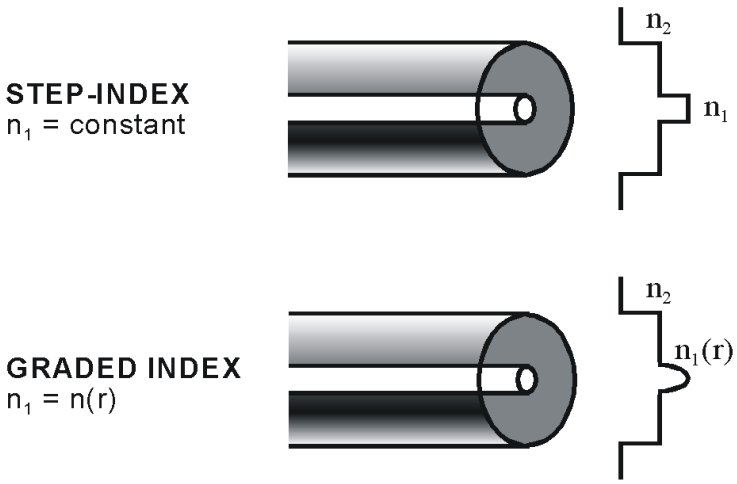
# Unit-1

# Lecture -7

Fiber structure : Step Index, Graded Index Fibers, Modal Theory

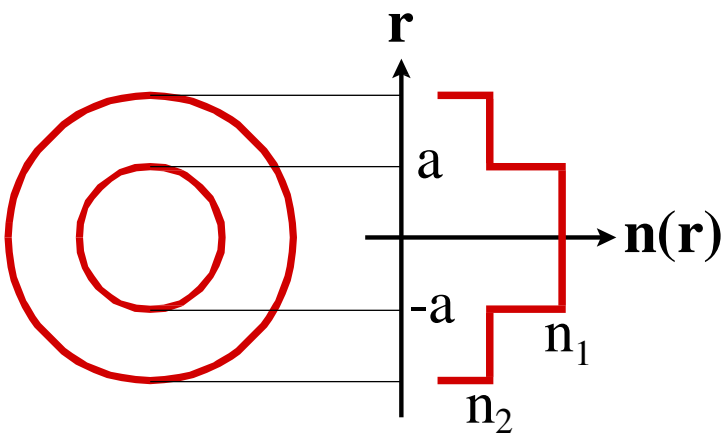
# Structure



# • Step-Index Fiber

– $n_2$ = constant (cladding)

– $n_1$ = **constant** (core)



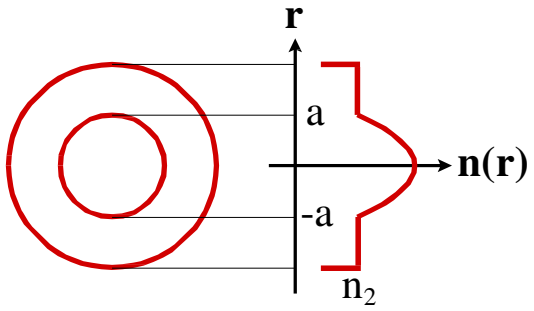

$$\Delta_{SI} = \frac{n_1 - n_2}{n_1}$$

$$r \le a \Rightarrow n(r) = n_1 = n(0)$$

$$r > a \Rightarrow n(r) = n_1\left[1 - \Delta\right] = n_2$$

# • Graded Index Fiber

- $n_2$ = constant (cladding)
- $n_1$ = n(r) **variable!** (core)




$$r \le a \Rightarrow n(r) = n(0)\left[1 - 2\Delta\left(\frac{r}{a}\right)^{g}\right]^{1/2}$$

$$r > a \Rightarrow n(r) = n(0)\left[1 - 2\Delta\right]^{1/2} = n_2$$

$$\Delta_{GI} = \frac{n^2(0) - n_2^{\,2}}{2 \cdot n^2(0)} = \frac{\left[n(0) - n_2\right] \cdot \left[n(0) + n_2\right]}{2 \cdot n^2(0)} \approx \frac{n(0) - n_2}{n(0)}$$

since $\quad n(0) + n_2 \approx 2 \cdot n(0)$

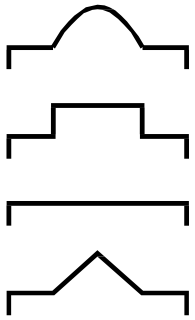
- Index profile parameter "g"

$$g = 2 \Rightarrow \text{parabolic profile (approximately)}$$

$$g \to \infty \Rightarrow IF: \quad r < a \Rightarrow (r/a)^{\infty} \to 0$$

$$g = 0 \Rightarrow n(r) = n_2 \quad \text{It does not work as a guide!}$$

$$g = 1 \Rightarrow n(r) \text{ straight: triangular profile (approx.)}$$



# • Modal Theory

– Maxwell eqs.

