UNIT-5

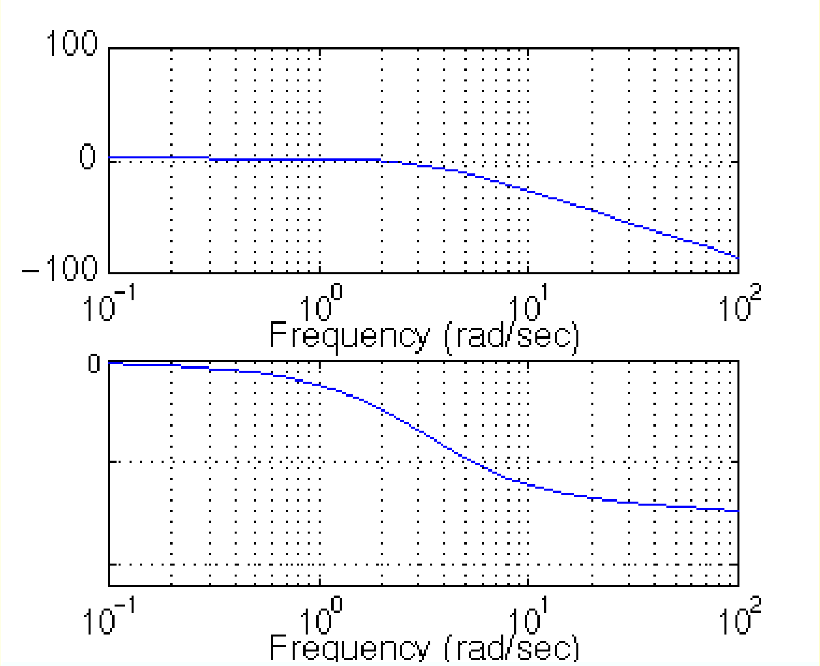
(Lecture-9)

Bode Plots (Cont…)

Bode Plots

Bode plot is the representation of the magnitude and phase of G(j*w) (where the frequency vector w contains only positive frequencies).

To see the Bode plot of a transfer function, you can use the MATLAB bode command.



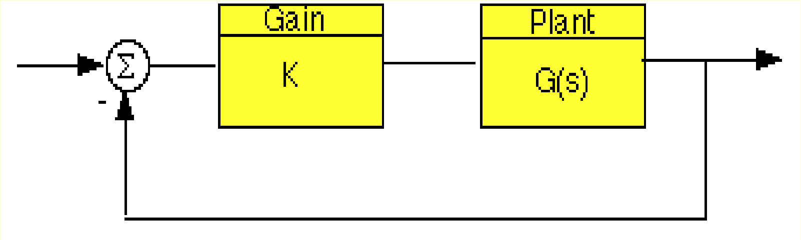
For example,

bode(50,[1 9 30 40])

displays the Bode plots for the transfer function:

50 / (s^3 + 9 s^2 + 30 s + 40)

Gain and Phase Margin

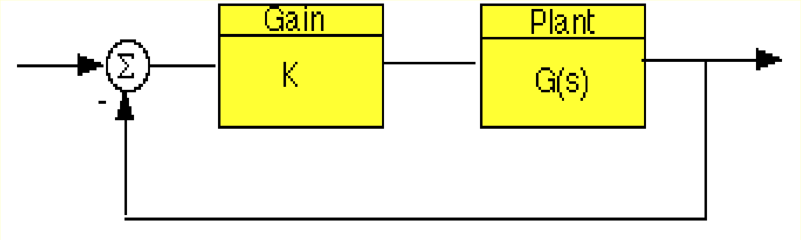


Let's say that we have the following system:

where K is a variable (constant) gain and G(s) is the plant under consideration.

<u>The gain margin is defined as the change in open loop gain required to make the system unstable.</u> Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed loop. Keep in mind that unity gain in magnitude is equal to a gain of zero in dB.

