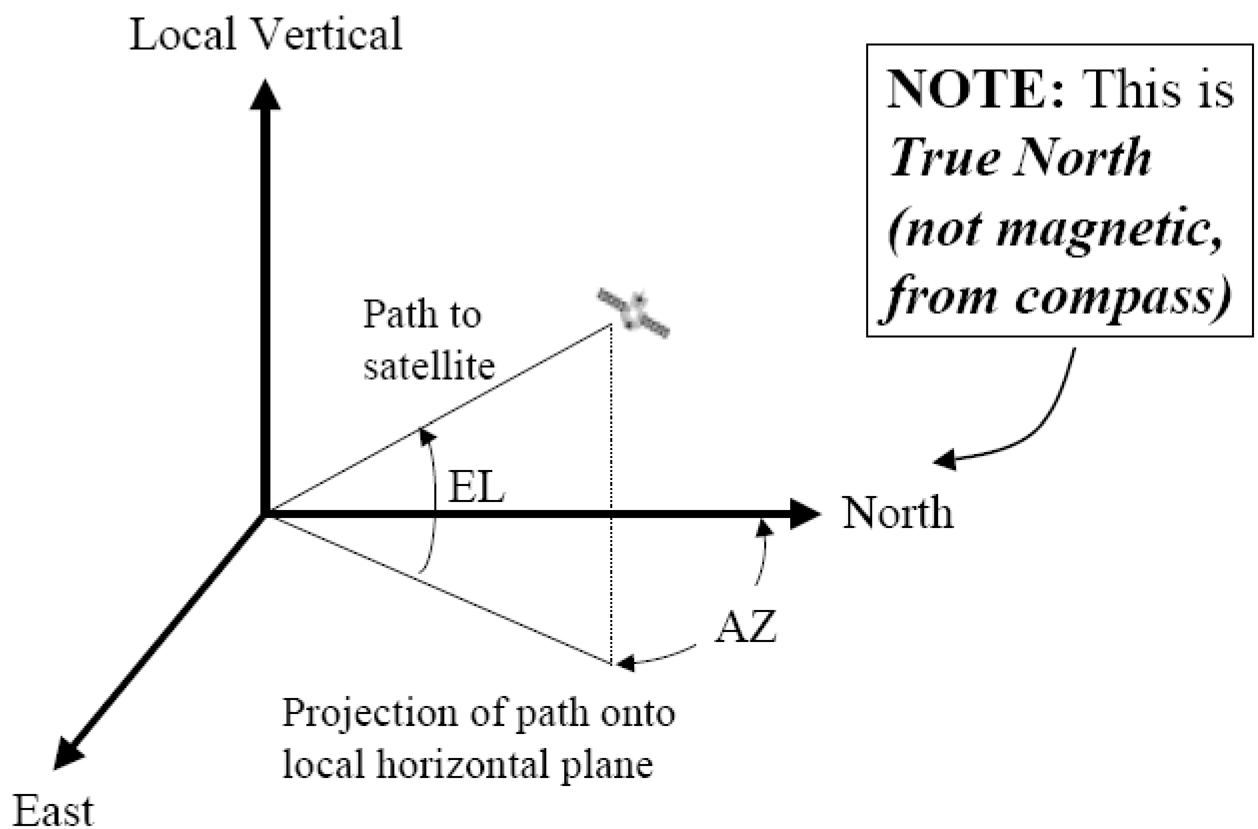
Satellite Communications

Look Angle

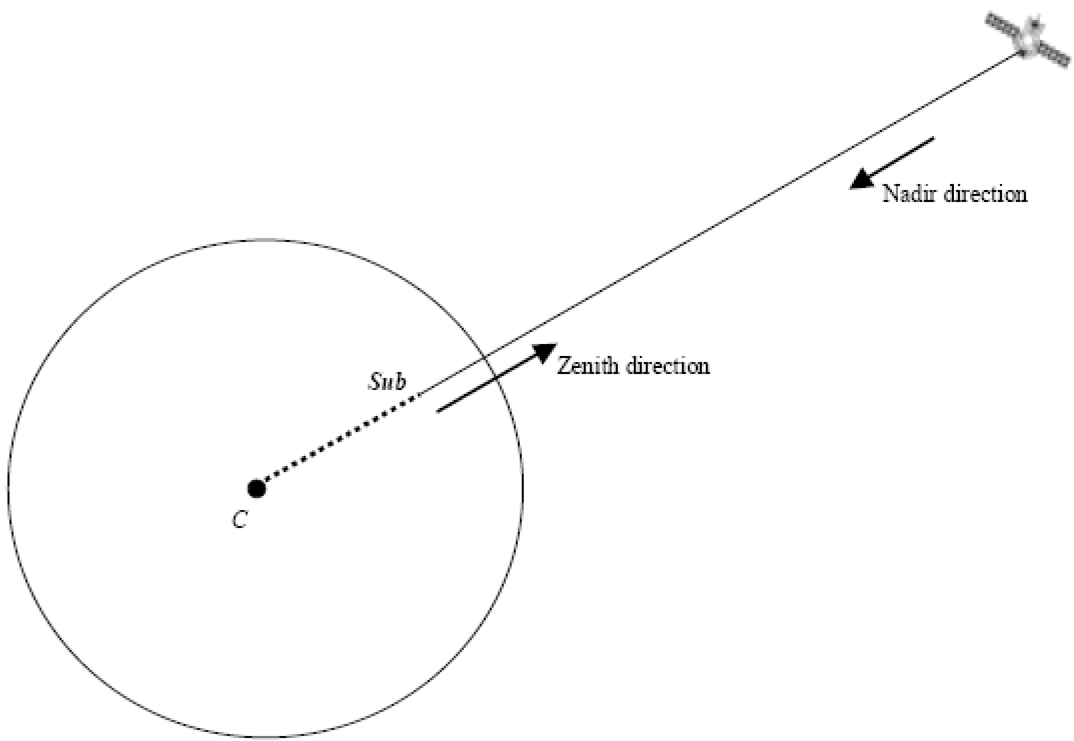
Look Angle Determination

- Look angles: The coordinates to which an ES must point to communicate with a satellite. These are *azimuth* (AZ) and *elevation angle* (EL)
  - AZ: The angle measured from N to E to projection of satellite path onto horizontal plane
  - EL: The angle measured from the horizontal plane to the orbit plane
- The *subsatellite point*: The point, on the earth's surface, of intersection between a line from the earth's center to the satellite

Look Angle Definition



