PRINCIPLES OF COMMUNICATIONS

UNIT-2

LECTURE-3

Basics of Probability Theory and Random Processes

Basics of probability theory [a]

- Probability of an event E represented by $P(E)$ and is given by

$$P(E) = \frac{N_E}{N_S} \tag{1}$$

where, N_S is the number of times the experiment is performed and N_E is number of times the event E occured.

Equation 1 is only an approximation. For this to represent the exact probability $N_S \to \infty$.

The above estimate is therefore referred to as *Relative Probability*

Clearly, $0 \leq P(E) \leq 1$.

[a] required for understanding communication systems

- Mutually Exclusive Events

  Let S be the sample space having N events $E_1, E_2, E_3, \cdots, E_N$.

  Two events are said to be mutually exclusive or statistically independent if $A_i \cap A_j = \varphi$ and $\bigcup_{i=1} A_i = S$ for all i and j.

- Joint Probability

  Joint probability of two events A and B represented by $P(A \cap B)$ and is defined as the probability of the occurence of both the events A and B is given by

$$P(A \cap B) = \frac{N_{A \cap B}}{N_S}$$

- Conditional Probability

  Conditional probability of two events A and B represented as

P (A|B) and defined as the probability of the occurence of event A after the occurence of B.

$$P(A|B) = \frac{N_{A \cap B}}{N_B}$$

Similarly,

$$P(B|A) = \frac{N_{A \cap B}}{N_B}$$

This implies,

$$P(B|A)P(A) = P(A|B)P(B) = P(A \cap B)$$

- Chain Rule

  Let us consider a chain of events A_1, A_2, A_3, $\cdots$, A_N which are dependent on each other. Then the probability of occurence of the sequence

$$
\begin{aligned}
P(A_N, A_{N-1}, A_{N-2}, \\
\cdots, A_2, A_1) \\
&= P(A_N | A_{N-1}, A_{N-2}, \cdots, A_1). \\
&\quad P(A_{N-1} | A_{N-2}, A_{N-3}, \cdots, A_1). \\
&\quad \cdots .P(A_2 | A_1).P(A_1)
\end{aligned}
$$

Bayes Rule

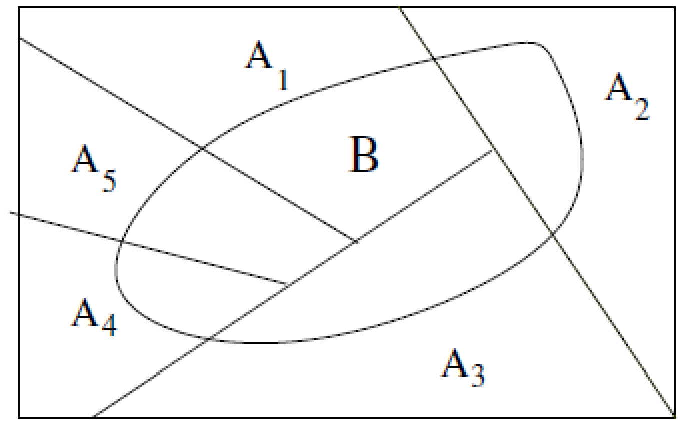

Figure 1: The partition space

In the above figure, if A_1, A_2, A_3, A_4, A_5 partition the sample space $\mathcal{S}$, then $(A_1 \cap B), (A_2 \cap B), (A_3 \cap B), (A_4 \cap B)$, and $(A_5 \cap B)$ partition B.

Therefore,

$$\begin{aligned} P(B) &= \sum_{i=1}^{n} P(A_i \cap B) \\ &= \sum_{i=1}^{n} P(B|A_i).P(A_i) \end{aligned}$$

In the example figure here, $n = 5$.

$$\begin{aligned} P(A_i|B) &= \frac{P(A_i \cap B)}{P(B)} \\ &= \frac{P(B|A_i).P(A_i)}{\sum_{i=1}^{n} P(B|A_i).P(A_i)} \end{aligned}$$

In the above equation, $P(A_i|B)$ is called posterior probability,

$P(B|A_i)$ is called likelihood, $P(A_i)$ is called prior probability and $\sum_{i=1}^{n} P(B|A_i).P(A_i)$ is called evidence.

Random Variables

Random variable is a function whose domain is the sample space and whose range is the set of real numbers Probabilistic description of a random variable

- Cummulative Probability Distribution:

  It is represented as $F_X(x)$ and defined as

$$F_X(x) = P(X \leq x)$$

  If $x_1 < x_2$, then $F_X(x_1) < F_X(x_2)$ and $0 \leq F_X(x) \leq 1$.

- Probability Density Function:

  It is represented as $f_X(x)$ and defined as

$$f_X(x) = \frac{dF_X(x)}{dx}$$

This implies,

$$\begin{aligned} P(x_1 \leq X \leq x_2) &= \int_{x_1}^{x_2} f_X(x)\, dx \\ f_X(x) &= \frac{dF_X(x)}{dx} \end{aligned}$$