Gain and Phase Margin

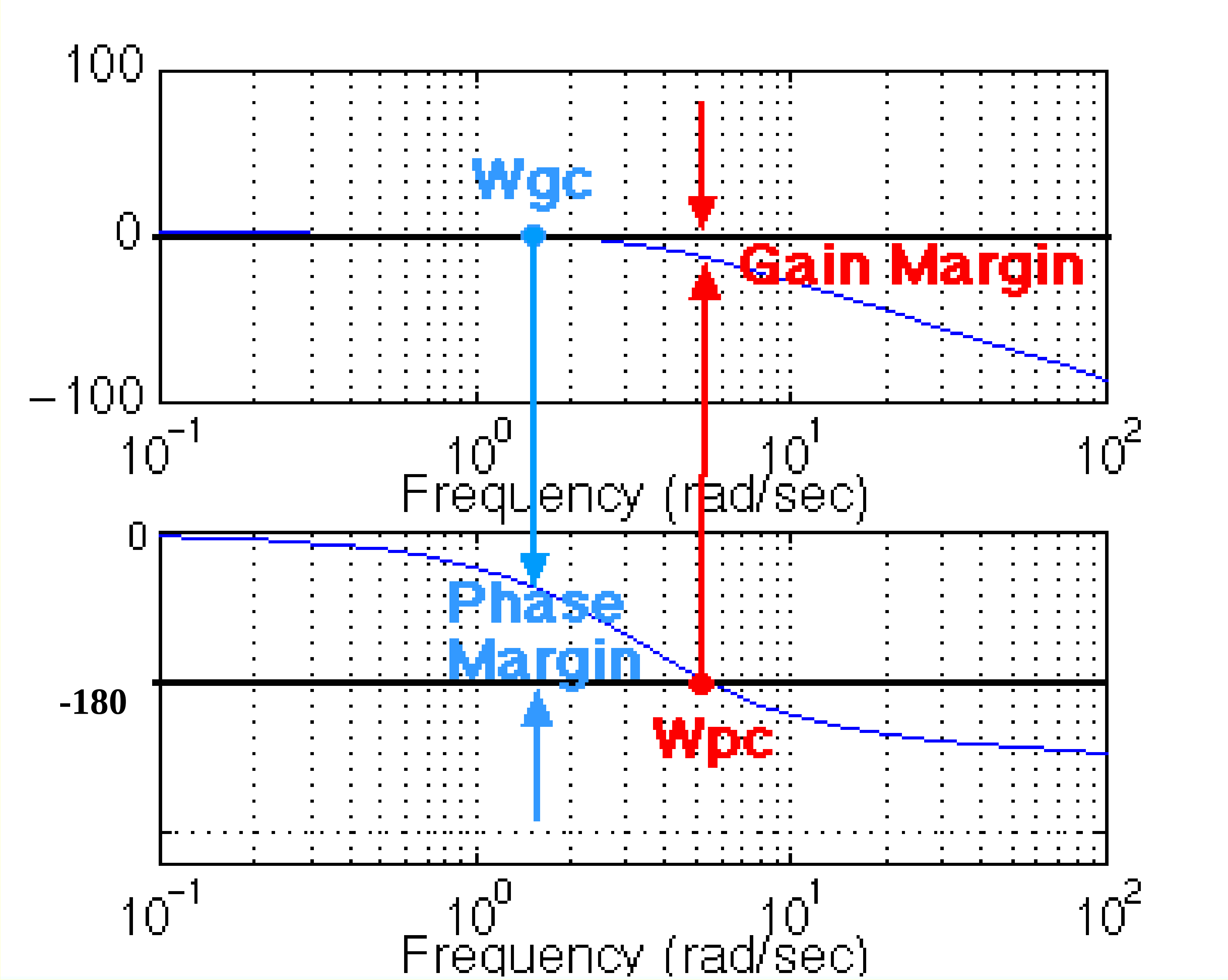


The phase margin is defined as the change in open loop phase shift required to make a closed loop system unstable.

The phase margin is the difference in phase between the phase curve and -180 deg at the point corresponding to the frequency that gives us a gain of 0dB (the gain cross over frequency, Wgc).

