PRINCIPLES OF COMMUNICATIONS

UNIT-2

LECTURE-6

Random Process: Some Examples

- Quadrature Modulation Process Given two random variables $X_1(t)$ and $X_2(t)$

$$\begin{aligned} X_1(t) &= X(t)\cos(2\omega_0 t + \Theta) \\ X_2(t) &= X(t)\sin(2\omega_0 t + \Theta) \end{aligned}$$

where ω_0 is a constant, and Θ is a random variable that is uniformly distributed over a range of 0 to 2π, that is,

$$\begin{aligned} f_\Theta(\theta) &= \frac{1}{2\pi}, 0 \leq \theta \leq 2\pi \\ &= 0, elsewhere \end{aligned}$$

The correlation function of $X_1(t)$ and $X_2(t)$ is

$$
\begin{aligned}
R_{12}(\tau) &= E[X_1(t)X_2(t+\tau)] \\
&= E[X(t)\cos(\omega_0 t + \Theta)X(t-\tau)\sin(2\omega_0(t-\tau)+\Theta)] \\
&= \int_0^{2\pi} \frac{1}{2\pi} X(t)X(t-\tau)\cos(\omega_0 t+\Theta)\sin(\omega_0(t-\tau)+\Theta)d\Theta \\
&= \frac{1}{2} R_X(\tau)\sin(\omega_0\tau)
\end{aligned}
$$

Random Process: Time vs. Ensemble Averages

Ensemble averages

- Difficult to generate a number of realisations of a random process
- $\Longrightarrow$ use time averages
- Mean

$$\mu_x(T) = \frac{1}{2T}\int_{-T}^{+T} x(t)\,dt$$

- Autocorrelation

$$R_x(\tau, T) = \frac{1}{2T}\int_{-T}^{+T} x(t)x(t+\tau)\,dt$$

- Ergodicity A random process is called ergodic if
  1. it is ergodic in mean:

$$\begin{aligned}\lim_{T\to+\infty}\mu_x(T) &= \mu_X \\ \lim_{T\to+\infty} var[\mu_x(T)] &= 0\end{aligned}$$

  2. it is ergodic in autocorrelation:

$$\begin{aligned}\lim_{T\to+\infty}R_x(\tau,T) &= R_X(\tau) \\ \lim_{T\to+\infty} var[R_x(\tau,T)] &= 0\end{aligned}$$

  where μ_X and $R_X(\tau)$ are the ensemble averages of the same random process.

Random Processes and Linear Shift Invariant Systems(LSI)

- The communication channel can be thought of as a system
- The signal that is transmitted through the channel is a realisation of the random process
- It is necessary to understand the behaviour of a signal that is input to a system.
- For analysis purposes it is assumed that a system is LSI
- Linear Shift Invariant(LSI) Systems

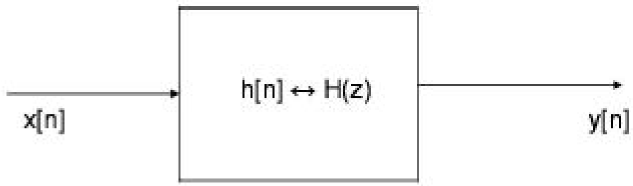

Figure 1: An LSI system

In Figure 1 , $h[n]$ is an LSI system if it satisfies the following properties

- Linearity The system is called linear, if the following equation holds for all signals $x_1[n]$ and $x_2[n]$ and any a and b:

$$\begin{aligned} x_1[n] &\rightarrow y_1[n] \\ x_2[n] &\rightarrow y_2[n] \\ \Longrightarrow a.x_1[n] + b.x_2[n] &\rightarrow a.y_1[n] + b.y_2[n] \end{aligned}$$

- Shift Invariance The system is called Shift Invariant, if the following equation holds for any signal $x[n]$

$$\begin{aligned} x[n] &\rightarrow y[n] \\ \Longrightarrow x[n-n_0] &\rightarrow y[n-n_0] \end{aligned}$$

* The assumption is that the output of the system is linear, in that if the input scaled, the output is scaled by the same factor.
* The system supports superposition
  - When two signals are added in the time domain, the output is equal to the sum of the individual responses
* If the input to the system is delayed by n_0, the output is also delayed by n_0.