PRINCIPLES OF COMMUNICATIONS

UNIT-1

LECTURE-8

Sampling Theorem and its Importance

- Sampling Theorem:

  “A bandlimited signal can be reconstructed exactly if it is sampled at a rate atleast twice the maximum frequency component in it.”

  Figure 1 shows a signal g(t) that is bandlimited.

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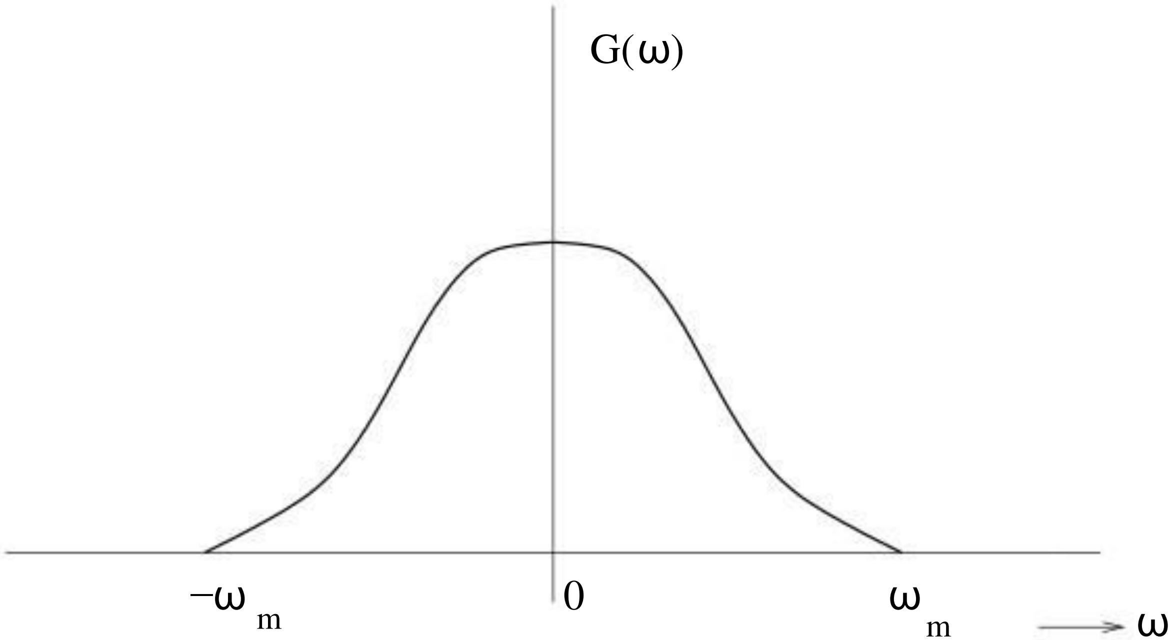

Figure 1: Spectrum of bandlimited signal g(t)

- The maximum frequency component of g(t) is f_m. To recover the signal g(t) exactly from its samples it has to be sampled at a rate $f_s \geq 2f_m$.
- The minimum required sampling rate $f_s = 2f_m$ is called

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Nyquist rate.

Proof: Let $g(t)$ be a bandlimited signal whose bandwidth is f_m ($\omega_m = 2\pi f_m$).

Figure 2: (a) Original signal $g(t)$ (b) Spectrum $G(\omega)$

$\delta_T(t)$ is the sampling signal with $f_s = 1/T > 2f_m$.

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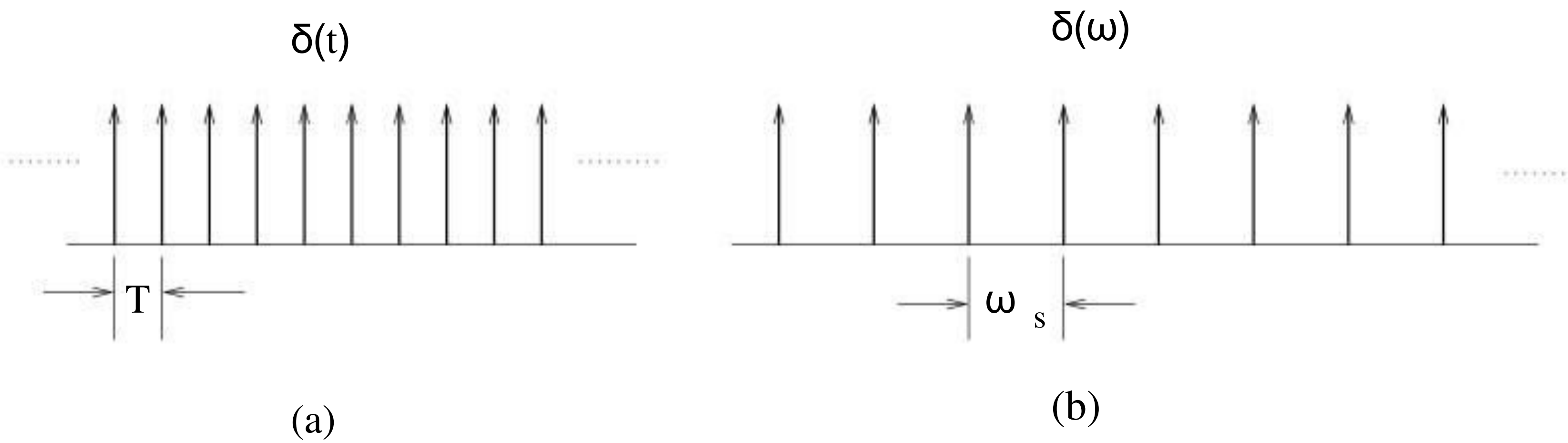