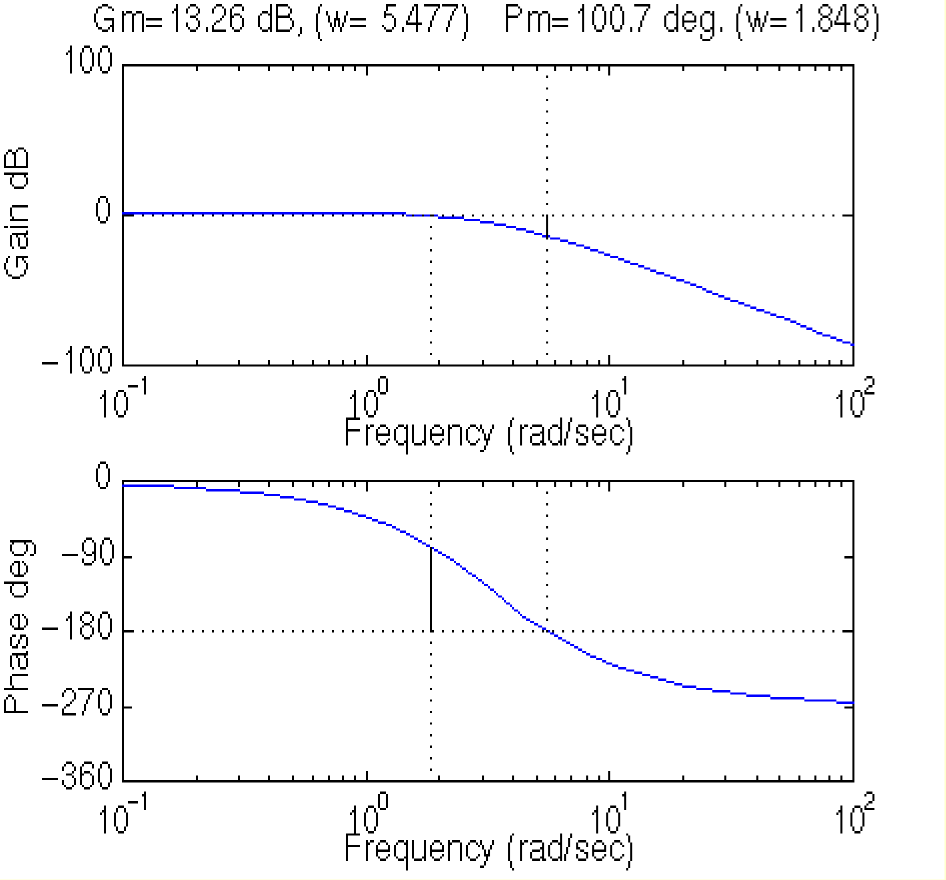
Likewise, the gain margin is the difference between the magnitude curve and 0dB at the point corresponding to the frequency that gives us a phase of -180 deg (the phase cross over frequency, Wpc).

Gain and Phase Margin



Gain and Phase Margin

We can find the gain and phase margins for a system directly, by using MATLAB. Just enter the margin command. This command returns the gain and phase margins, the gain and phase cross over frequencies, and a graphical representation of these on the Bode plot.



margin(50,[1 9 30 40])

Gain and Phase Margin

Magnitude:

$$db(G,\omega) := 20\cdot\log(|G(j\cdot\omega)|)$$

Phase shift:

$$ps(G,\omega) := \frac{180}{\pi}\cdot\arg(G(j\cdot\omega)) - 360\cdot(\text{if}(\arg(G(j\cdot\omega)) \geq 0,1,0))$$

Assume

$K := 2$ $\qquad$ $G(s) := \dfrac{K}{s\cdot(1+s)\cdot\left(1+\dfrac{s}{3}\right)}$

Next, choose a frequency range for the plots (use powers of 10 for convenient plotting):

lowest frequency (in Hz): $\omega_{start} := .01$ number of points: $N := 50$

highest frequency (in Hz): $\omega_{end} := 100$

step size: $r := \log\left(\dfrac{\omega_{start}}{\omega_{end}}\right)\cdot\dfrac{1}{N}$

range for plot: $i := 0..N$ range variable: $\omega_i := \omega_{end}\cdot 10^{i\cdot r}$ $\qquad$ $s_i := j\cdot\omega_i$

Gain and Phase Margin

Guess for **crossover frequency**: $\omega_c := 1$

Solve for the gain crossover frequency:

$$\omega_c := \text{root}\left(\text{db}\left(G, \omega_c\right), \omega_c\right) \qquad \omega_c = 1.193$$

Calculate the **phase margin**

$$pm := ps\left(G, \omega_c\right) + 180 \qquad pm = 18.265 \text{ degrees}$$

Gain Margin

Now using the phase angle plot, estimate the frequency at which the phase shift crosses 180 degr

$$\omega_{gm} := 1.8$$

Solve for **ω** at the phase shift point of 180 degrees:

$$\omega_{gm} := \text{root}\left(ps\left(G, \omega_{gm}\right) + 180, \omega_{gm}\right)$$

$$\omega_{gm} = 1.732$$

Calculate the **gain margin**

$$gm := -\text{db}\left(G, \omega_{gm}\right) \qquad gm = 6.021$$