– Wave eqs. MODES

medium medium

– MODES: different ways for light to propagate

– Number of solutions —depends on

  • $n_1$, $n_2$
  • $\lambda$
  • a (core radius), diameter Ø = 2a

$$V_{SI} = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

SI: Step Index

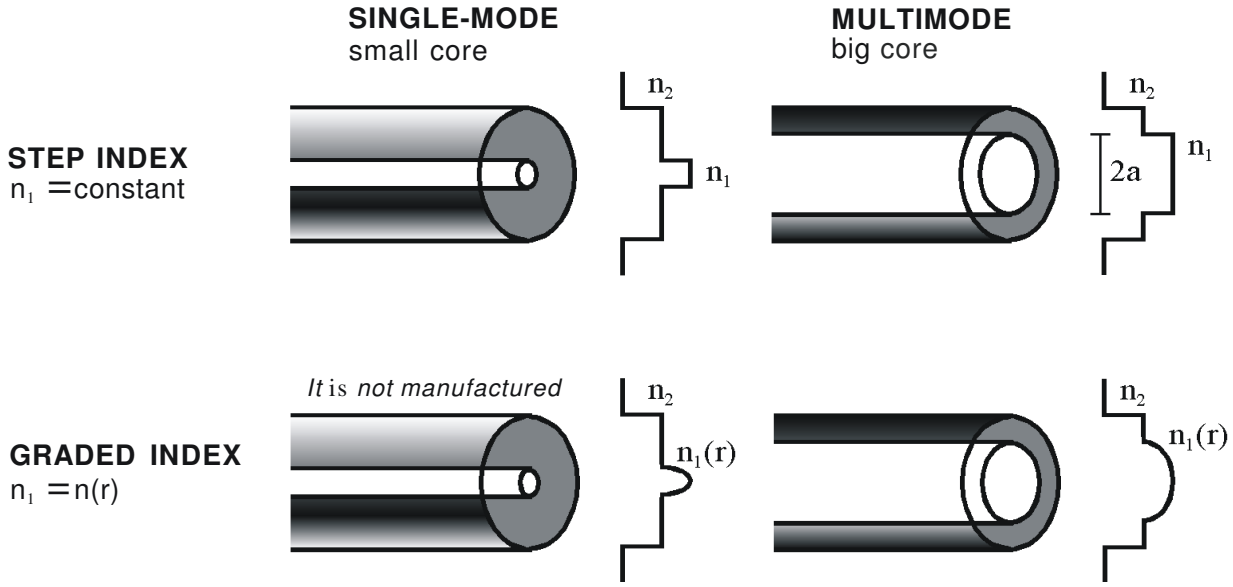
$$V_{GI} = \frac{2\pi a}{\lambda}\sqrt{[n_1(0)]^2 - n_2^2}$$

GI: Graded Index

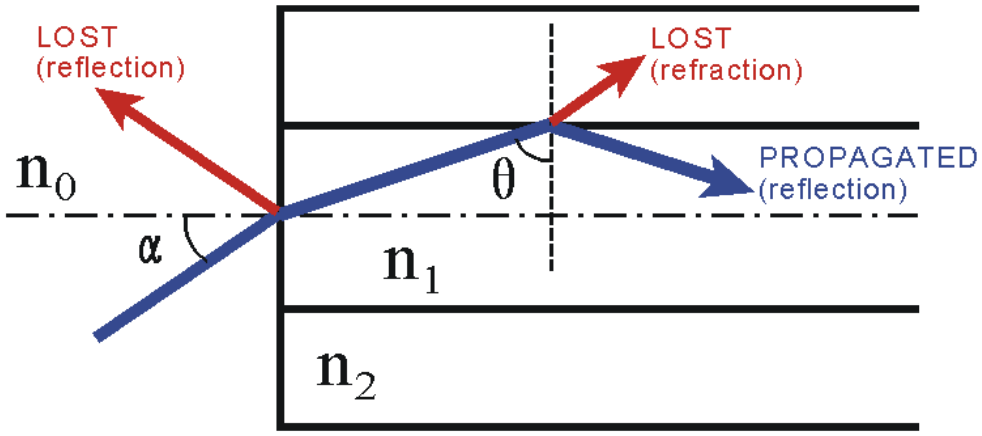
Fiber types: according to the number of propagating modes

**SINGLE-MODE: M = 1**

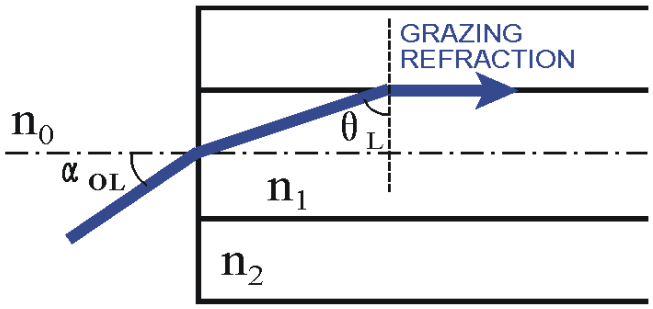
**MULTIMODE: M > 1**


SINGLE-MODE
small core
MULTIMODE
big core
STEP INDEX
n1 =constant
n2
n1
n2
2a
n1
It is not manufactured
GRADED INDEX
n1 =n(r)
n2
n1(r)
n2
n1(r)

