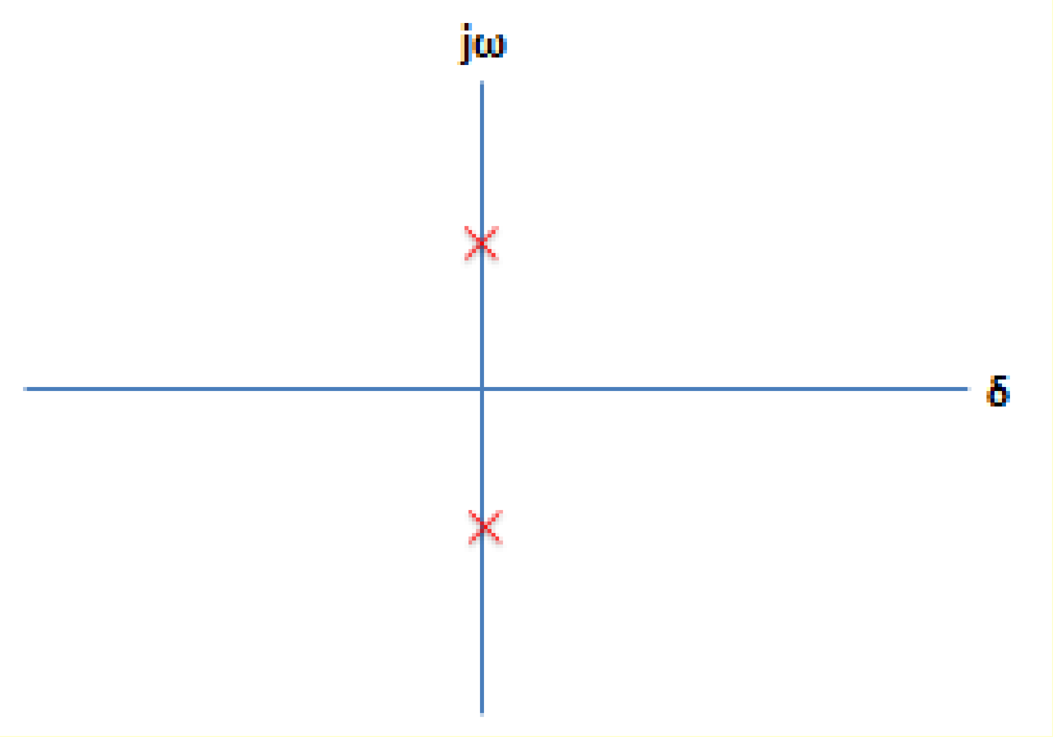
S-Plane

- For Underdamped system $0^\circ < \theta < 90^\circ$ therefore,



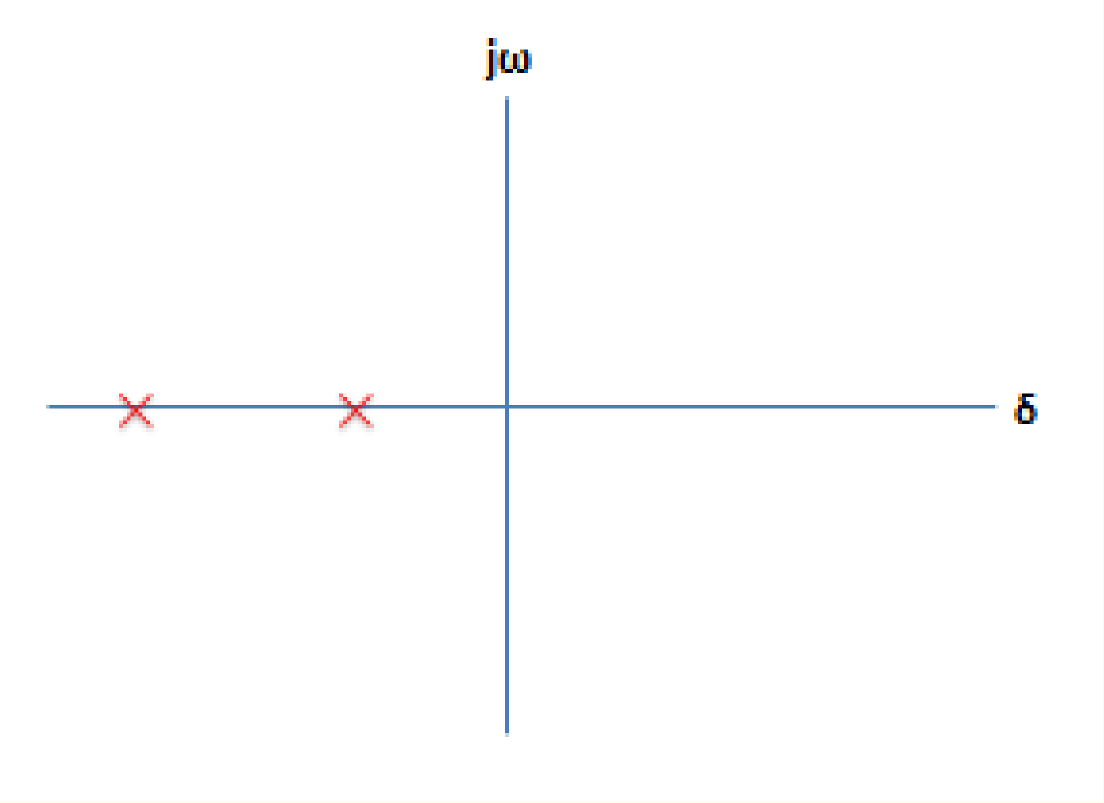
S-Plane

- For Undamped system $\theta = 90^{\circ}$ therefore, $\zeta = 0$



