PRINCIPLES OF COMMUNICATIONS

UNIT-1

LECTURE-10

Z-transforms

Computation of the Z-transform for discrete-time signals:

- Enables analysis of the signal in the frequency domain.
- Z - Transform takes the form of a polynomial.
- Enables interpretation of the signal in terms of the roots of the polynomial.
- z^{-1} corresponds to a delay of one unit in the signal.

The Z - Transform of a discrete time signal $x[n]$ is defined as

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n].z^{-n} \tag{1}$$

where $z = r.e^{j\omega}$

The discrete-time Fourier Transform (DTFT) is obtained by evaluating Z-Transform at $z = e_{j\omega}$.

or

The DTFT is obtained by evaluating the Z-transform on the unit circle in the z-plane.

The Z-transform converges if the sum in equation 1 converges

Region of Convergence(RoC)

Region of Convergence for a discrete time signal x[n] is defined as a continuous region in z plane where the Z-Transform converges.

In order to determine RoC, it is convenient to represent the Z-Transform as:[a]

$$X(z) = \frac{P(z)}{Q(z)}$$

- The roots of the equation P (z) = 0 correspond to the 'zeros' of X(z)
- The roots of the equation Q(z) = 0 correspond to the 'poles' of X(z)
- The RoC of the Z-transform depends on the convergence of the

[a] Here we assume that the Z-transform is rational

polynomials $P(z)$ and $Q(z)$,

- Right-handed Z-Transform
  - Let $x[n]$ be causal signal given by

$$x[n] = a^n u[n]$$

  - The Z - Transform of $x[n]$ is given by

$$
\begin{aligned}
X(z) &= \sum_{n=-\infty}^{+\infty} x[n]z^{-n} \\
&= \sum_{n=-\infty}^{+\infty} a^n u[n]z^{-n} \\
&= \sum_{n=0}^{+\infty} a^n z^{-n} \\
&= \sum_{n=0}^{+\infty} (az^{-1})^n \\
&= \frac{1}{1-az^{-1}} \\
&= \frac{z}{z-a}
\end{aligned}
$$

– The ROC is defined by $|az^{-1}| < 1$ or $|z| > |a|$.

– The RoC for $x[n]$ is the entire region outside the circle $z = ae^{j\omega}$ as shown in Figure 1.

