PRINCIPLES OF COMMUNICATIONS

UNIT-4

LECTURE-4

Channel Coding

Channel Coding is done to ensure that the signal transmitted is recovered with very low probability of error at the destination.

Let X and Y be the random variables of symbols at the source and destination respectively. The description of the channel is shown in the Figure. 1

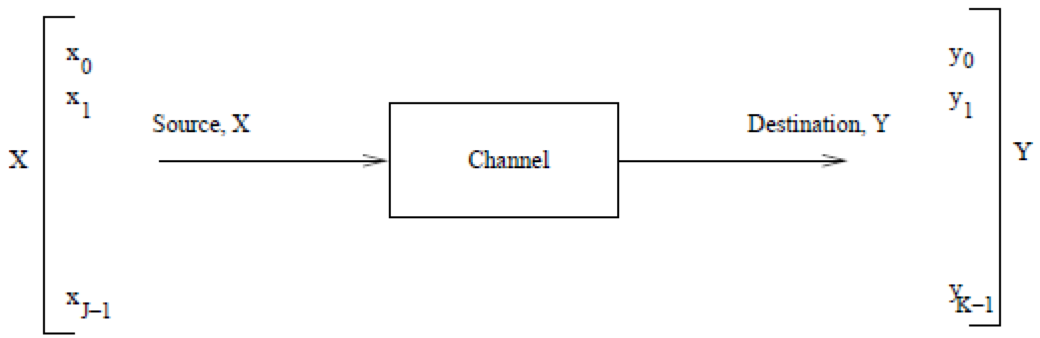

Figure 1: Description of a channel

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The channel is described by a set of transition probabilities

$$P(Y = y_k | X = x_j) = p(y_k | x_j), \forall j, k$$

such that

$$\sum_{k=0} p(y_k | x_j) = 1, \forall j.$$

The joint probability is now given by

$$\begin{aligned} p(x_j, y_k) &= P(X = x_j, Y = y_k) \\ &= P(Y = y_k | X = x_j) P(X = x_j) \\ &= p(y_k | x_j) p(x_j) \end{aligned}$$

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- Binary Symmetric Channel

  - A discrete memoryless channel with $J = K = 2$.

  - The Channel has two input symbols($x_0 = 0$, $x_1 = 1$) and two output symbols($y_0 = 0$, $y_1 = 1$).

  - The channel is symmetric because the probability of receiving a 1 if a 0 is sent is same as the probability of receiving a 0 if a 1 is sent.

  - The conditional probability of error is denoted by p. A binary symmetric channel is shown in Figure. 2 and its transition probability matrix is given by

$$\begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$

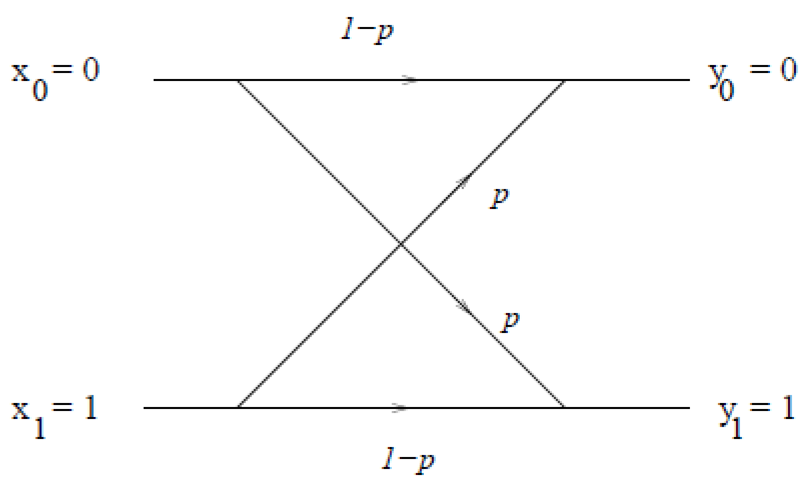

Figure 2: Binary Symmetric Channel

- Mutual Information

If the output Y is the noisy version of the channel input X and $H(\mathcal{X})$ is the uncertainity associated with X, then the uncertainity about X after observing Y, $H(\mathcal{X}|\mathcal{Y})$ is given by

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\begin{aligned}
H(\mathcal{X}|\mathcal{Y}) &= \sum_{k=0}^{K-1} H(\mathcal{X}|Y = y_k)p(y_k) &(1)\\
&= \sum_{k=0}^{K-1}\sum_{j=0}^{J-1} p(x_j|y_k)p(y_k)log_2\left[\frac{1}{p(x_j|y_k)}\right] &(2)\\
&= \sum_{k=0}^{K-1}\sum_{j=0}^{J-1} p(x_j, y_k)log_2\left[\frac{1}{p(x_j|y_k)}\right] &(3)
\end{aligned}
$$

The quantity $H(\mathcal{X}|\mathcal{Y})$ is called *Conditional Entropy*. It is the amount of uncertainity about the channel input after the channel output is observed. Since $H(\mathcal{X})$ is the uncertainity in channel input before observing the output, $H(\mathcal{X}) - H(\mathcal{X}|\mathcal{Y})$ represents the uncertainity in channel input that is resolved by observing the channel output. This uncertainity measure is termed as *Mutual Information* of the channel and is denoted by