Definitions (Contd.)



Calculating Look Angle

- ***Need*** six Orbital Elements
- ***Calculate*** the orbit from these Orbital Elements
- ***Define*** the orbital plane
- ***Locate*** satellite at time ***t*** with respect to the ***First Point of Aries***
- ***Find location*** of the Greenwich Meridian relative to the first point of Aries
- ***Use Spherical Trigonometry*** to find the position of the satellite relative to a point on the earth's surface

Coordinate System

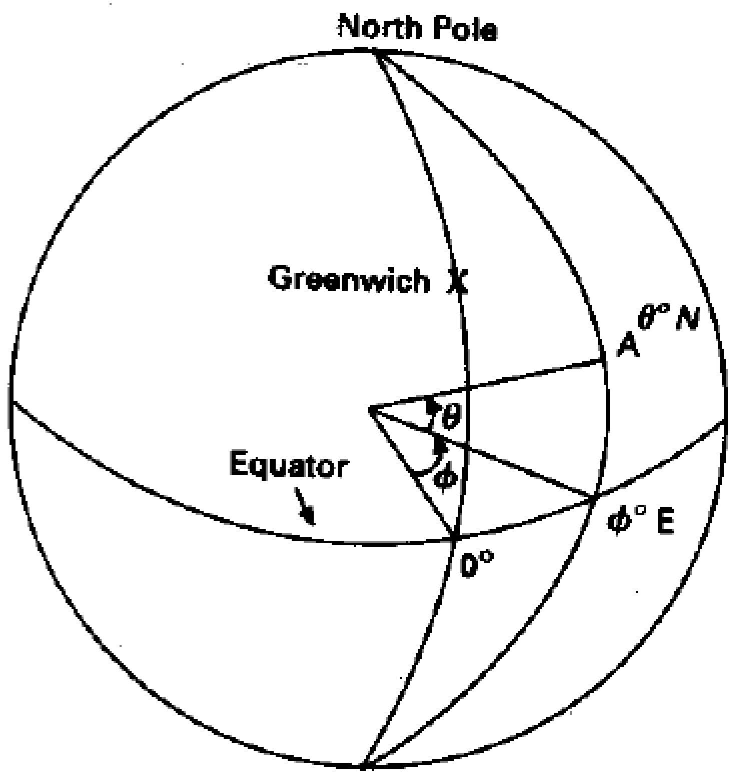
- **Latitude:** Angular distance, measured in degrees, north or south of the equator.

