PRINCIPLES OF COMMUNICATIONS

UNIT-1

LECTURE-7

Continuous Fourier transforms of Periodic Functions

- Fourier transform of $e^{jn\omega_0 t}$ Using the frequency shifting property of the Fourier transform

$$
\begin{aligned}
e^{jn\omega_0 t} &= 1.e^{jn\omega_0 t} \\
\mathcal{F}(e^{jn\omega_0 t}) &= \mathcal{F}(1)\ shifted\ by\ \omega_0 \\
&= 2\pi\delta(\omega - n\omega_0)
\end{aligned}
$$

- Fourier transform of $\cos\omega_0 t$

' $

$$\cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$F(e^{j\omega_0 t}) = F(1) \text{ shifted by } \omega_0$$

$$= 2\pi\delta(\omega - \omega_0)$$

$$F(\cos \omega_0 t) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

- Fourier transform of a periodic function $f(t)$
  - The periodic function is not absolutely summable.
  - The Fourier transform can be represented by a Fourier series.
  - The Fourier transform of the Fourier series representation of the periodic function (period T) can be computed

& %

$$
\begin{aligned}
f(t) &= \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}, \omega_0 = \frac{2\pi}{T}
\end{aligned}
$$

$$
F_n = \frac{1}{T}
$$

$$
\begin{aligned}
\mathcal{F}(f(t)) &= \frac{1}{T}\mathcal{F}(\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}) \\
&= \sum_{i=-\infty}^{\infty} F_n \mathcal{F}(e^{jn\omega_0 t}) \\
&= 2\pi\frac{1}{T}\sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) \\
&= \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)
\end{aligned}
$$

Note: A periodic train of impulses results in a Fourier

$$
\begin{aligned}
f(t) &= \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}, \omega_0 = \frac{2\pi}{T} \\
\mathcal{F}(f(t)) &= \mathcal{F}(\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}) \\
&= \sum_{i=-\infty}^{\infty} F_n \mathcal{F}(e^{jn\omega_0 t}) \\
&= 2\pi \sum_{n=-\infty}^{\infty} F_n \delta(\omega - n\omega_0)
\end{aligned}
$$

Note: The Fourier transform is made up of components at discrete frequencies.

- Fourier transform of a periodic function
  $f(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ (a periodic train of impulses)

transform which is also a periodic train of impulses (see Figure 1) .

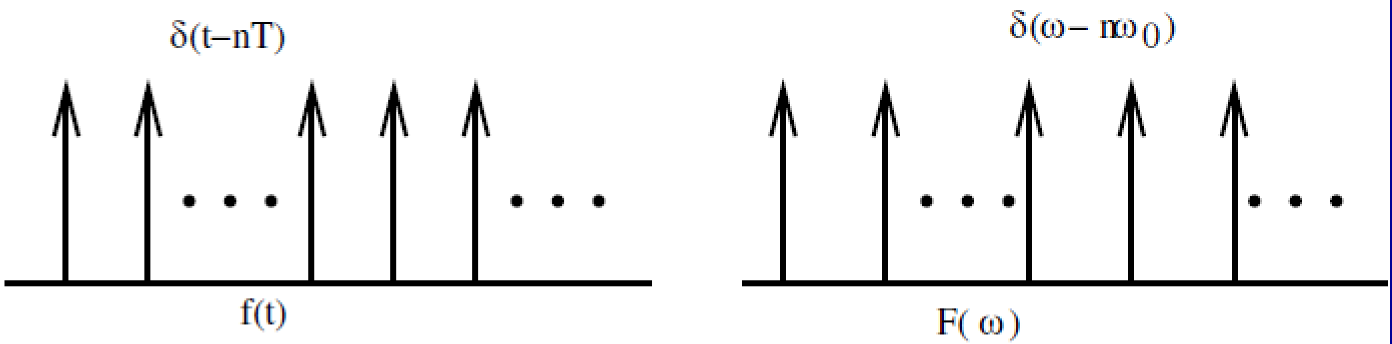

Figure 1: The periodic pulse train and its Fourier transform