UNIT-4
(Lecture-2)

Routh-Hurwitz's stability criterion

Routh-Hurwitz's stability criterion

A necessary (but not sufficient) condition for stability:

(1) All the coefficients of the characteristic equation have the same sign.

(2) None of the coefficients vanishes.

Consider the characteristic equation of a LTI system

$$D(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n = 0, \quad a_0 > 0$$

Note 1: The above conditions are based on the laws of algebra .

$$(s-s_1)(s-s_2)(s-s_3)=0 \quad \rightarrow a_0s^3+a_1s^2+a_2s+a_3=0$$

$$s(s-s_2)(s-s_3)-s_1(s-s_2)(s-s_3)=0$$

$$s(s^2-(s_2+s_3)s+s_2s_3)-s_1(s^2-(s_2+s_3)s+s_2s_3)=0$$

$$s^3-(s_1+s_2+s_3)s^2+(s_1s_2+s_1s_3+s_2s_3)s-s_1s_2s_3=0$$

$$\frac{a_1}{a_0}=-\sum_{i=1}^{3}s_i>0 \qquad \frac{a_2}{a_0}=\sum_{\substack{i,j=1\\ i\neq j}}^{3}s_is_j>0 \qquad \frac{a_3}{a_0}=-\sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{3}s_is_js_k>0$$

If all roots of the system characteristic equation have negative real parts, all the coefficients have the same sign

Note 2: These conditions are not sufficient.

Routh's Tabulation

Consider the characteristic equation of a LTI system

$$D(s) = a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n = 0, \quad a_0 > 0$$

s^n	a_0	a_2	a_4	a_6	$\cdots$
s^{n-1}	a_1	a_3	a_5	a_7	$\cdots$
s^{n-2}	b_1	b_2	b_3	b_4	$\cdots$
s^{n-3}	c_1	c_2	c_3	c_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$			
s^2	e_1	e_2			
s^1	f_1				
s^0	g_1				

$$b_1 = \frac{-1}{a_1}\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} \qquad c_1 = \frac{-1}{b_1}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix}$$

$$b_2 = \frac{-1}{a_1}\begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix} \qquad c_2 = \frac{-1}{b_1}\begin{vmatrix} a_1 & a_5 \\ b_1 & b_3 \end{vmatrix}$$

$$b_3 = \frac{-1}{a_1}\begin{vmatrix} a_0 & a_6 \\ a_1 & a_7 \end{vmatrix} \qquad c_3 = \frac{-1}{b_1}\begin{vmatrix} a_1 & a_7 \\ b_1 & b_4 \end{vmatrix}$$