  L from -90 to +90 (or from 90S to 90N)

- **Longitude:** Angular distance, measured in degrees, from a given reference longitudinal line (Greenwich, London).

  l from 0 to 360E (or 180W to 180E)

Coordinate System



Latitude (θ°N) and longitude (ϕ°E) of a point A.

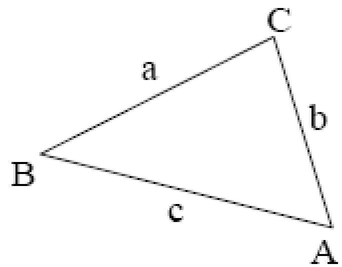
Satellite Coordinates

- SUB-SATELLITE POINT
  Latitude $\mathbf{L_s}$
  Longitude $\mathbf{l_s}$
- EARTH STATION LOCATION
  Latitude $\mathbf{L_e}$
  Longitude $\mathbf{l_e}$
- Calculate γ, ANGLE AT EARTH CENTER

Between the line that connects the earth-center to the satellite and the line from the earth-center to the earth station.

Review of Geometry

- Review of plane trigonometry
  - Law of Sines
  - Law of Cosines
  - Law of Tangents



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

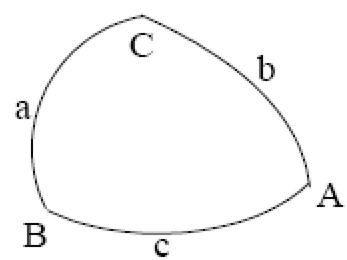
$$\tan\frac{C}{2} = \sqrt{\frac{(d-a)(d-b)}{d(d-c)}}, d = \frac{a+b+c}{2}$$

- Review of spherical trigonometry
  - Law of Sines
  - Law of Cosines for angles
  - Law of Cosines for sides

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$



Geometry of Elevation Angle

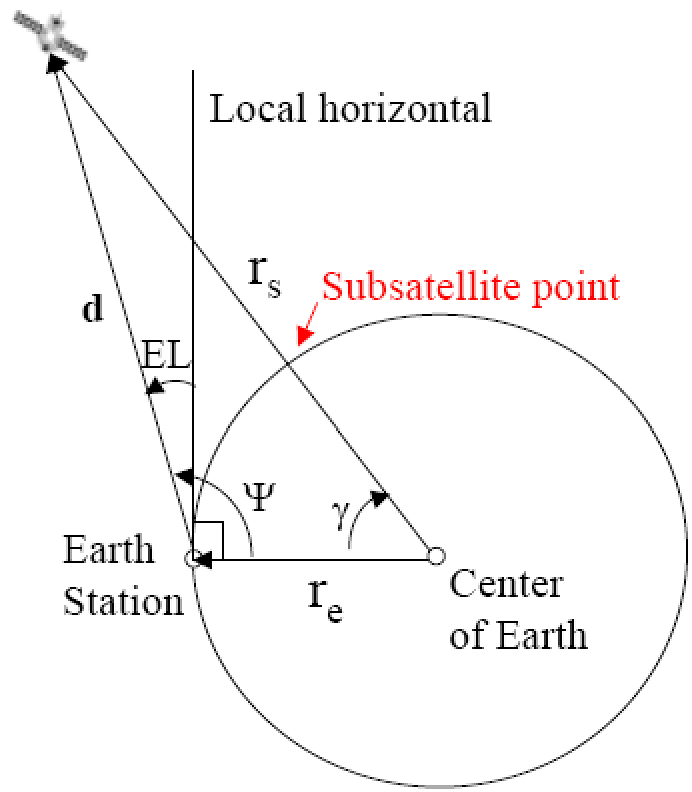
- Plane in picture is the one that includes center of the earth, Earth Station and Satellite.
- Subsatellite point will also be on the same plane.

