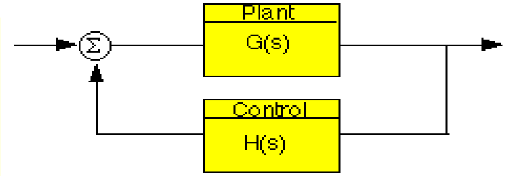
UNIT-5

(Lecture-5)

Nyquist Stability Criterion (Cont…)

Consider the Negative Feedback System:



Remember from the Cauchy criterion that the number N of times that the plot of G(s)H(s) encircles -1 is equal to the number Z of zeros of 1 + G(s)H(s) enclosed by the frequency contour minus the number P of poles of 1 + G(s)H(s) enclosed by the frequency contour (N = Z - P).

Keeping careful track of open- and closed-loop transfer functions, as well as numerators and denominators, you should convince yourself that:

- the zeros of 1 + G(s)H(s) are the poles of the closed-loop transfer function
- the poles of 1 + G(s)H(s) are the poles of the open-loop transfer function.

The Nyquist criterion then states that:

- **P = the number of open-loop (unstable) poles of G(s)H(s)**
- **N = the number of times the Nyquist diagram encircles -1**
- **clockwise encirclements of -1 count as positive encirclements**
- **counter-clockwise (or anti-clockwise) encirclements of -1 count as negative encirclements**
- **Z = the number of right half-plane (positive, real) poles of the closed-loop system**

The important equation which relates these three quantities is:

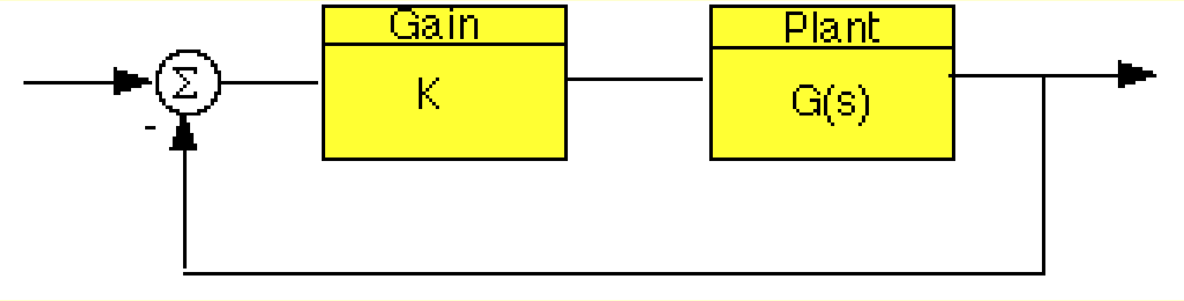
Z = P + N

The Nyquist Stability Criterion - Application

Knowing the number of right-half plane (unstable) poles in open loop (P), and the number of encirclements of -1 made by the Nyquist diagram (N), we can determine the closed-loop stability of the system.

If Z = P + N is a positive, nonzero number, the closed-loop system is unstable.

We can also use the Nyquist diagram to find the range of gains for a closed-loop unity feedback system to be stable. The system we will test looks like this:



where G(s) is :

$$G(s) = \frac{s^2 + 10s + 24}{s^2 - 8s + 15}$$

The Nyquist Stability Criterion

This system has a gain K which can be varied in order to modify the response of the closed-loop system. However, we will see that we can only vary this gain within certain limits, since we have to make sure that our closed-loop system will be stable. This is what we will be looking for: the range of gains that will make this system stable in the closed loop.

The first thing we need to do is find the number of positive real poles in our open-loop transfer function:

roots([1 -8 15]) ***ans =5***
3

The Nyquist Stability Criterion

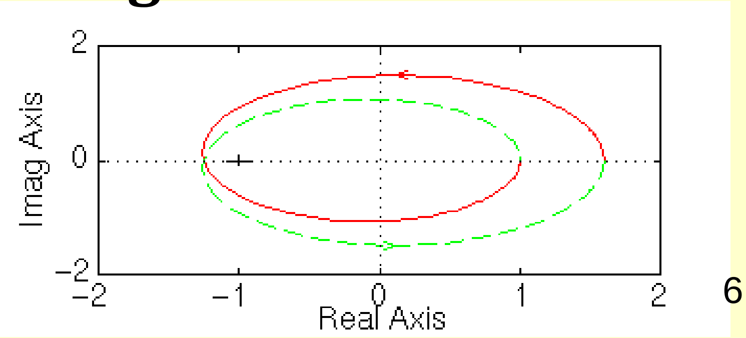
The poles of the open-loop transfer function are both positive. Therefore, we need two anti-clockwise (N = -2) encirclements of the Nyquist diagram in order to have a stable closed-loop system (Z = P + N). If the number of encirclements is less than two or the encirclements are not anti-clockwise, our system will be unstable.

Let's look at our Nyquist diagram for a gain of 1:

nyquist([1 10 24], [1 -8 15])

There are two anti-clockwise encirclements of -1.

Therefore, the system is stable for a gain of 1.

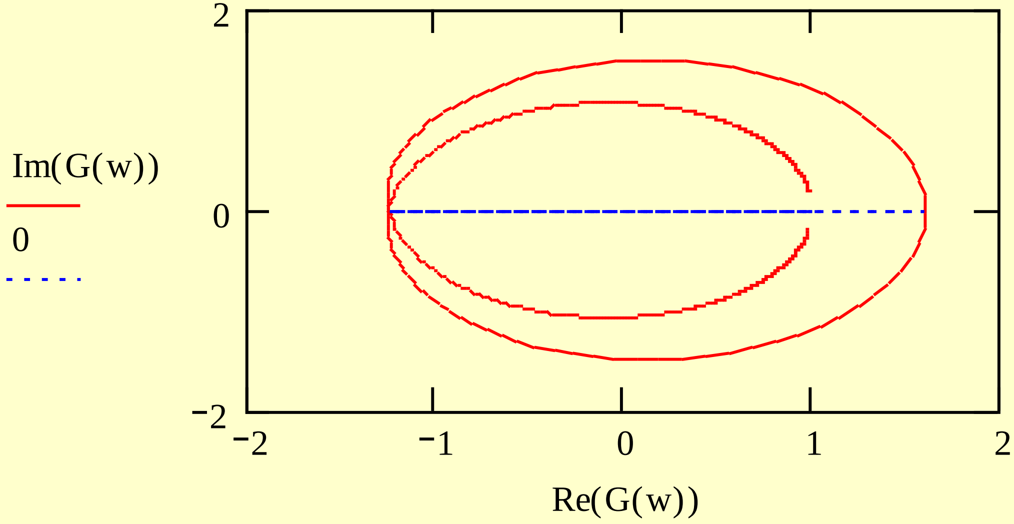


The Nyquist Stability Criterion

MathCAD Implementation

$w := -100, -99.9 .. 100$ $j := \sqrt{-1}$ $s(w) := j \cdot w$

$$G(w) := \frac{s(w)^2 + 10 \cdot s(w) + 24}{s(w)^2 - 8 \cdot s(w) + 15}$$



There are two anti-clockwise encirclements of -1. Therefore, the system is stable for a gain of 1.

The Nyquist Stability Criterion