PRINCIPLES OF COMMUNICATIONS

UNIT-5

LECTURE-3

Error Control Coding

- Channel is noisy
- Channel output prone to error
- $\Rightarrow$ we need measure to ensure correctness of the bit stream transmitted

Error control coding aims at developing methods for coding to check the correctness of the bit stream transmitted.

The bit stream representation of a symbol is called the codeword of that symbol.

Different error control mechanisms:

- Linear Block Codes
- Repetition Codes
- Convolution Codes

Linear Block Codes

A code is linear if two codes are added using modulo-2 arithmetic produces a third codeword in the code.

Consider a (n, k) linear block code. Here,

1. n represents the codeword length
2. k is the number of message bit
3. $n - k$ bits are error control bits or parity check bits generated from message using an appropriate rule.

We may therefore represent the codeword as

$$c_i = \begin{cases} b_i, & i = 0, 1, \cdots, n-k-1 \\ m_{i+k-n}, & i = n-k, n-k+1, \cdots, n-1 \end{cases} \quad (1)$$

The (n − k) parity bits are linear sums of the k message bits.

$$b_i = p_{0i}m_0 + p_{1i}m_1 + \cdots + p_{k-1,i}m_{k-1} \tag{2}$$

where the coefficients are

$$p_{ij} = \begin{cases} 1, & \text{if } b_i \text{ depends on } m_j \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

We define the 1-by-k message vector, or information vector, m, the 1-by-(n − k) parity vector b, and the 1-by-n codevector c as follows:

$$m = [m_0, m_1, \cdots, m_{k-1}]$$
$$b = [b_0, b_1, \cdots, b_{n-k-1}]$$
$$n = [n_0, n_1, \cdots, n_{n-1}]$$

We may thus write simultaneous equations in matrix equation form as

$$\mathbf{b} = \mathbf{mP}$$

where $\mathbf{P}$ is a k by $n-k$ matrix defined by

$$\mathbf{P} = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0,n-k-1} \\ p_{10} & p_{11} & \cdots & p_{1,n-k-1} \\ \vdots & \vdots & \vdots & \vdots \\ p_{k-1,0} & p_{k-1,1} & \cdots & p_{k-1,n-k-1} \end{bmatrix}$$

$\mathbf{c}$ matrix can be expressed as a partitioned row vector in terms of the vectors $\mathbf{m}$ and $\mathbf{b}$ as follows

$$c = [b : m]$$
$$\implies c = m[P : I_k]$$

where I_k is a k-by-k identity matrix. Now, we define k-by-n generator matrix as

$$\mathbf{G} = [\mathbf{P} : I_k]$$
$$\implies \mathbf{c} = \mathbf{mG}$$

Closure property of linear block codes:
Consider a pair of code vectors c_i and c_j corresponding to a pair of message vectors m_i and m_j respectively.

$$c_i + c_j = m_iG + m_jG$$
$$= (m_i + m_j)G$$