# PRINCIPLES OF COMMUNICATIONS

UNIT-1

LECTURE-3

## Representation of Signals by a set of Mutually Orthogonal Real Functions

Let $g_1(t), g_2(t), ..., g_n(t)$ be $n$ real functions that are orthogonal to each other over an interval $t_1, t_2$:

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} g_i(t) g_j(t) dt = 0, \quad i \neq j \tag{1}$$

Let

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} g_j(t) g_j(t) dt = K_j \tag{2}$$

$$f(t) = C_1 g_1(t) + C_2 g_2(t) + ... + C_n g_n(t) \tag{3}$$

$$f(t) = \sum_{r=1}^{n} C_r g_r(t) \tag{4}$$

$$f_e(t) = f_(t) - \sum_{r=1}^{n} C_r g_r(t) \tag{5}$$

$$\varepsilon = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f(t) - \sum_{r=1}^{n} C_r g_r(t)]^2 \, dt \tag{6}$$

To find $C_r$,

$$\frac{\partial \varepsilon}{\partial C_1} = \frac{\partial \varepsilon}{\partial C_2} = ... = \frac{\partial \varepsilon}{\partial C_r} = 0 \tag{7}$$

When $\varepsilon$ is expanded we have

$$\varepsilon = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f^{(}t) - 2f(t) \sum_{i=1}^{n} C_r g_r(t) + \sum_{r=1}^{n} C_r g_r(t) \sum_{k=1}^{n} C_k g_k(t)\, dt \tag{8}$$

Now all cross terms disappear

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} C_i g_i(t) C_j g_j(t) dt = 0, \quad i \neq j \tag{9}$$

since $g_i(t)$ and $g_j(t)$ are orthogonal to each other.

Solving the above equation we get

$$C_j = \frac{\frac{1}{t_2-t_1}\int_{t_1}^{t_2} f(t).g_j(t)\,dt}{\frac{1}{t_2-t_1}\int_{t_1}^{t_2} g_j^2(t)\,dt} \tag{10}$$

Analogy to Vector Spaces: Projection of $f(t)$ along the signal $g_j(t) = C_j$

# Representation of Signals by a set of Mutually Orthogonal Complex Functions

When the basis functions are complex. [a]

$$E_x = \int_{t_1}^{t_2} |x(t)|^2 dt \tag{11}$$

represents the energy of a signal.

Suppose $g(t)$ is represented by the complex signal $x(t)$

[a] $|u+v|^2 = (u+v)(u^* + v^*) = |u|^2 + |v|^2 + u^*v + uv^*$

$$
\begin{aligned}
E_e &= \int_{t_1}^{t_2} |g(t) - cx(t)|^2 dt && (12) \\
&= \int_{t_1}^{t_2} |g(t)|^2 dt - \left| \frac{1}{\sqrt{E_x}} \int_{t_1}^{t_2} g(t)x^*(t)dt \right|^2 + && (13) \\
& \left| c\sqrt{E_x} - \frac{1}{\sqrt{E_x}} \int_{t_1}^{t_2} g(t)x^*(t)dt \right|^2 && (14)
\end{aligned}
$$

Minimising the second term yields

$$
c = \frac{1}{E_x} \int_{t_1}^{t_2} g(t)x^*(t)dt \qquad (15)
$$

Thus the coefficients can be determined by projection $g(t)$ along $x^*(t)$.