PRINCIPLES OF COMMUNICATIONS

UNIT-1

LECTURE-9

Discrete-Time Signals and their Fourier Transforms

Generation of Discrete-time signals

- Discrete time signals are obtained by sampling a continuous time signal.
- The continuous time signal is sampled with an impulse train with sampling period T
- which is usually taken greaterthan or equal to Nyquist Rate to avoid Aliasing.

The Discrete-Time Fourier Transform (DTFT) of a discrete time signal $g(nT)$is represented by[a]

$$G(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} g(nT).e^{-j\omega nT}$$

[a] $G(e^{j\omega})$ represents the DTFT. It signifies the periodicity of the DTFT

In practice, it is assumed that signals are adequately sampled and hence T is dropped to yield:

$$G(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} g(n).e^{-j\omega n}$$

The inverse DTFT is given by:

$$g(n) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} G(\omega).e^{j\omega n} d\omega$$

Some important Discrete-Time Signals

- Discrete time impulse or unit sample function Unit sample function is similar to impulse function in continuous time (see Figure 1. It is defined as follows

$$\delta(n) = \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{elsewhere} \end{cases}$$

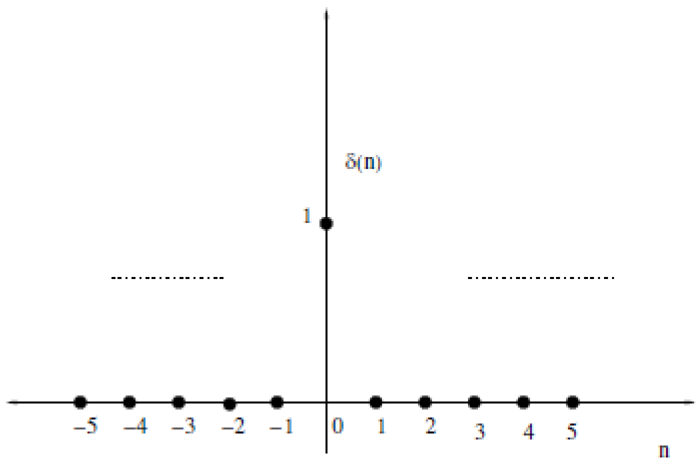

Figure 1: The unit sample function

- Unit step function This is similar to unit step function in continuous time domain (see Figure 2) and is defined as follows

$$u(n) = \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

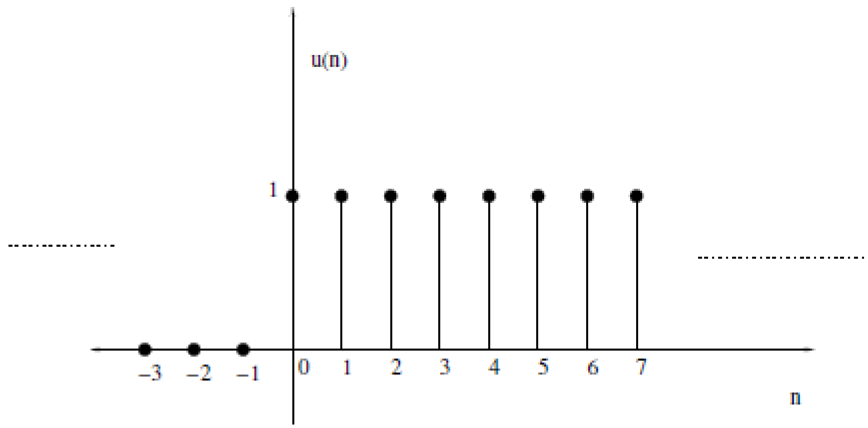

Figure 2: The unit step function

Properties of the Discrete-time Fourier transform

- Time shift property [a]

$$\mathcal{F}(f(n-n_0)) \longleftrightarrow F(e^{j\omega})e^{-j\omega n_0}$$

- Modulation property

$$\mathcal{F}(f(n)e^{-j\omega_0 n}) \longleftrightarrow F(e^{j(\omega-\omega_0)})$$

- Differentiation in the frequency domain

$$\mathcal{F}(nf(n)) \longleftrightarrow \frac{dF(e^{j\omega})}{d\omega}$$

- Convolution in the time domain

$$\mathcal{F}(f(n) * g(n)) \longleftrightarrow \frac{1}{2\pi}F(e^{j\omega})G(e^{j\omega})$$

- Prove that the forward and inverse DTFTs form a pair

[a] A tutorial on this would be appropriate

$$
\begin{aligned}
F(e^{j\omega}) &= \sum_{n=-\infty}^{+\infty} f(n).e^{-j\omega n} \\
f(n) &= \frac{1}{2\pi}\int_{-\pi}^{+\pi} F(\omega).e^{j\omega n}d\omega \\
f(n) &= \frac{1}{2\pi}\int_{-\pi}^{+\pi} \sum_{l=-\infty}^{+\infty} f(l).e^{-j\omega l}e^{j\omega n}d\omega \\
&= \sum_{l=-\infty}^{+\infty} f(l).\frac{1}{2\pi}\int_{-\pi}^{+\pi})e^{-j(\omega l-\omega n)}d\omega \\
f(n) &= \sum_{l=-\infty}^{+\infty} f(l)\delta(l-n) \\
&= f(n)
\end{aligned}
$$