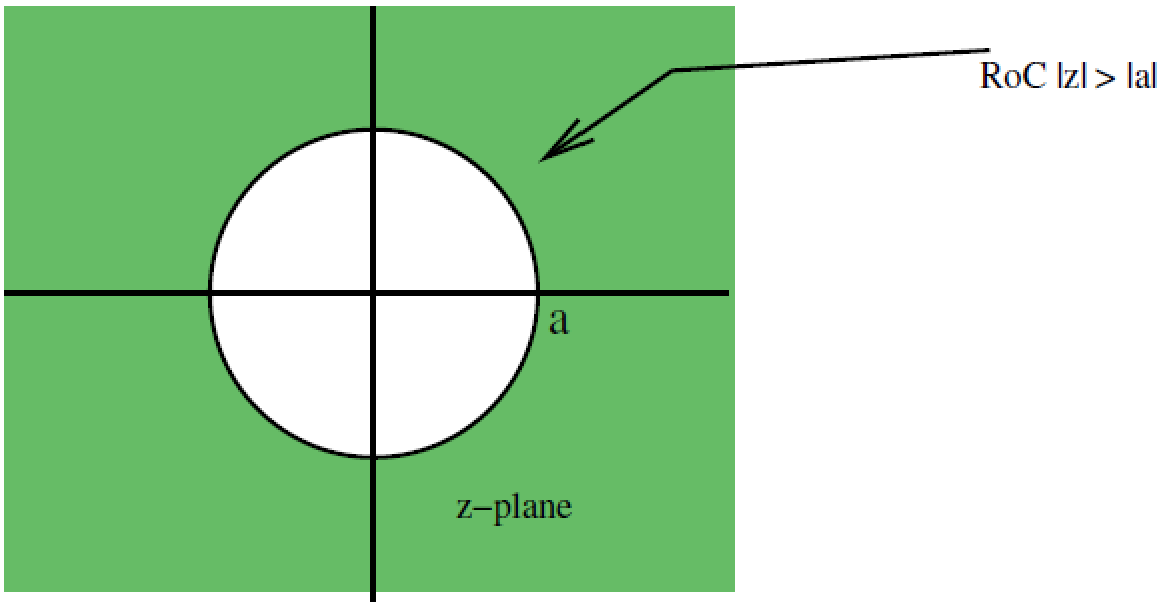

Figure 1: RoC(green region) for a causal signal

- Left-handed Z-Transform

– Let $x[n]$ be an anti-causal signal given by

$$y[n] = -b^n u[-n-1]$$

– The Z - Transform of $y[n]$ is given by

$$
\begin{aligned}
Y(z) &= \sum_{n=-\infty}^{+\infty} y[n] z^{-n} \\
&= \sum_{n=-\infty}^{+\infty} -b^n u[-n-1] z^{-n} \\
&= \sum_{n=-\infty}^{-1} -b^n z^{-n} \\
&= \sum_{n=0}^{+\infty} -(b^{-1}z)^n + 1 \\
&= \frac{1}{1-\frac{z}{b}} + 1 \\
&= \frac{z}{z-b}
\end{aligned}
$$

– $Y(z)$ converges when $|b^{-1}z < 1$ or $|z| < |b|$.

– The RoC for y[n] is the entire region inside the circle $z = be^{j\omega}$ as shown in Figure 2

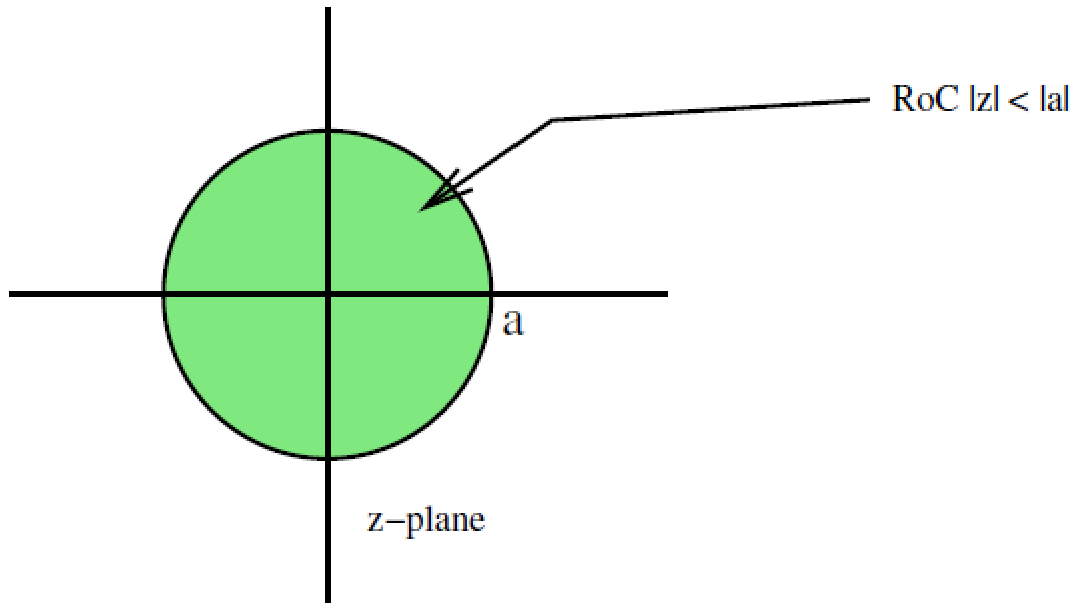

Figure 2: RoC(green region) for an anti-causal signal