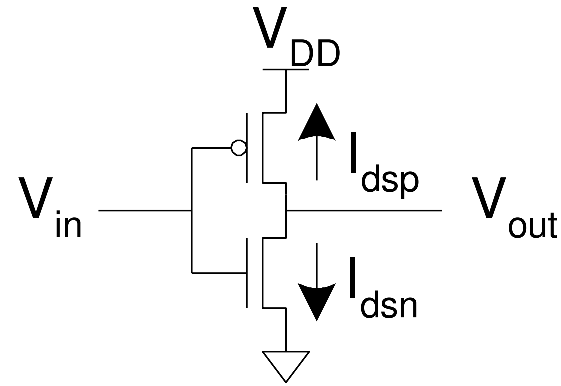
Unit 2
MOS Inverters

DC Response

- DC Response: V_{out} vs. V_{in} for a gate
- Ex: Inverter
  - When $V_{in} = 0$ -> $V_{out} = V_{DD}$
  - When $V_{in} = V_{DD}$ -> $V_{out} = 0$
  - In between, V_{out} depends on transistor size and current
  - By KCL, must settle such that $I_{dsn} = |I_{dsp}|$
  - We could solve equations
  - But graphical solution gives more insight

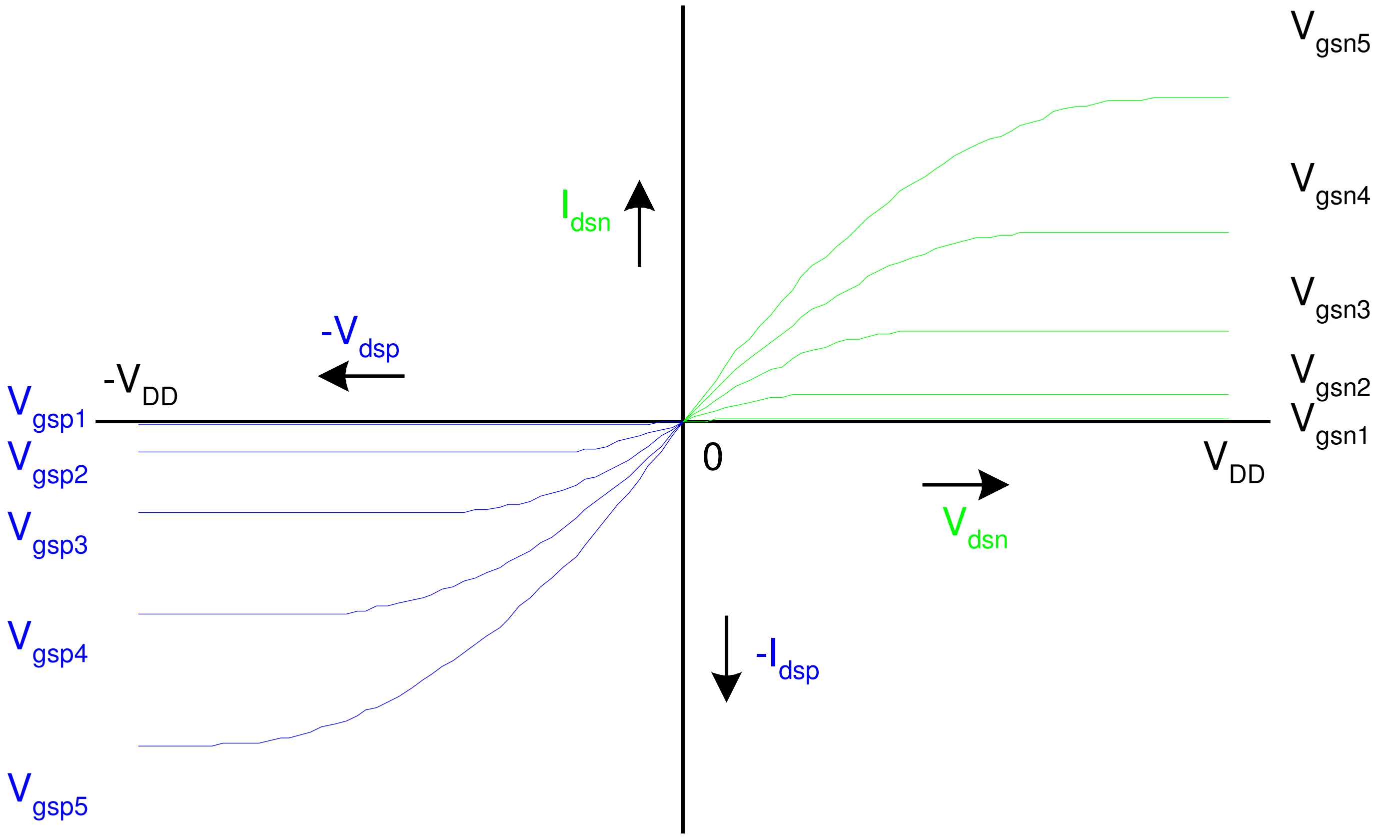


Transistor Operation

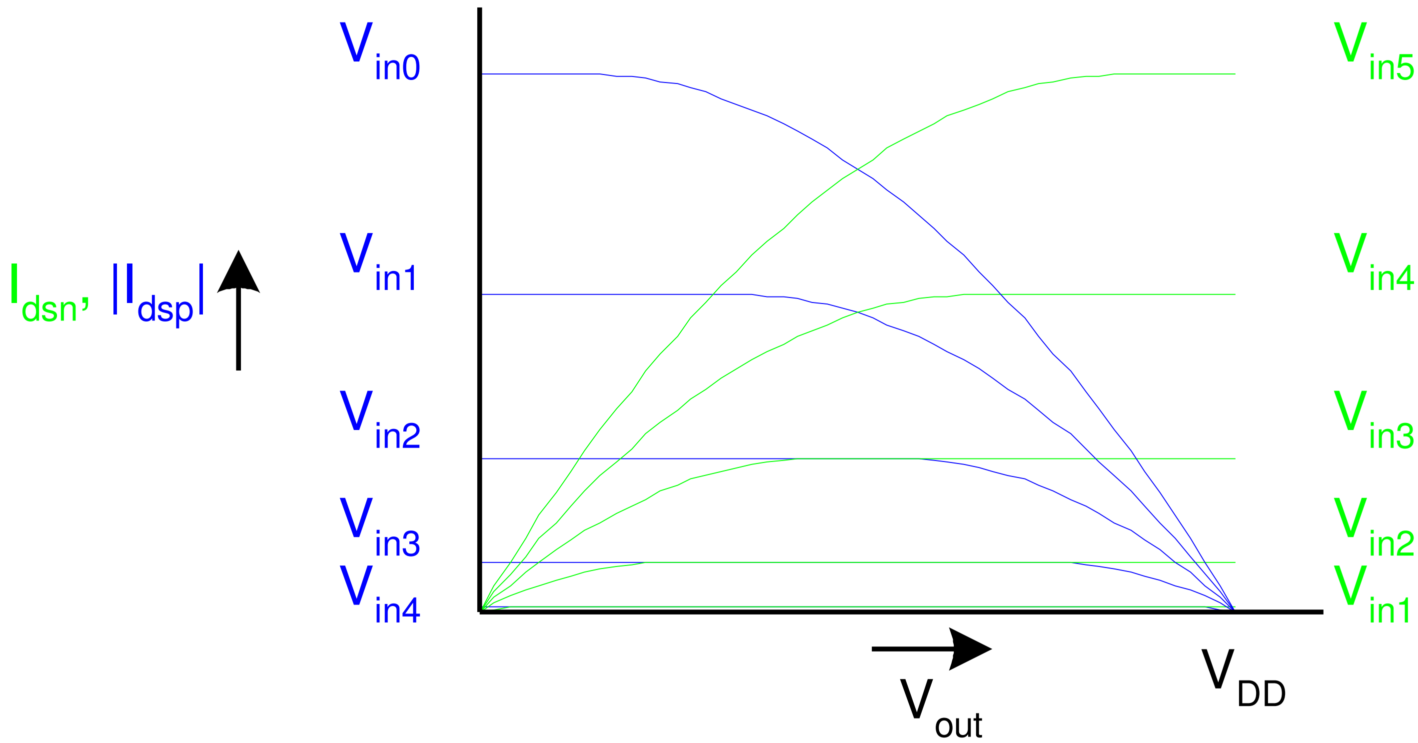
- Current depends on region of transistor behavior
- For what V_{in} and V_{out} are nMOS and pMOS in
  – Cutoff?
  – Linear?
  – Saturation?