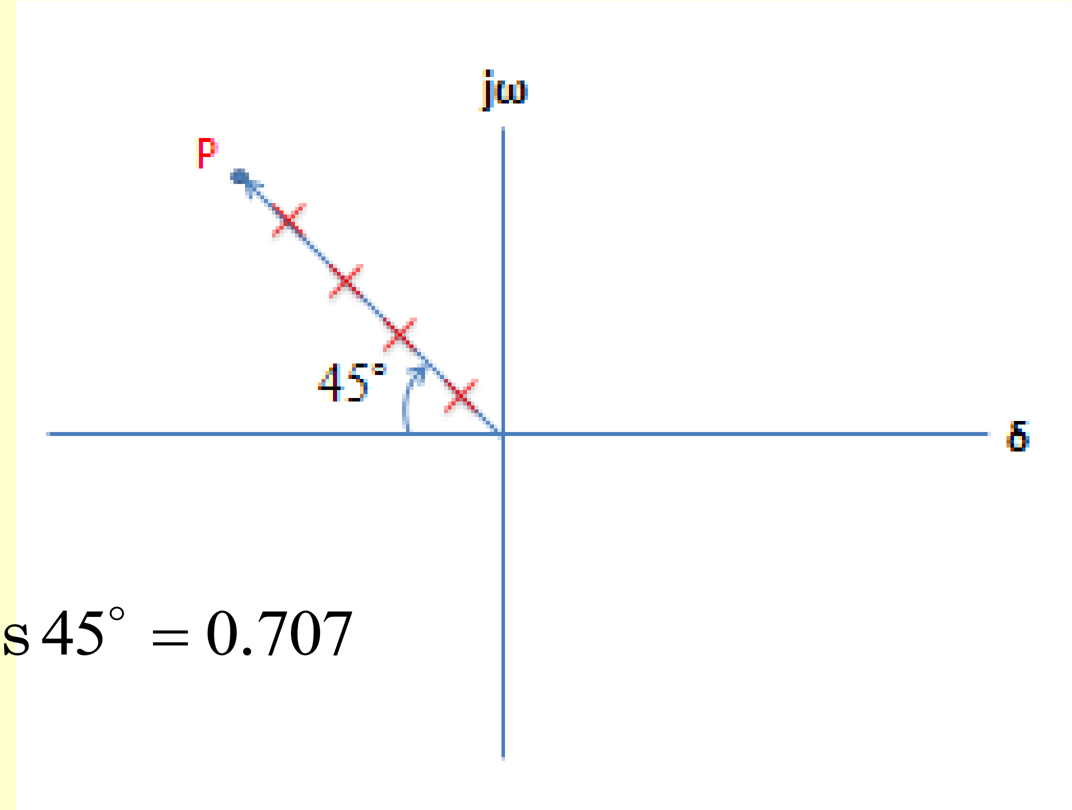
S-Plane

- For overdamped and critically damped systems therefore, $\zeta \geq 0$



S-Plane

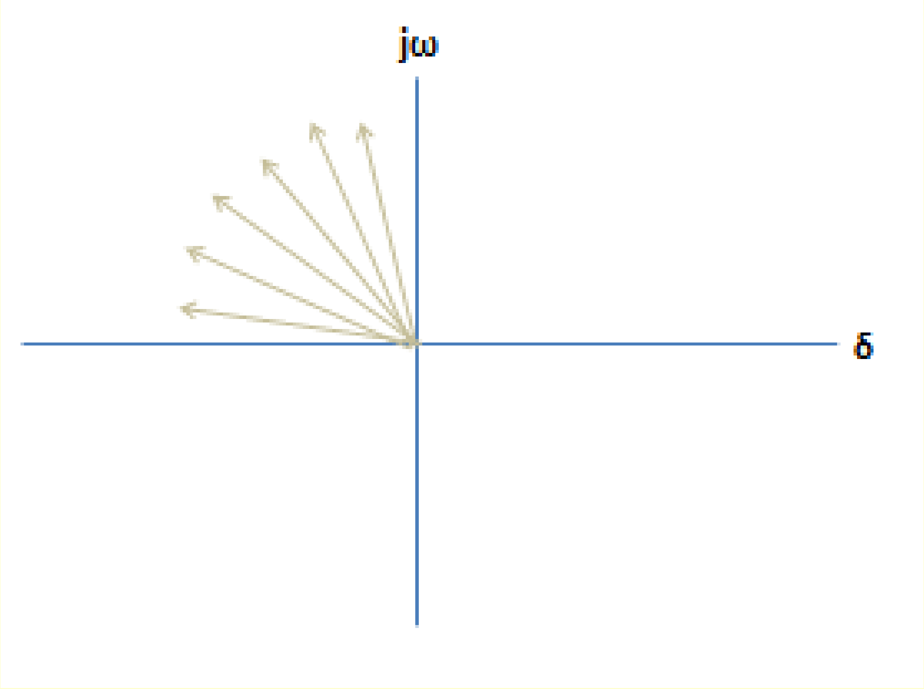
- Draw a vector connecting origin of s-plane and some point P.



