# PRINCIPLES OF COMMUNICATIONS

UNIT-2

LECTURE-5

## Random Process: Some Examples

- A sinusoid with random phase

  Consider a sinusoidal signal with random phase, defined by

$$X(t) = a\sin(\omega_0 t + \Theta)$$

where $\omega_0$ and $a$ are constants, and $\Theta$ is a random variable that is uniformly distributed over a range of 0 to $2\pi$ (see Figure 1)

$$\begin{aligned} f_\Theta(\theta) &= \frac{1}{2\pi}, 0 \le \theta \le 2\pi \\ &= 0, elsewhere \end{aligned}$$

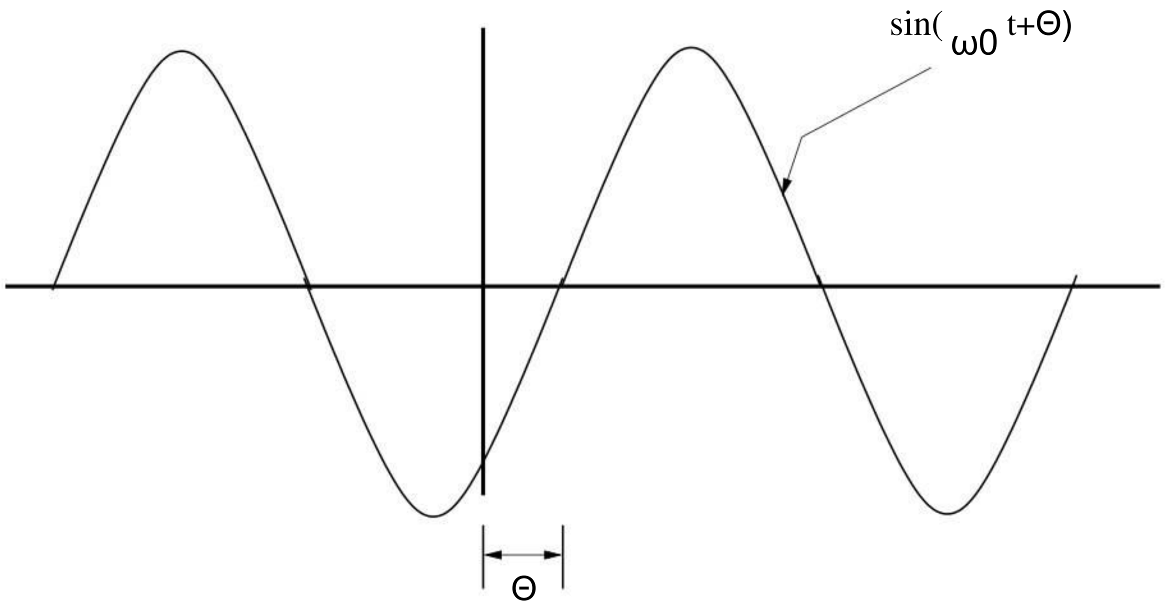


Figure 1: A sinusoid with random phase

This means that the random variable Θ is equally likely to have any value in the range 0 to 2π. The autocorrelation function of X(t) is

$$R_X(t) = E[X(t+\tau)X(t)]$$

$$= E[\sin(\omega_0 t + \omega_0 \tau) + \Theta)\sin(\omega_0 t + \Theta)]$$

$$= \frac{1}{2}E[\sin(2\omega_0 t+\omega_0\tau+2\Theta)]+2E[\sin(\overline{\omega}_0\tau)]$$

$$= \frac{1}{2}\int_0^{2\pi}\frac{1}{2\pi}\cos(4\pi f_c t + \omega_0\tau + 2\Theta)] - \cos(\omega_0\tau)\,d\Theta$$

The first term intergrates to zero, and so we get

$$R_X(\tau) = \frac{1}{2}\cos(\omega_0\tau)$$

The autocorrelation function is plotted in Figure 2.

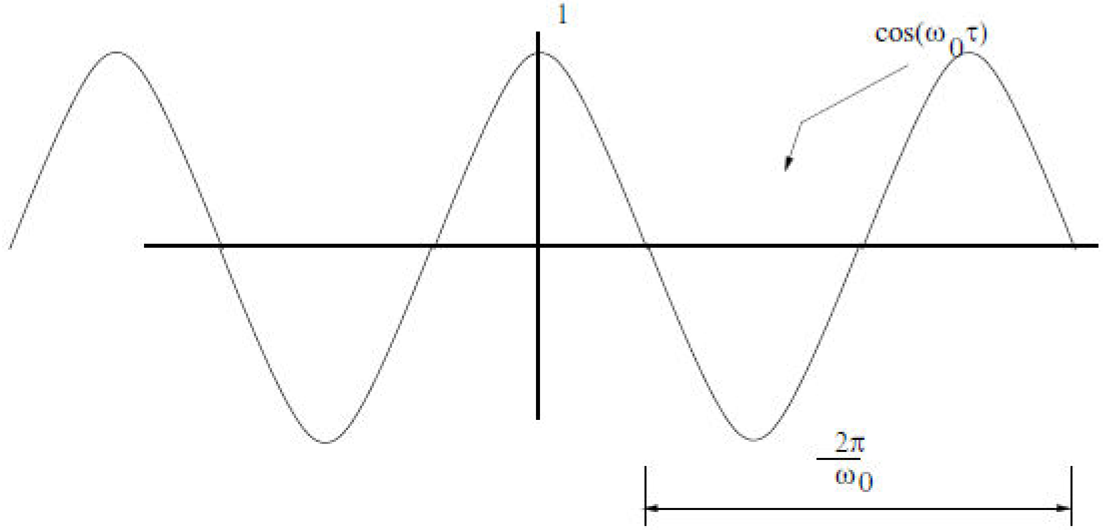


Figure 2: Autocorrelation function of a sinusoid with random phase

The autocorrelation function of a sinusoidal wave with random phase is another sinusoid at the same frequency in the $\tau$ -domain