I-V Characteristics

- Make pMOS is wider than nMOS such that $\beta_n = \beta_p$

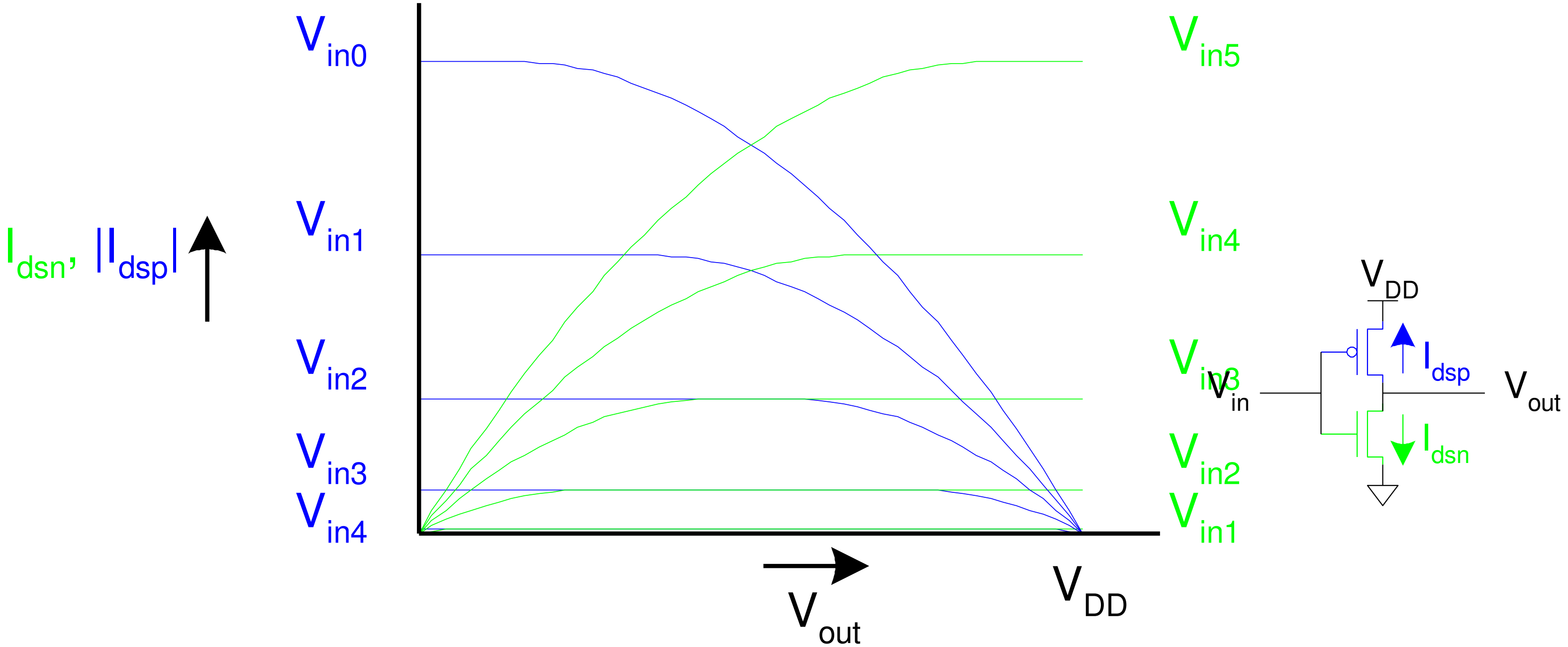


Current vs. V_{out}, V_{in}