$$\zeta = \cos 45^\circ = 0.707$$

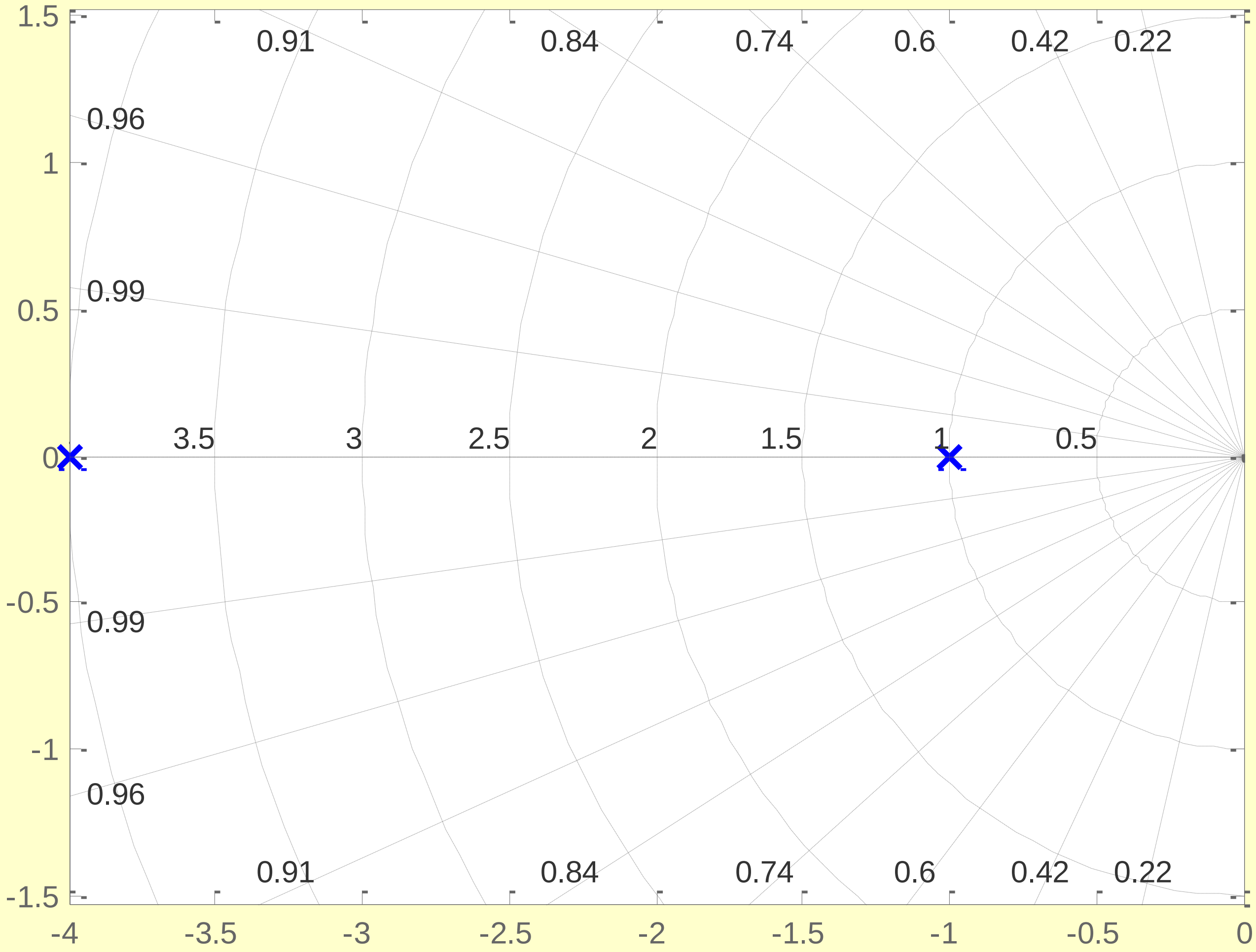
S-Plane

- Therefore, s-plane is divided into sections of constant damping ratio lines.



