UNIT-4

(Lecture-1)

Stability of Linear Control Systems

The concept of stability

Two types of response for LTI systems:

--Zero-state response: the response is due to input only; all the initial conditions are zero;

Bounded-input-bounded-output(BIBO) stability: with zero initial conditions, the system's output y(t) is bounded to a bounded input u(t).

--Zero-input response: the response is due to the initial conditions only; all the inputs are zero;

Asymptotic stability: with zero input, for finite initial conditions $y(t_0), \dot{y}(t_0), \cdots, y^{(n)}(t_0)$ **, a LTI system is asymptotic stable there exist a positive number M which depends on the initial conditions, such that**

$$(1) \quad |y(t)| \leq M < \infty \quad \textit{for all } t > t_0; \text{ and} \quad (2) \quad \lim_{t\to\infty} |y(t)| = 0.$$

Time-domain definition：

The initial condition of the system is zero. When system input is unit impulse function δ(t), the system output is g(t).
If $\lim_{t\to\infty} g(t)=0$, then the system is stable。

$$G(s)=\frac{Y(s)}{U(s)}=\sum_{i=1}^{n}\frac{C_i}{s+p_i} \qquad g(t)=L^{-1}[G(s)]=\sum_{i=1}^{n}C_i e^{-p_i t}$$

$$\lim_{t\to\infty} g(t)=0 \qquad \Rightarrow e^{-p_i t} \text{ decay with time}$$

All poles should locate in the left side of s-plane

Stability criterion in complex plane

A system is stable if and only if

all roots of the system characteristic equation have negative real parts

or equivalently

all poles of closed-loop transfer functions must locate in the left half of s-plane.

For LTI systems, both BIBO stability and asymptotic stability have the same requirement on pole location. Thus if a system is BIBO stable , it must also e asymptotic stable.

So we simply refer to the stability condition of a LTI system as stable or unstable.

LTI systems

Stability Conditions	Location of poles
stable	all poles in LHP
marginally stable	simple poles on the jω axis and no poles in RHF
unstable	at least one simple pole in RHF or at least one multi-order pole on the jw-axis