PRINCIPLES OF COMMUNICATIONS

UNIT-1

LECTURE-4

Fourier Representation of continuous time signals

Any periodic signal $f(t)$ can be represented with a set of complex exponentials as shown below.

$$\begin{aligned} f(t) &= F_0 + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t} + \cdots + F_n e^{jn\omega_n t} + \quad (1) \\ &\cdots + F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + F_{-n} e^{-jn\omega_n t} + \cdots \quad (2) \end{aligned}$$

The exponential terms are orthogonal to each other because

$$\int_{-\infty}^{+\infty} (e^{jn\omega t})(e^{jm\omega t})^* \, dt = 0, \; m \neq n$$

The energy of these signals is unity since

$$\int_{-\infty}^{+\infty} (e^{jn\omega t})(e^{jm\omega t})^* \, dt = 1, \; m = n$$

Representing a signal in terms of its exponential Fourier series components is called *Fourier Analysis.*
The weights of the exponentials are calculated as

$$F_n = \frac{\int_{t_0}^{t_0+T} f(t).(e^{jn\omega_0 t})^* \, dt}{\int_{t_0}^{t_0+T} (e^{jn\omega_0} t).(e^{jn\omega_0} t)^* \, dt}$$

$$= \frac{1}{T} \int_{t_0}^{t_0+T} f(t).(e^{jn\omega_0 t})^* \, dt$$

Extending this representation to aperiodic signals:

When $T \longrightarrow \infty$ and $\omega_0 \longrightarrow 0$, the sum becomes an *integral* and ω_0 becomes continuous.

The resulting represention is termed as the Fourier Transform $(F(\omega))$ and is given by

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}\, dt$$ [a]

The signal $f(t)$ can recovered from $F(\omega)$ as

$$f(t) = \int_{-\infty}^{+\infty} F(\omega)e^{j\omega t}\, d\omega$$ [b]

[a] Analysis equation

[b] Synthesis equation

Some Important Functions

Delta function is a very important signal in signal analysis. It is defined as

$$\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

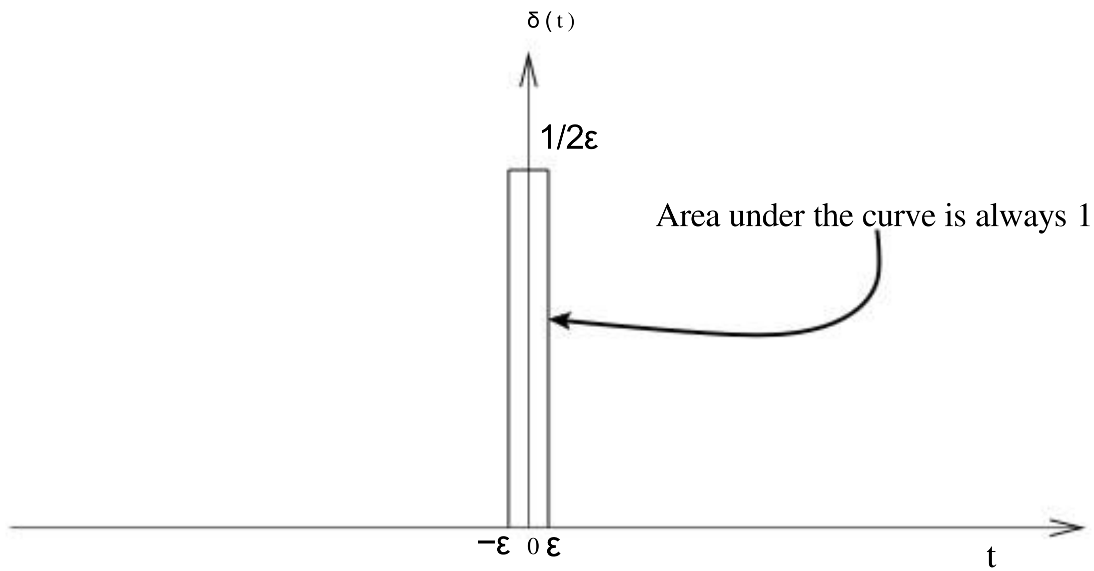

Figure 1: The Dirac delta function

The Dirac delta function is also called the Impulse function. This function can be represented as the limiting function of a number of sampling functions:

1. Gaussian Pulse

$$\delta(t) = \lim_{T \to 0} \frac{1}{T} e^{-\frac{t^2}{\pi T^2}}$$

2. Triangular Pulse

$$\delta(t) = \lim_{T \to 0} \frac{1}{T}\left[1 - \frac{|t|}{T}\right], \quad |t| \le T \tag{3}$$

$$= 0, \quad |t| > T \tag{4}$$

3. Exponential Pulse

$$\delta(t) = \lim_{T \to 0} \frac{1}{2T} e^{-\frac{|t|}{T}}$$

4. Sampling Function

$$\int_{-\infty}^{\infty} \frac{k}{\pi} Sa(kt)\,dt = 1$$

$$\delta(t) = \lim_{k \to \infty} \frac{k}{\pi} Sa(kt)$$

5. Sampling Square function

$$\delta(t) = \lim_{k\to\infty} \frac{k}{\pi} Sa^2(kt)$$

The unit step function is another important function signal processing. It is defined by

$$u(t) = \begin{cases} 1, & t > 0 \\ \frac{1}{2}, & t = 0 \\ 0, & t < 0 \end{cases}$$

The Fourier transform of the unit step can be found only in the limit. Some common Fourier transforms will be discussed.