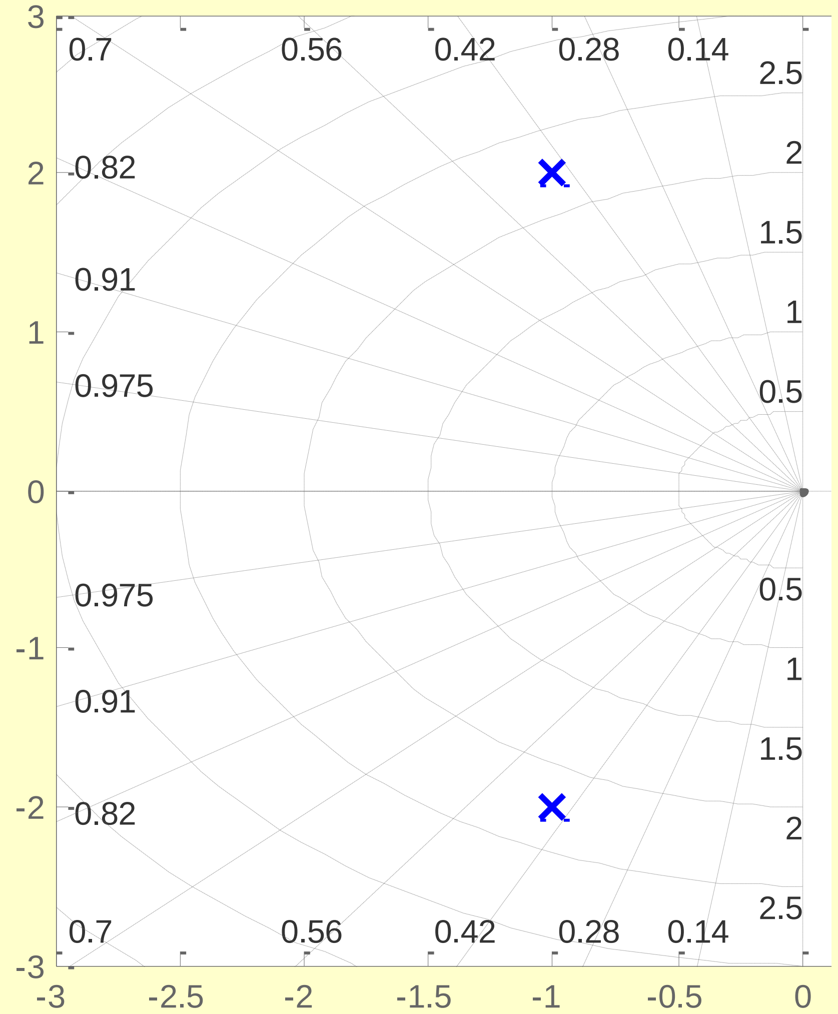
Example-2

- Determine the natural frequency and damping ratio of the poles from the following pz-map.