Figure 3: (a) sampling signal δ_T (t) (b) Spectrum δ_T (ω)

- Let $g_s(t)$ be the sampled signal. Its Fourier Transform $G_s(\omega)$ is given by

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$$\mathcal{F}(g_s(t)) = \mathcal{F}[g(t)\delta_T(t)]$$

$$= \mathcal{F}\left[g(t) \sum_{n=-\infty} \delta(t - nT)\right]$$

$$= \frac{1}{2\pi}\left[G(\omega) * \omega_0 \sum_{n=-\infty} \delta(\omega - n\omega_0)\right]$$

$$G_s(\omega) = \frac{1}{T} \sum_{n=-\infty} G(\omega) * \delta(\omega - n\omega_0)$$

$$G_s(\omega) = \mathcal{F}[g(t) + 2g(t)\cos(\omega_0 t) + 2g(t)\cos(2\omega_0 t) + \cdots]$$

$$G_s(\omega) = \frac{1}{T} \sum_{n=-\infty} G(\omega - n\omega_0)$$

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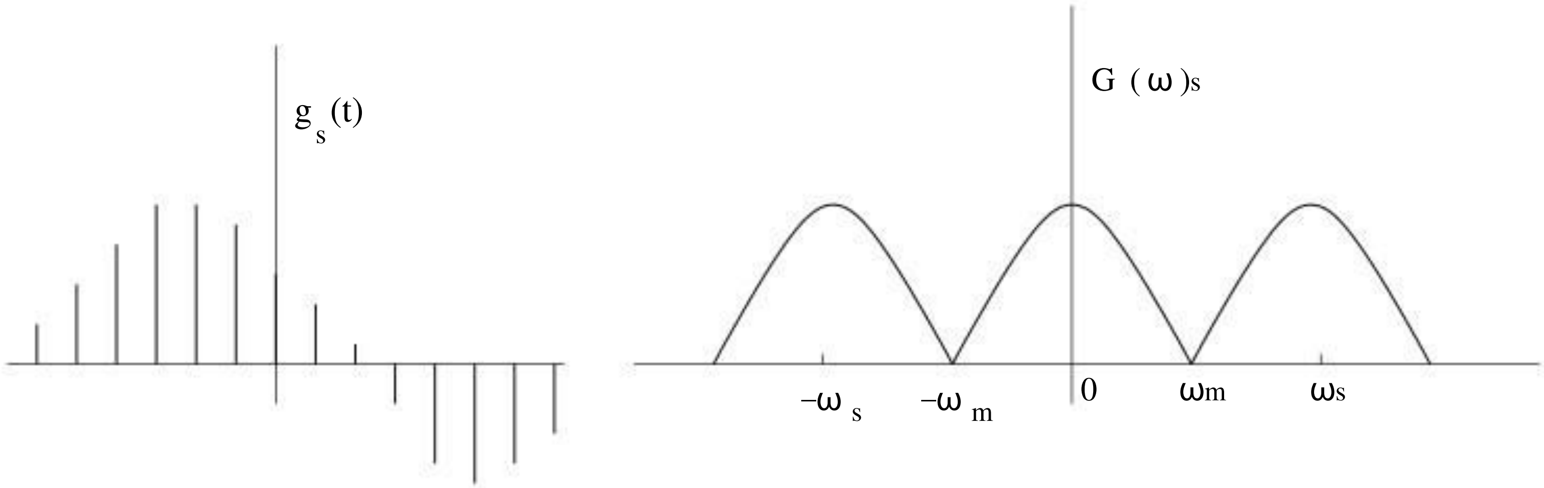

