PRINCIPLES OF COMMUNICATIONS

UNIT-1

LECTURE-6

Some Example Continuous Fourier transforms

- $\mathcal{F}(\delta(t))$

$$\mathcal{F}(\delta(t)) = \int_{-\infty}^{+\infty} \delta(t) e^{-j\omega t}\, dt$$

Given that

$$\begin{aligned}
\int_{-\infty}^{+\infty} f(t)\delta(t)\, dt &= f(0) \int_{-\infty}^{+\infty} \delta(t)\, dt \\
&= f(0) \\
\int_{-\infty}^{+\infty} f(t)\delta(t-t_0)\, dt &= f(t_0)
\end{aligned}$$

Therefore $\mathcal{F}(\delta(t)) = 1$

' $

- Linearity of the Fourier transform

$$F\,(a_1 f_1(t) + a_2 f_2(t)) = a_1 F_1(\omega) + a_2 F_2(\omega)$$

- F(A), where A is constant Using the duality property and the linearity property of the Fourier transform

$$F\,(A) = 2\pi A \delta(\omega)$$

- Fourier transform of e^{-at}, $t > 0$ (see Figure 1)

& %

$$f(t) = e^{-at}, \; t > 0$$

$$F(\omega) = \int_0^\infty e^{-(a+j\omega)t}\,dt$$

$$= \frac{1}{a+j\omega}$$

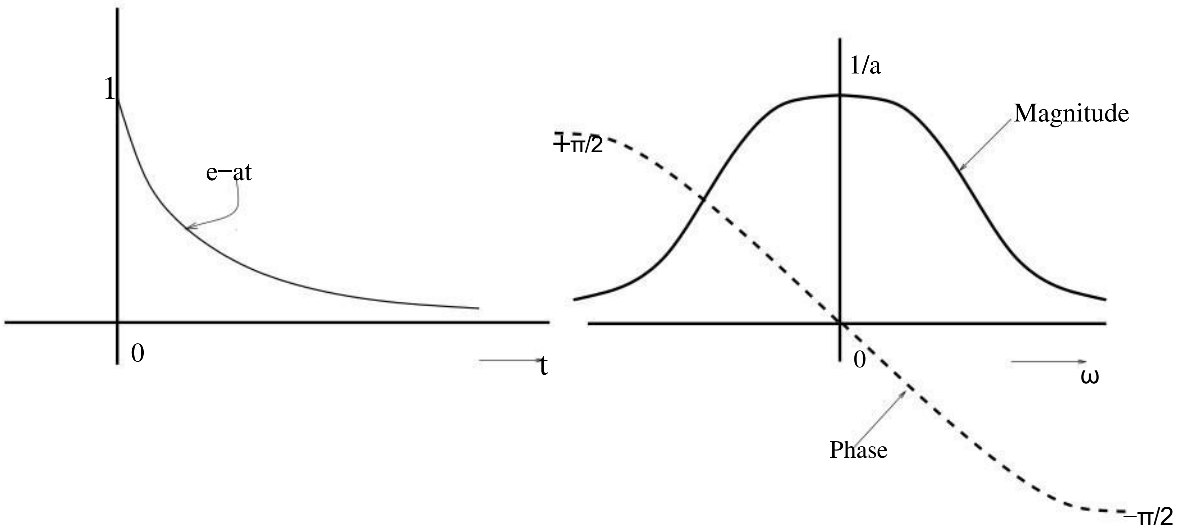

Figure 1: The exponential function and its Fourier transform

- Fourier transform of the unit step function
  The Fourier transform of the unit step function can be obtained only in the limit

$$
\begin{aligned}
\mathcal{F}(u(t)) &= \lim_{a \to 0} \mathcal{F}(e^{-at}) \\
&= \frac{1}{j\omega}
\end{aligned}
$$

- Fourier transform of $e^{-a|t|}$(see Figure 2)

