PRINCIPLES OF COMMUNICATIONS

UNIT-1

LECTURE-2

Analogy between Signal Spaces and Vector Spaces

Consider two vectors V_1 and V_2 as shown in Fig. 1. If V_1 is to be represented in terms of V_2

$$V_1 = C_{12}V_2 + V_e \tag{1}$$

where V_e is the error.

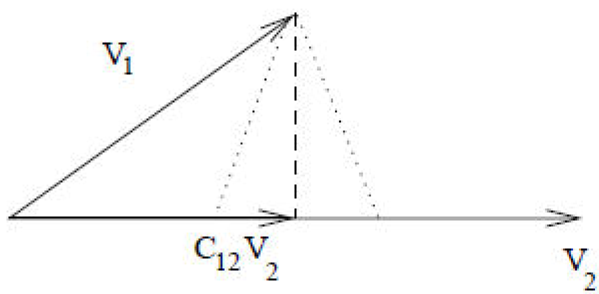

Figure 1: Representation in vector space

The error is minimum when V_1 is projected perpendicularly onto V_2. In this case, C_{12} is computed using dot product between V_1 and V_2.

Component of V_1 along V_2 is

$$= \frac{V_1.V_2}{\|V_2\|} \qquad (2)$$

Similarly, component of V_2 along V_1 is

$$= \frac{V_1.V_2}{\|V_1\|} \qquad (3)$$

Using the above discussion, analogy can be drawn to signal spaces also.

Let $f_1(t)$ and $f_2(t)$ be two real signals. Approximation of $f_1(t)$ by $f_2(t)$ over a time interval $t_1 < t < t_2$ can be given by

$$f_e(t) = f_1(t) - C_{12} f_2(t) \tag{4}$$

where $f_e(t)$ is the error function.

The goal is to find C_{12} such that $f_e(t)$ is minimum over the interval considered. The energy of the error signal ε given by

$$\varepsilon = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_1(t) - C_{12} f_2(t)]^2 \, dt \tag{5}$$

To find C_{12},

$$\frac{\partial \varepsilon}{\partial C_{12}} = 0 \tag{6}$$

Solving the above equation we get

$$C_{12} = \frac{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f_1(t).f_2(t)\, dt}{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f_2^2(t)\, dt} \tag{7}$$

The denominator is the energy of the signal $f_2(t)$.

When $f_1(t)$ and $f_2(t)$ are orthogonal to each other $C_{12} = 0$.

Example: $\sin n\omega_0 t$ and $\sin m\omega_0 t$ be two signals where m and n are integers. When $m \neq n$

$$\int_{\frac{-\pi}{\omega_0}}^{\frac{\pi}{\omega_0}} \sin n\omega_0 t . \sin m\omega_0 t \, dt = 0 \tag{8}$$

Clearly $\sin n\omega_0 t$ and $\sin m\omega_0 t$ are orthogonal to each other.