Figure 4: (a) sampled signal $g_s(t)$ (b) Spectrum $G_s(\omega)$

- If $\omega_s = 2\omega_m$, i.e., $T = 1/2f_m$. Therefore, $G_s(\omega)$ is given by

$$G_s(\omega) = \frac{1}{T} \sum_{n=-\infty} G(\omega - n\omega_m)$$

- To recover the original signal $G(\omega)$:

  1. Filter with a Gate function, $H_{2\omega_m}(\omega)$ofwidth$2\omega_m$.

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2. Scale it by T .

$$G(\omega) = T\, G_s(\omega) H_{2\omega_m}(\omega).$$

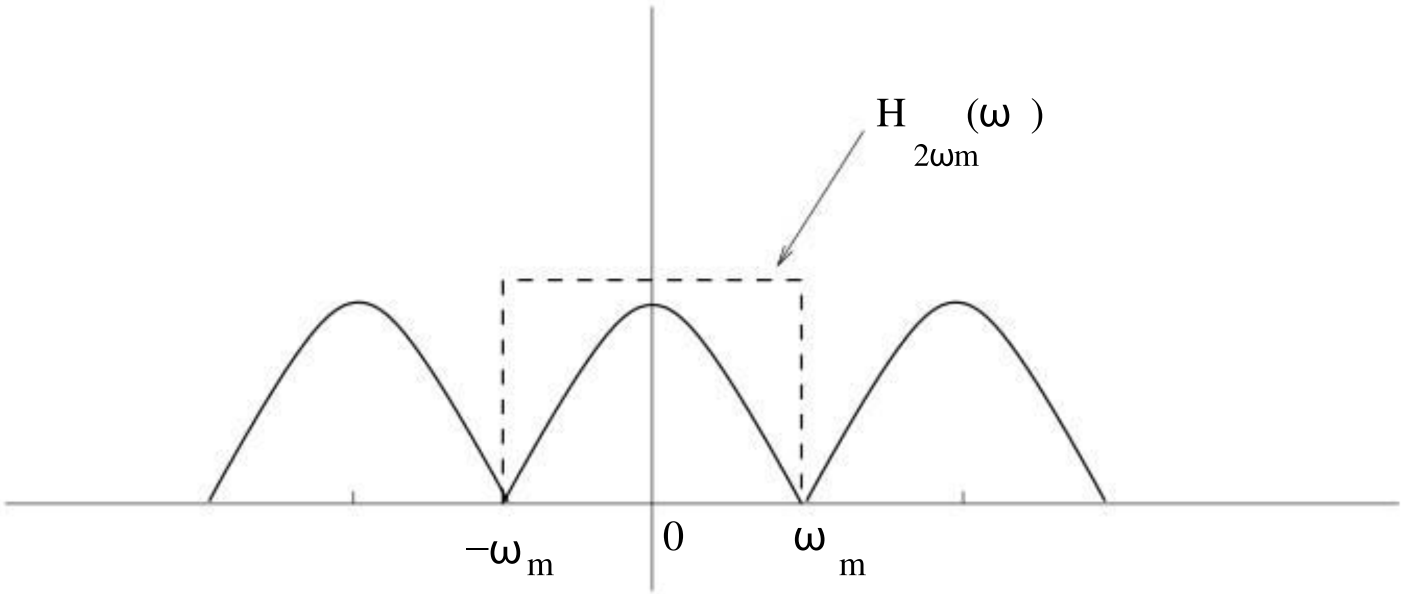

Figure 5: Recovery of signal by filtering with a filter of width $2\omega_m$

- Aliasing
  - Aliasing is a phenomenon where the high frequency components of the sampled signal interfere with each other

because of inadequate sampling $\omega_s < 2\omega_m$.