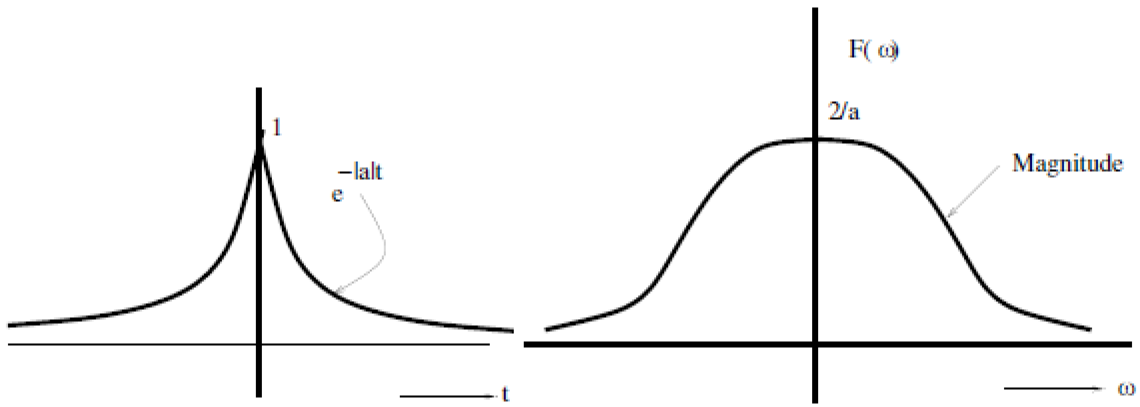

Figure 2: $e^{-|a|t}$ and its Fourier transform

$$
\begin{aligned}
f(t) &= e^{-at},\ t > 0 \\
f(t) &= 1,\ t = 0 \\
&= e^{at},\ t < 0 \\
F(\omega) &= = \int_0^{\infty} e^{-(a+j\omega)t} dt \int_0^{-\infty} e^{-(a-j\omega)t} dt \\
&= \frac{1}{a + j\omega} + \frac{1}{a - j\omega}
\end{aligned}
$$

- Fourier transform of the rectangular function

$$
\begin{aligned}
f(t) &= A, -T/2 \leq t \leq T/2 \\
&= 0,\ otherwise
\end{aligned}
$$

$$
\begin{aligned}
F(\omega) &= \int_{-\frac{T}{2}}^{+\frac{T}{2}} Ae^{-j\omega t}\, dt \\
&= A\frac{e^{-j\omega T/2} - e^{+j\omega T/2}}{-j\omega} \\
&= AT\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \\
&= sinc(\frac{\omega T}{2})
\end{aligned}
$$

The rectangular function $rect(t)$ and its Fourier transform $F(\omega)$ are shown in Figure 3

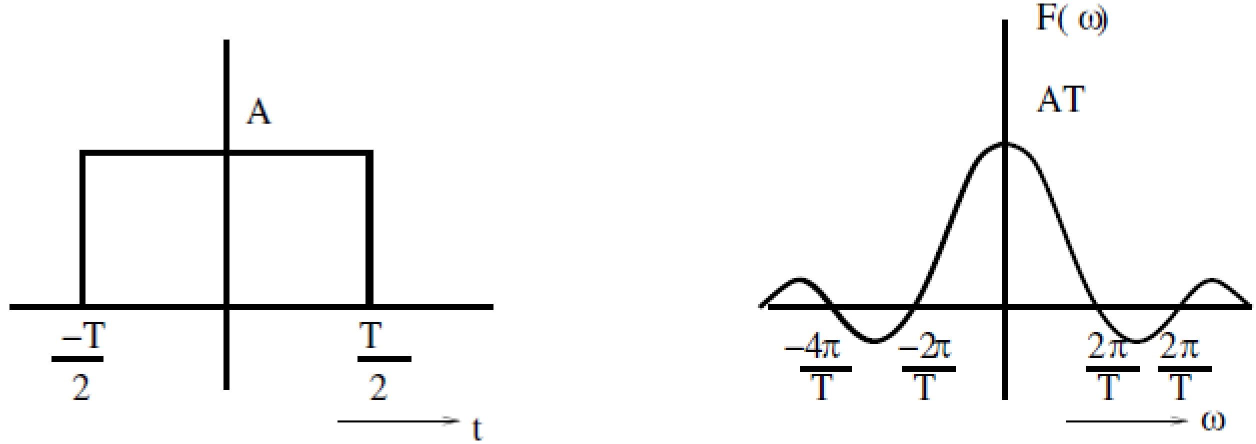

Figure 3: rect(t) and its Fourier transform

- Fourier transform of the *sinc* function
  - Using the duality property, the Fourier transform of the *sinc* function can be determined (see Figure 4).

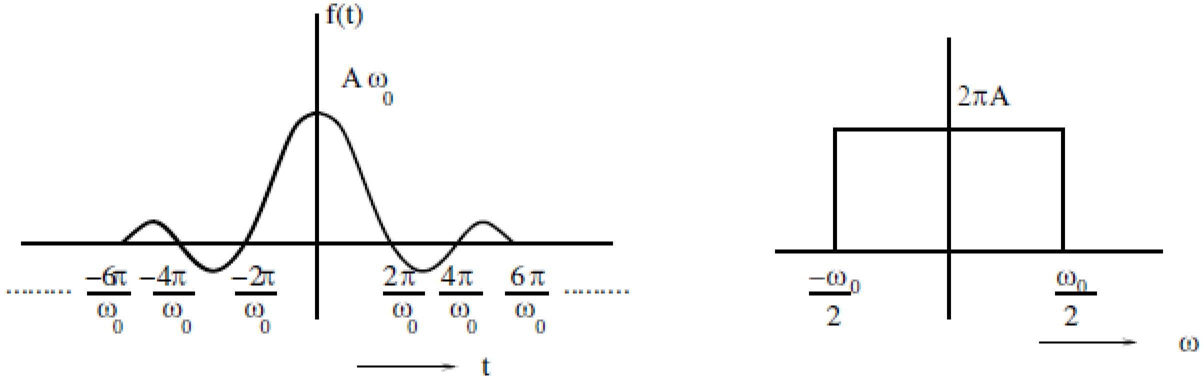

Figure 4: sinc(t) and its Fourier transform

– An important point is that a signal that is "bandlimited" is not "time-limited" while a signal that is "time-limited" is not "bandlimited"