$$El = \psi - 90^{o}$$

γ = central angle

r_s = radius to the satellite

r_e = radius of the earth



Central Angle

γ is defined so that it is non-negative and

$$\cos(\gamma) = \cos(Le)\cos(L_s)\cos(l_s - l_e) + \sin(L_e)\sin(L_s)$$

The magnitude of the vectors joining the center of the earth, the satellite and the earth station are related by the law of cosine:

$$d = r_s\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]^{1/2}$$

Elevation Angle Calculation

By the sine law we have

$$\frac{r_s}{\sin(\psi)} = \frac{d}{\sin(\gamma)}$$

Which yields

$$\cos(El) = \frac{\sin(\gamma)}{\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]^{1/2}}$$

Elevation Angle for GEO Satellite

Using r_s = 42,164 km and r_e = 6,378.14 km gives

d = 42,164 [1.0228826 -0.3025396 cos(γ)]1/2 km

Which finally gives the elevation angle

$$\cos(El) = \frac{\sin(\gamma)}{\left[1.0228826 - 0.3025396\,\cos(\gamma)\right]^{1/2}}$$

Azimuth Angle Calculation

- More complex approach for non-geo satellites.
- Different formulas and corrections apply depending on the combination of positions of the earth station and subsatellite point with relation to each of the four quadrants (NW, NE, SW, SE).
- Its calculation is simple for GEO satellites

Azimuth Angle Calculation for GEO Satellites

- SUB-SATELLITE POINT
  - Equatorial plane, Latitude **Ls = 0o**
  - Longitude **ls**
- EARTH STATION LOCATION
  - Latitude **Le**
  - Longitude **le**

Azimuth Angle for GEO sat.

The original calculation previously shown:

$$\cos(\gamma) = \cos(Le)\cos(L_s)\cos(\mathrm{l_s} - \mathrm{l_e}) + \sin(L_e)\sin(L_s)$$

Simplifies using $L_s = 0^{\mathrm{o}}$ since the satellite is over the equator:

$$\cos(\gamma) = \cos(Le)\cos(\mathrm{l_s} - \mathrm{l_e})$$

Azimuth Angle for GEO sat.

To find the azimuth angle, an ***intermediate angle, α***, must first be found. The intermediate angle allows the correct quadrant (see Figs. 2.10 & 2.13) to be found since the azimuthal direction can lie anywhere between 0° (true North) and clockwise through 360° (back to true North again). The intermediate angle is found from

$$\alpha = \tan^{-1}\left[\frac{\tan\left|\left(l_s - l_e\right)\right|}{\sin\left(L_e\right)}\right]$$

Azimuth Angle for GEO sat.

Case 1: Earth station in the Northern Hemisphere with

(a) Satellite to the SE of the earth station: $Az = 180^o - \alpha$

(b) Satellite to the SW of the earth station: $Az = 180^o + \alpha$

Case 2: Earth station in the Southern Hemisphere with

(c) Satellite to the NE of the earth station: $Az = \alpha$

(d) Satellite to the NW of the earth station: $Az = 360^o - \alpha$

Example for Look Angle Calculation of a GEO satellite

FIND the **Elevation** and **Azimuth** Look Angles for the following case:

Earth Station Latitude	52^{o} N	London, England Dockland region
Earth Station Longitude	0^{o}	
Satellite Latitude	0^{o}	Geostationary INTELSAT IOR Primary
Satellite Longitude	66^{o} E	

Example (Contd.)

Step 1. Find the central angle γ

$$\cos(\gamma) = \cos(L_e)\cos(l_s\text{-}l_e)$$
$$= \cos(52)\cos(66)$$
$$= 0.2504$$

yielding $\gamma = 75.4981^o$

Step 2. Find the elevation angle *El*

$$\cos(El) = \frac{\sin(\gamma)}{\left[1.0228826 - 0.3025396\cos(\gamma)\right]^{1/2}}$$

$El = 5.85^o$

Example (Contd.)

Step 3. Find the intermediate angle, α

$$\alpha = \tan^{-1}\left[\frac{\tan\left|(l_s - l_e)\right|}{\sin(L_e)}\right]$$

$= \tan^{-1}$ [(tan (66 - 0)) / sin (52)]

= 70.6668

Example (Contd.)

The earth station is in the Northern hemisphere and the satellite is to the South East of the earth station. This gives

$Az = 180^{o} - \alpha$

$= 180 – 70.6668 = 109.333^{o}$ (clockwise from true North)

ANSWER: The look-angles to the satellite are

Elevation Angle = 5.85^{o}

Azimuth Angle = 109.33^{o}