Example-3

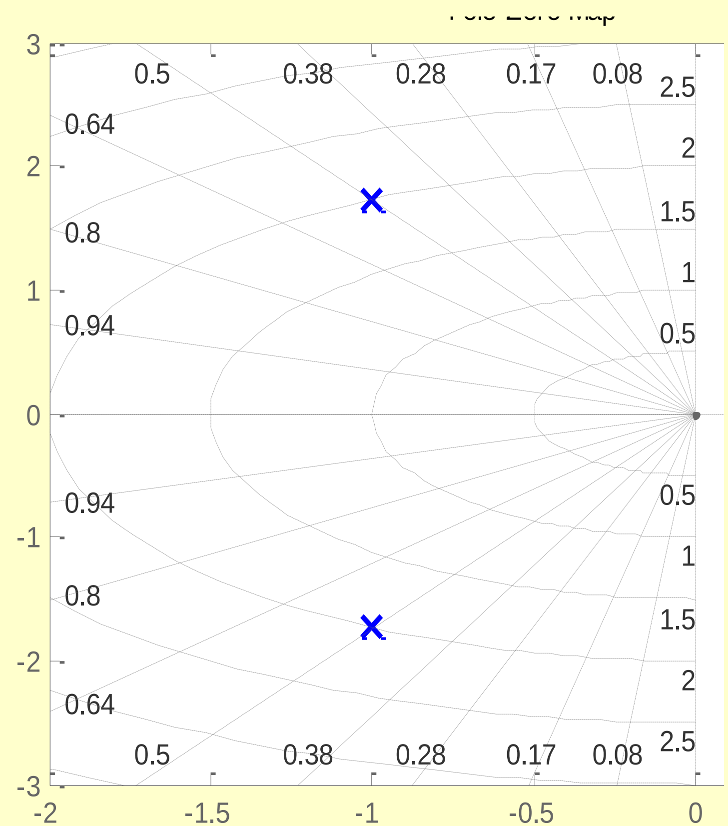
- Determine the natural frequency and damping ratio of the poles from the given pz-map.
- Also determine the transfer function of the system and state whether system is underdamped, overdamped, undamped or critically damped.



Example-4

- The natural frequency of closed loop poles of 2nd order system is 2 rad/sec and damping ratio is 0.5.

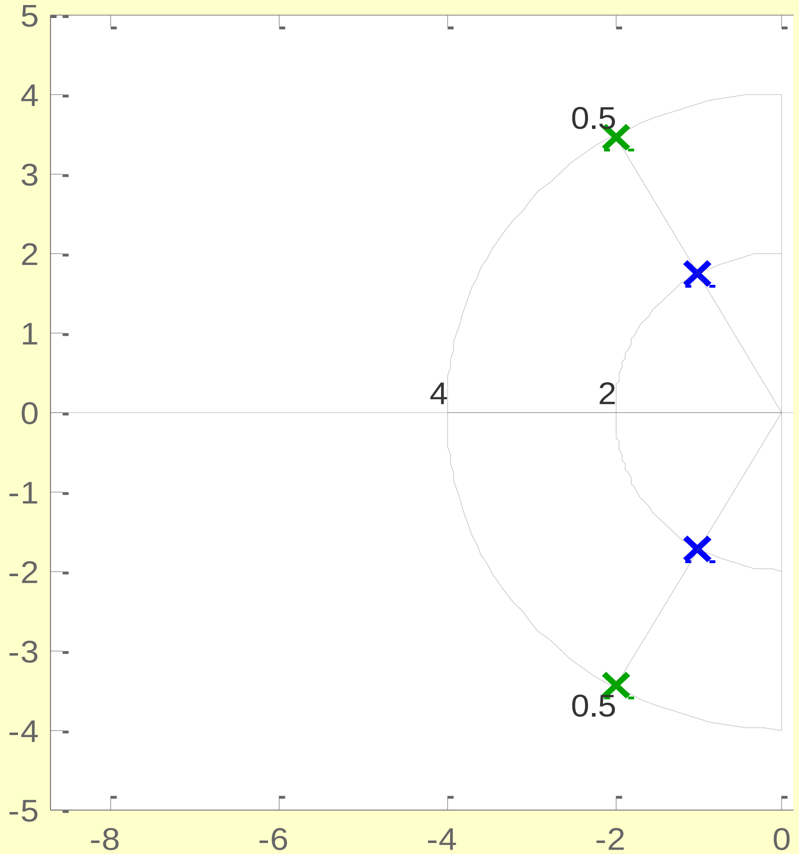
- Determine the location of closed loop poles so that the damping ratio remains same but the natural undamped frequency is doubled.



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{4}{s^2 + 2s + 4}$$

Example-4

- Determine the location of closed loop poles so that the damping ratio remains same but the natural undamped frequency is doubled.



S-Plane

$$-\omega_n\zeta + \omega_n\sqrt{\zeta^2 - 1}$$

$$-\omega_n\zeta - \omega_n\sqrt{\zeta^2 - 1}$$