PRINCIPLES OF COMMUNICATIONS

UNIT-5

LECTURE-2

Channel Coding

- Channel Capacity

  Channel Capacity, C is defined as

  'the maximum mutual information $I(\mathcal{X};\mathcal{Y})$ in any single use of the channel, where the maximization is over all possible input probability distributions $\{p(x_j)\}$ on X "

$$C = \max_{p(x_j)} I(\mathcal{X};\mathcal{Y}) \tag{1}$$

  C is measured in bits/channel-use, or bits/transmission.

  Example:

  For, the binary symmetric channel discussed previously, $I(\mathcal{X};\mathcal{Y}$ will be maximum when $p(x_0) = p(x_1) = \frac{1}{2}$. So, we have

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$$C = I(X;Y|_{p(x_0)=p(x_1)=\frac{1}{2}} \tag{2}$$

Since, we know

$$p(y_0|x_1) = p(y_1|x_0) = p \tag{3}$$

$$p(y_0|x_0) = p(y_1|x_1) = 1 - p \tag{4}$$

Using the probability values in Eq. 3 and Eq. 4 in evaluating Eq. 2, we get

$$C = 1 + p\log_2 p + (1-p)\log_2(1-p) \Longrightarrow 1 - H(p) \tag{5}$$

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- Channel Coding Theorem:

  Goal: Design of channel coding to increase resistance of a digital communication system to channel noise.

  The channel coding theorem is defined as

  1. Let a discrete memoryless source
     - with an alphabet S
     - with an entropy H(S)
     - produce symbols once every T_s seconds
  2. Let a discrete memoryless channel
     - have capacity C
     - be used once every T_c seconds.
  3. Then if,

$$\frac{H(S)}{T_s} \leq \frac{C}{T_c} \tag{6}$$

There exists a *coding scheme* for which the source output can be *transmitted over the channel* and be *reconstructed* with an *arbitrarily small probability of error.* The parameter $\frac{C}{T_c}$ is called *critical rate.*

4. Conversly, if

$$\frac{H(S)}{T_s} > \frac{C}{T_c} \tag{7}$$

it is not possible to transmit information over the channel and reconstruct it with an arbitrarily small probability of error.

Example:

Considering the case of a binary symmetric channel, the source entropy $H(S)$ is 1. Hence, from Eq. 6, we have

$$\frac{1}{T_s} \leq \frac{C}{T_c} \tag{8}$$

But the ratio $\frac{T_c}{T_s}$ equals the code rate, r of the channel encoder. Hence, for a binary symmetric channel, if $r \leq C$, then there exists a code capable of achieving an arbitrarily low probability of error.