PRINCIPLES OF COMMUNICATIONS

UNIT-2

LECTURE-4

Random Process

A random process is defined as the ensemble(collection) of time functions together with a probability rule (see Figure 2)

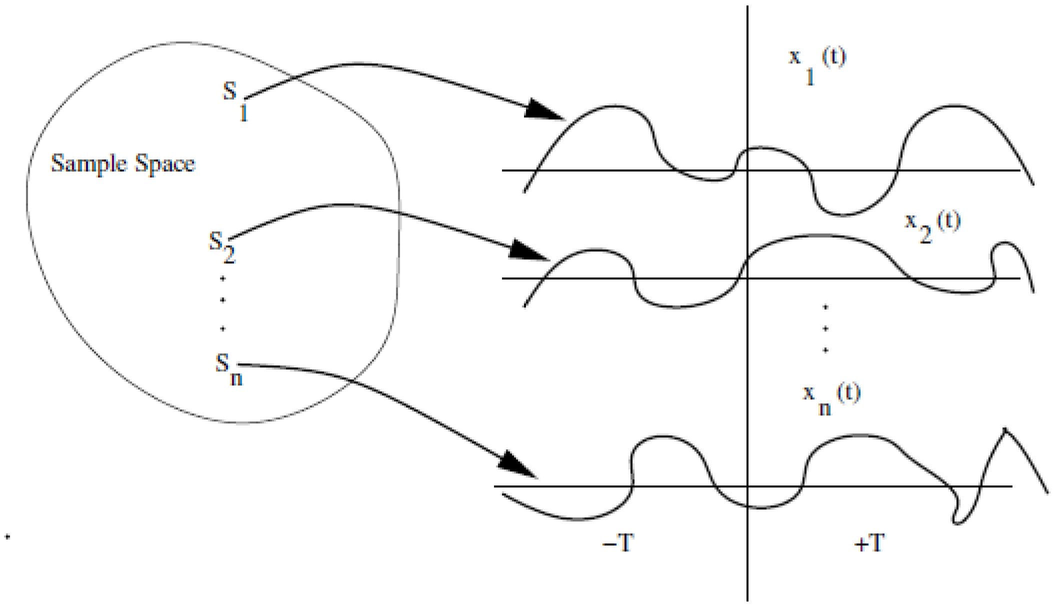

Figure 1: Random Processes and Random Variables

$x_1(t)$ is an outcome of experiment 1

$x_2(t)$ is the outcome of experiment 2

$x_n(t)$ is the outcome of experiment n

- Each sample point in S is associated with a sample function $x(t)$
- $X(t,s)$ is a random process
  - is an ensemble of all time functions together with a probability rule
  - $X(t,s_j)$ is a realisation or sample function of the random process
  - Probability rules assign probability to any meaningful event associated with an observation An observation is a sample function of the random process
- A random variable:

$\{x_1(t_k), x_2(t_k), \ldots, x_n(t_k)\} = \{X(t_k, s_1), X(t_k, s_2), \ldots, X(t_k, s_n)\}$
$X(t_k, s_j)$ constitutes a random variable.

- Outcome of an experiment mapped to a real number
  - An oscillator with a frequency ω_0 with a tolerance of 1%
  - The oscillator can take values between $\omega_0(1 \pm 0.01)$
  - Each realisation of the oscillator can take any value between $(\omega_0)(0.99)$ to $(\omega_0)(1.01)$
  - The frequency of the oscillator can thus be characterised by a random variable
- Stationary random process A random process is said to be stationary if its statistical characterization is independent of the observation interval over which the process was initiated. Mathematically,

$$F_{X(t_1+T)\cdots X(t_k+T)} = F_{X(t_1)\cdots X(t_k)}$$

- Mean, Correlation and Covariance Mean of a stationary random process is independent of the time of observation.

$$\mu_X(t) = E[X(t)] = \mu_x$$

Autocorrelation of a random process is given by:

$$\begin{aligned} R_X(t_1, t_2) &= E[X(t_1)X(t_2)] \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_1.x_2 f_{X(t_1)X(t_2)}(x_1, x_2)\, dx_1\, dx_2 \end{aligned}$$

For a stationary process the autocorrelation is dependent only on the time shiftand not on the time of observation.

Autocovariance of a stationary process is given by

$$C_X(t_1, t_2) = E[(X(t_1 - \mu_x)(X(t_2 - \mu_x)]$$

- Properties of Autocorrelation
  1. $R_X(\tau) = E[X(t + \tau)X(t)]$
  2. $R_X(0) = E[X^2(t)]$
  3. The autocorrelation function is an even function i.e, $R_X(\tau) = R_X(-\tau)$.
  4. The autocorrelation value is maximum for zero shift i.e, $R_X(\tau) \leq R_X(0)$.

Proof:

$$E[(X(t+\tau) \pm X(t))^2] \geq 0$$

$$\implies E[X^2(t+\tau)] + E[X^2(t)] \pm 2E[X(t+\tau)X(t)] \geq 0$$

$$\implies R_X(0) + R_X(0) + 2R_X(\tau) \geq 0$$

$$\implies -R_X(0) \leq R_X(\tau) \leq R_X(0)$$

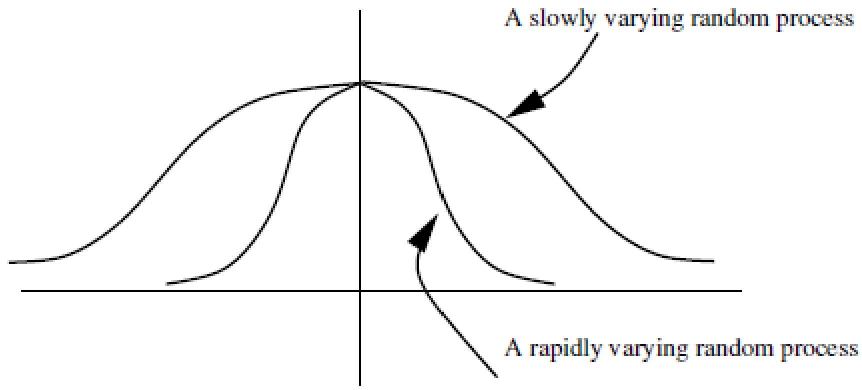

Figure 2: Autocorrelation function of a random process