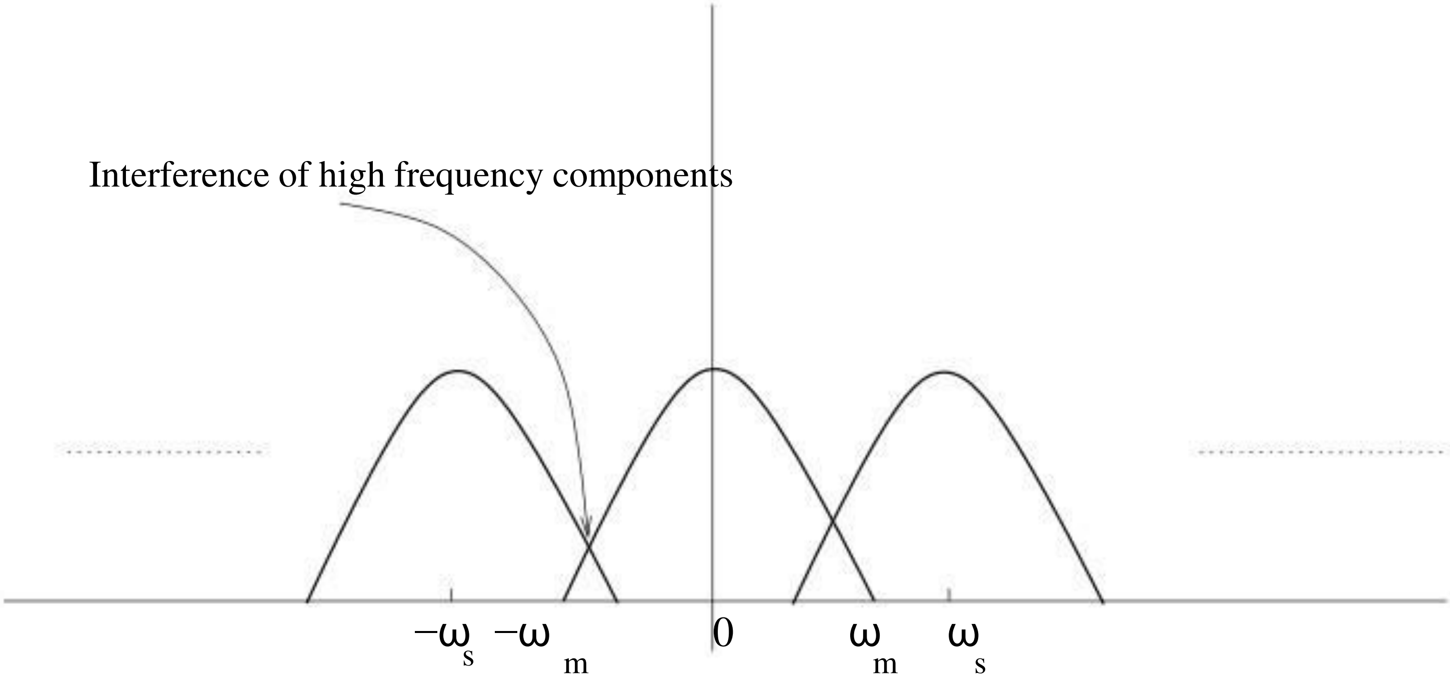

Figure 6: Aliasing due to inadequate sampling

Aliasing leads to distortion in recovered signal. This is the reason why sampling frequency should be atleast twice the bandwidth of the signal.

- Oversampling
  - In practice signal are oversampled, where f_s is significantly higher than Nyquist rate to avoid aliasing.

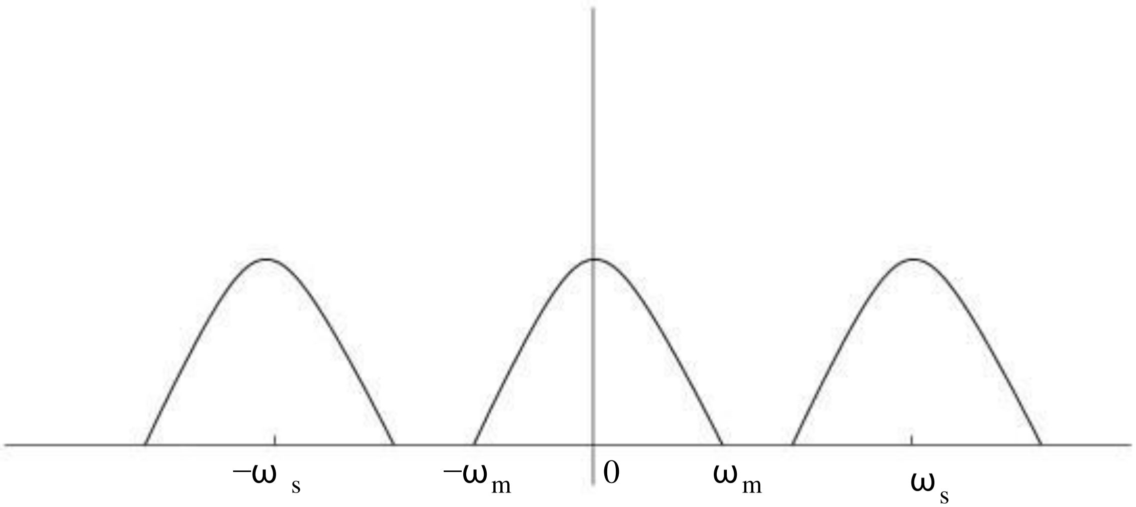

Figure 7: Oversampled signal-avoids aliasing

Problem: Define the frequency domain equivalent of the Sampling Theorem and prove it.