PRINCIPLES OF COMMUNICATIONS

UNIT-1 LECTURE-5

Fourier Representation of continuous time signals Properties of Fourier Transform^a

• Translation Shifting a signal in time domain introduces linear phase in the frequency domain.

 $f(t) \longleftrightarrow F(\omega)$

$$f(t-t_0) \longleftrightarrow e^{-j\omega t_0} F(\omega)$$

Proof:

 ${}^{\mathbf{a}}\mathcal{F}$ and F^{-1} correspond to the Forward and Inverse Fourier transforms

$$F(\omega) = \int_{-\infty}^{+\infty} f(t - t_0) e^{-j\omega t} dt$$

Put $\tau = t - t_0$

$$F(\omega) = \int_{-\infty}^{+\infty} f(\tau) e^{-j\omega(\tau+t_0)} dt$$
$$= e^{-j\omega t_0} \int_{-\infty}^{+\infty} f(\tau) e^{-j\omega\tau} d\tau \qquad (1)$$

$$= F(\omega)e^{-j\omega t_0} \tag{2}$$

• Modulation A linear phase shift introduced in time domain signals results in a frequency domain.

$$f(t) \longleftrightarrow F(\omega)$$

$$e^{j\omega_0 t}f(t) \longleftrightarrow F(\omega - \omega_0)$$

Proof:

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{j\omega_0 t}e^{-j\omega t} dt$$
$$= \int_{-\infty}^{+\infty} f(t)e^{-j(\omega-\omega_0)t} dt \qquad (3)$$

$$= F(\omega - \omega_0) \tag{4}$$

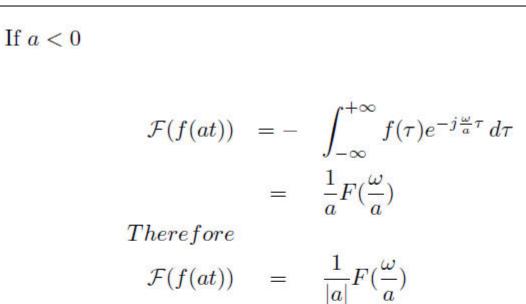
• Scaling Compression of a signal in the time domain results in an expansion in frequency domain and vice-versa.

$$egin{array}{rcl} f(t) & \longleftrightarrow & F(\omega) \ f(at) & \longleftrightarrow & rac{1}{|a|}F(rac{\omega}{a}) \end{array}$$

$$F(\omega) = \int_{-\infty}^{+\infty} f(at)e^{-j\omega t} dt$$

Put $\tau = at$ If a > 0

$$\mathcal{F}(f(at)) = \int_{-\infty}^{+\infty} f(\tau) e^{-j\frac{\omega}{a}\tau} d\tau$$
$$= \frac{1}{a} F(\frac{\omega}{a})$$



• Duality

$$egin{array}{rcl} f(t) & \longleftrightarrow & F(\omega) \ F(t) & \longleftrightarrow & 2\pi f(-\omega) \end{array}$$

Replace t with ω and ω with t in

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t} dt$$
$$F(t) = \int_{-\infty}^{+\infty} f(\omega)e^{-jt\omega} d\omega$$

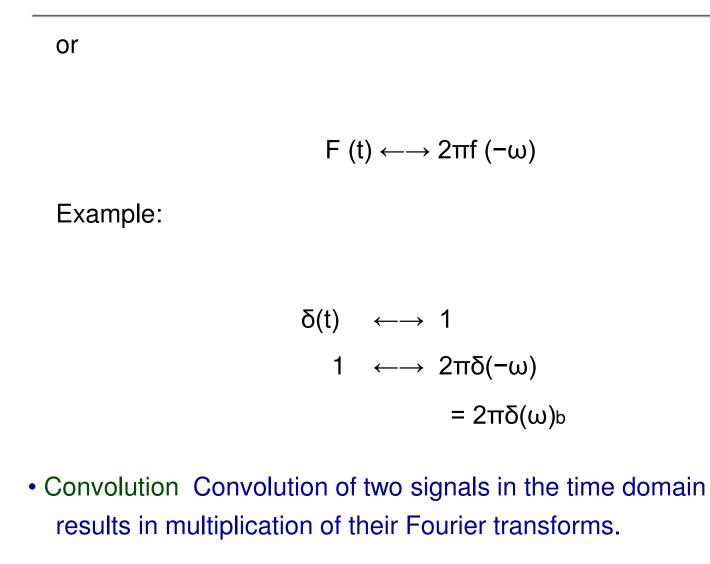
But the inverse Fourier transform of a given FT $f(\omega)$ is

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\omega) e^{j\omega t} \, d\omega$$

Therefore

$$F(t) = 2\pi \mathcal{F}^{-1}(f(-\omega))$$

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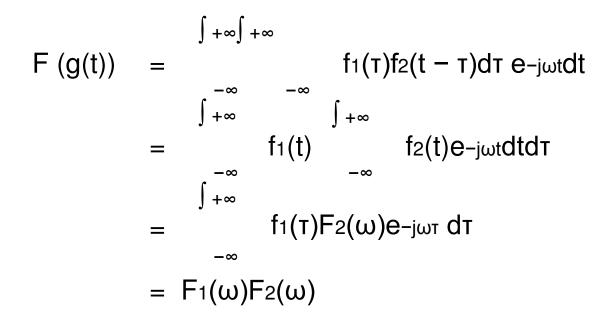


$$f_1(t) * f_2(t) \longleftrightarrow F_1(\omega)F_2(\omega)$$

I.

$$\int g(t) = f_1(t) * f_2(t) = -\infty_{+\infty}f_1(\tau)f_2(t - \tau)d\tau$$

Proof:



 Multiplication Multiplication of two signas in the time domain results in convolution of their Fourier transforms

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$$f_1(t)f_2(t) \longleftrightarrow \frac{1}{2\pi}F_1(\omega)*F_2(\omega)$$

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This can be easily proved using the Duality Property

• Differentiation in time

d dtf(t)←→jωF(ω)

Proof:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega)e_{j\omega t}d\omega$$

Differentiating both sides w.r.t t yields the result.

• Differentiation in Frequency

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This follows from the duality property.

• Integration in time

 $\int_{-\infty}^{t} f(t)dt \longleftrightarrow \frac{1}{j\omega} F(\omega)$

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