- **Ray Theory**
  - **Core: carries light**
  - Cladding: confines light into the core
  - In order to propagate light
    - Air/core interface: refraction (*lighting horizontally*)
    - Core/cladding interface: reflection ($n_1 > n_2$)



- **<u>Critical Angle</u>** —to propagate light




$\theta_L$: limit angle of propagation

$\alpha_{OL}$: limit angle of acceptance

$$n_1 \sin\theta_L = n_2 \cdot 1 \Rightarrow \theta_L = \arcsin\left(\frac{n_2}{n_1}\right)$$

$$n_0 \sin\alpha_{OL} = n_1 \sin(90 - \theta_L) = n_1 \cos(\theta_L)$$

$$\sin\alpha_{OL} = \frac{n_1}{n^0}\sqrt{1-\sin^2\theta_L} = \frac{n_1}{n_0}\sqrt{\frac{n_1^2}{n_1^2} - \frac{n_2^2}{n_1^2}} = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

$$Air:\ n_0 \approx 1 \quad \Rightarrow \quad \sin\alpha_{OL} \approx \sqrt{n_1^2 - n_2^2}$$

**Numerical Aperture**

Light capturing ability of the fiber

Step-Index Fiber

$$\left.\begin{aligned} \theta_L &= \arcsin\left(\frac{n_2}{n_1}\right) \\ \sin\alpha_L &= \frac{n_1}{n_0}\cos\theta_L \\ n_0 &\approx 1 \end{aligned}\right\} \quad AN = n_1\cos\left[\arcsin\left(\frac{n_2}{n_1}\right)\right] = n_1\cos(\theta_L)$$

$$\cos(\theta_L) = \sqrt{1-\sin^2\theta_L} = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

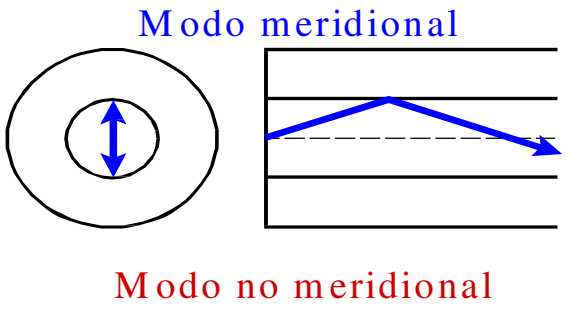
$$\left.\begin{aligned} \mathrm{AN} &= n_1\sqrt{1-\left(\frac{n_2}{n_1}\right)^2} = \sqrt{\mathrm{n}_1^2-\mathrm{n}_2^2} \\ \Delta_{SI} &= \frac{n_1-n_2}{n_1} \end{aligned}\right\} \quad AN_{SI} = n_1\sqrt{\frac{n_1-n_2}{n_1}\cdot\frac{n_1+n_2}{n_1}}$$

$$AN_{SI} \approx n_1\sqrt{\Delta\cdot 2} = n(0)\sqrt{2\Delta}$$

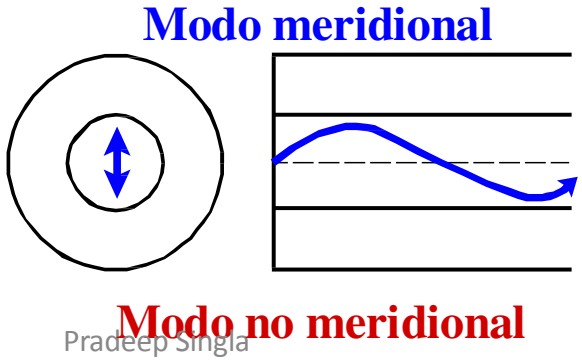
Graded Index Fiber —same expression is applied

$$\left.\begin{aligned} \mathbf{AN_{IG}(r)} &= \sqrt{\mathbf{n_1^2(r)-n_2^2}} \\ n_1(r) &= n(0)\left[1-2\Delta\left(\frac{r}{a}\right)^g\right]^{1/2} \\ n_2 &= n(0)\left[1-2\Delta\right]^{1/2} \end{aligned}\right\} \quad AN_{IG}(r) = n(0)\left[1-\left(\frac{r}{a}\right)^g\right]^{1/2}\cdot\sqrt{2\Delta}$$

- **Ray Paths**
  - SI Fiber
    - **Straight line paths**




city ($n_1$ constant)

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