UNIT-4

(Lecture-4)

Routh-Hurwitz's (Special Cases)

Special cases when applying Routh's Tabulation

Case 1: only the first element in one of the rows of Routh's tabulation is zero

Solution: replace the zero with a small positive constant ε and proceed as before by taking the limit as $\varepsilon \to 0$

$$s^4 + 3s^3 + 4s^2 + 12s + 16 = 0$$

s^4	1	4	16
s^3	3	12	
s^2	$0(\varepsilon)$	16	
s^1	$\dfrac{12\varepsilon - 48}{\varepsilon}$	0	
s^0	16		

when $\varepsilon \to 0$

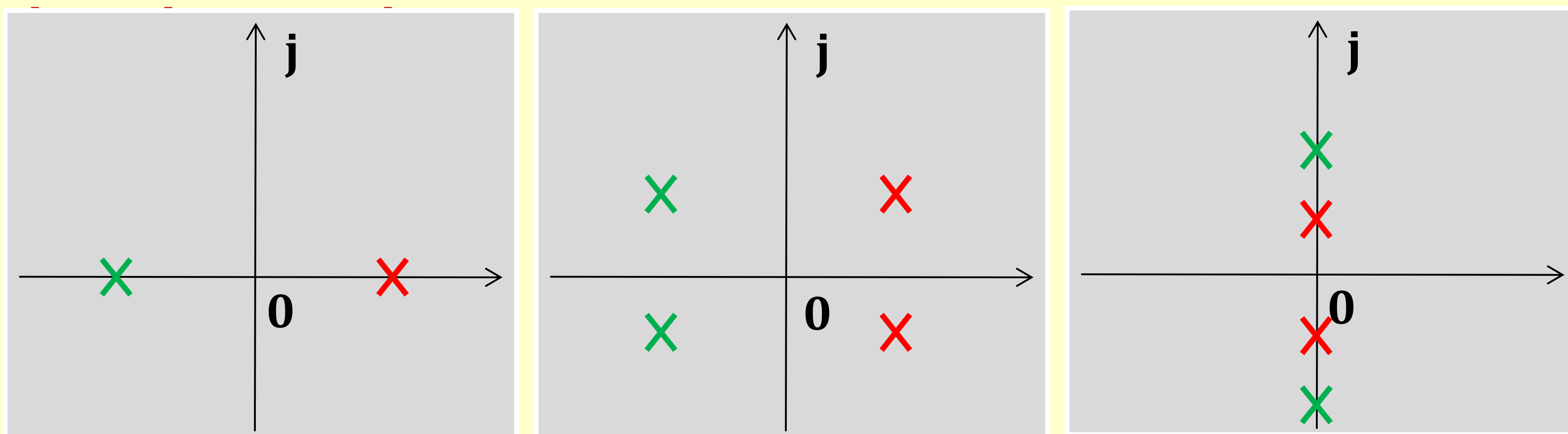
$$\frac{12\varepsilon - 48}{\varepsilon} = 12 - \frac{48}{\varepsilon} < 0$$

The system is unstable and has two roots not in the left-half s-plane.

Case 2: an entire row of Routh's tabulation is zero.

This indicates…

There are complex conjugate pairs of roots that are mirror images of each other with respect to the



e.g. $s_{1,2} = \pm 1$

e.g. $s_{1,2} = -1 \pm j1$

$s_{3,4} = 1 \pm j1$

e.g. $s_{1,2} = \pm 1j$

$s_{3,4} = \pm 2j$

Example

The characteristic equation of a system is：

$$s^5 + 3s^4 + 3s^3 + 9s^2 - 4s - 12 = 0$$

Determine whether there are any roots on the imaginary axis or in the RHP.

s^5	1	3	−4
s^4	3	9	−12
s^3	12	18	0
s^2	9 / 2	−12	
s^1	50	0	
s^0	−12		

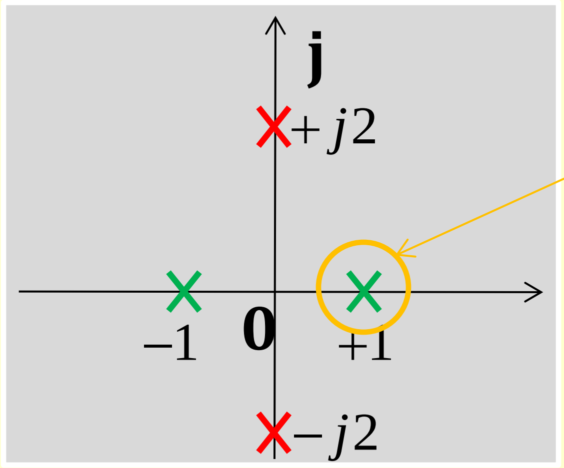
The sign in the first column changes once, so the system is unstable and there is one root outside LHP.

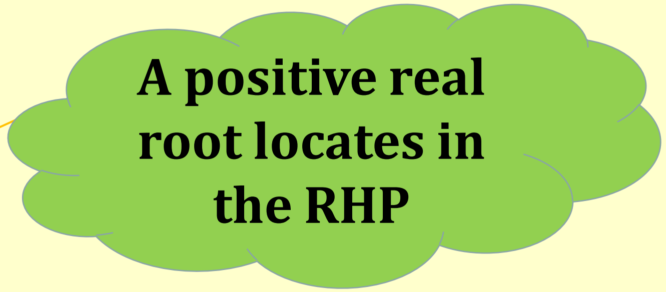
Solving the auxiliary equation

$$A(s) = 3s^4 + 9s^2 - 12 = 0$$

$$s^4 + 3s^2 - 4 = (s^2 - 1)(s^2 + 4) = 0$$

$$s_{1,2} = \pm 1 \qquad s_{3,4} = \pm j2$$





Do yourself

Determine the stability of the following systems:

(1) $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$

(2) $s^5 + 3s^4 + 12s^3 + 24s^2 + 32s + 48 = 0$