- Random binary wave

  Figure 3 shows the sample function $x(t)$ of a random process

$X(t)$ consisting of a random sequence of binary symbols 1 and 0.

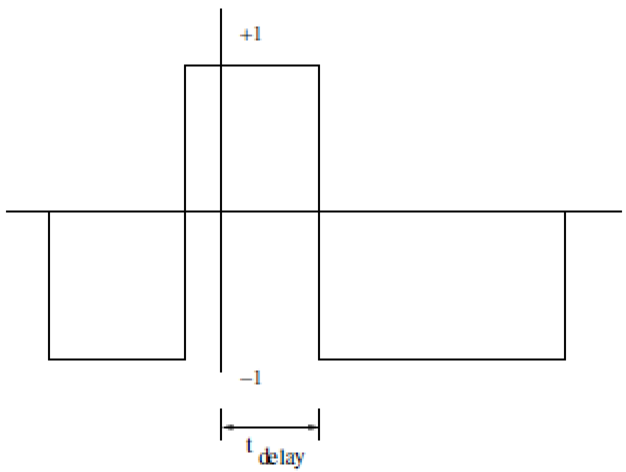


Figure 3: A random binary wave

1. The symbols 1 and 0 are represented by pulses of amplitude $+1$ and $-1$ volts, respectively and duration $T$ seconds.

2. The starting time of the first pulse, $t_{delay}$, is equally likely

to lie anywhere between zero and $T$ seconds

3. $t_{delay}$ is the sample value of a uniformly distributed random variable $T_{delay}$ with a probability density function

$$f_{T_{delay}}(t_{delay}) = \frac{1}{T}, 0 \leq t_{delay} \leq T$$
$$= 0, elsewhere$$

4. In any time interval $(n-1)T < t - t_{delay} < nT$, where $n$ is an interger, a 1 or a 0 is determined randomly (for example by tossing a coin: *heads* $\implies$ 1, *tails* $\implies$ 0

– $E[X(t)] = 0$, for all $t$ since 1 and 0 are equally likely.

– Autocorrelation function $R_X(t_k, t_l)$ is given by $E[X(t_k)X(t_l)]$, where $X(t_k)$ and $X(t_l)$ are random variables

Case 1: when $|t_k - t_l| > T$. $X(t_k)$ and $X(t_l)$ occur in different pulse intervals and are therefore independent:

$$E[X(t_k)X(t_l)] = E[X(t_k)]E[X(t_l)] = 0, for|t_k - t_l| > T$$

Case 2: when $|t_k - t_l| < T$, with $t_k = 0$ and $t_l < t_k$. $X(t_k)$ and $X(t_l)$ occur in the same pulse interval provided $t_{delay}$ satisfies the condition $t_{delay} < T - |t_k - t_l|$.

$$\begin{aligned} E[X(t_k)X(t_l)|t_{delay}] &= 1, t_{delay} < T - |t_k - t_l| \\ &= 0, elsewhere \end{aligned}$$

Averaging this result over all possible values of $t_{delay}$, we get

$$
\begin{aligned}
E[X(t_k)X(t_l)] &= \int_0^{T-|t_k-t_l|} f_{T_{delay}}(t_{delay})\, dt_{delay} \\
&= \int_0^{T-|t_k-t_l|} \frac{1}{T}\, dt_{delay} \\
&= (1 - \frac{|t_k - t_l|}{T}), |t_k - t_l| < T
\end{aligned}
$$

The autocorrelation function is given by

$$
\begin{aligned}
R_X(\tau) &= (1 - \frac{|\tau|}{T}), |\tau| < T \\
&= 0, |\tau| > T
\end{aligned}
$$

This result is shown in Figure 4

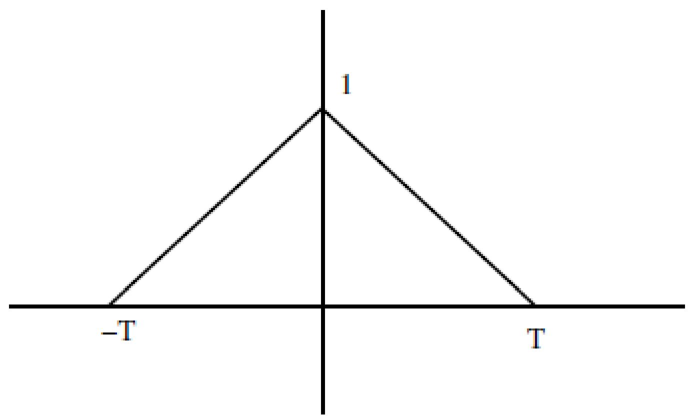


Figure 4: Autocorrelation of a random binary wave