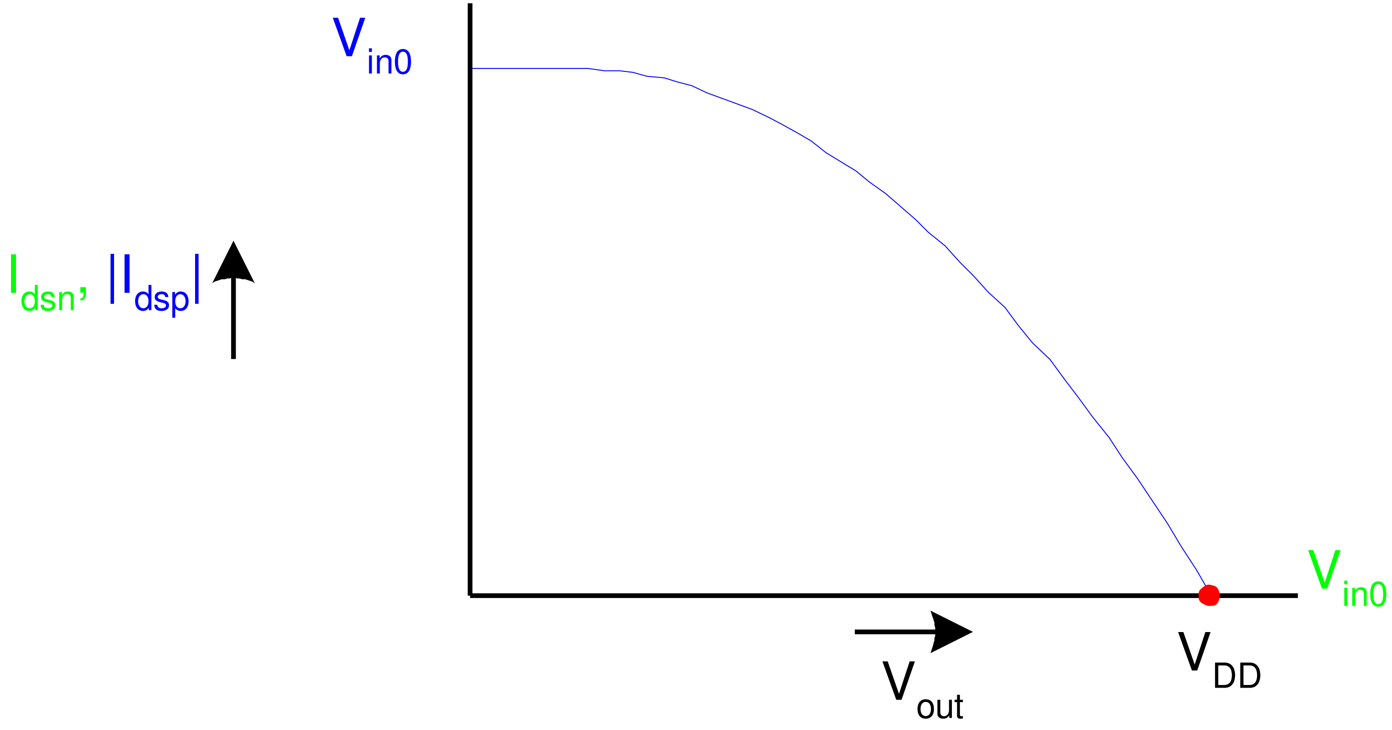
Load Line Analysis

- For a given V_{in}:
  - Plot I_{dsn}, I_{dsp} vs. V_{out}
  - V_{out} must be where |currents| are equal in



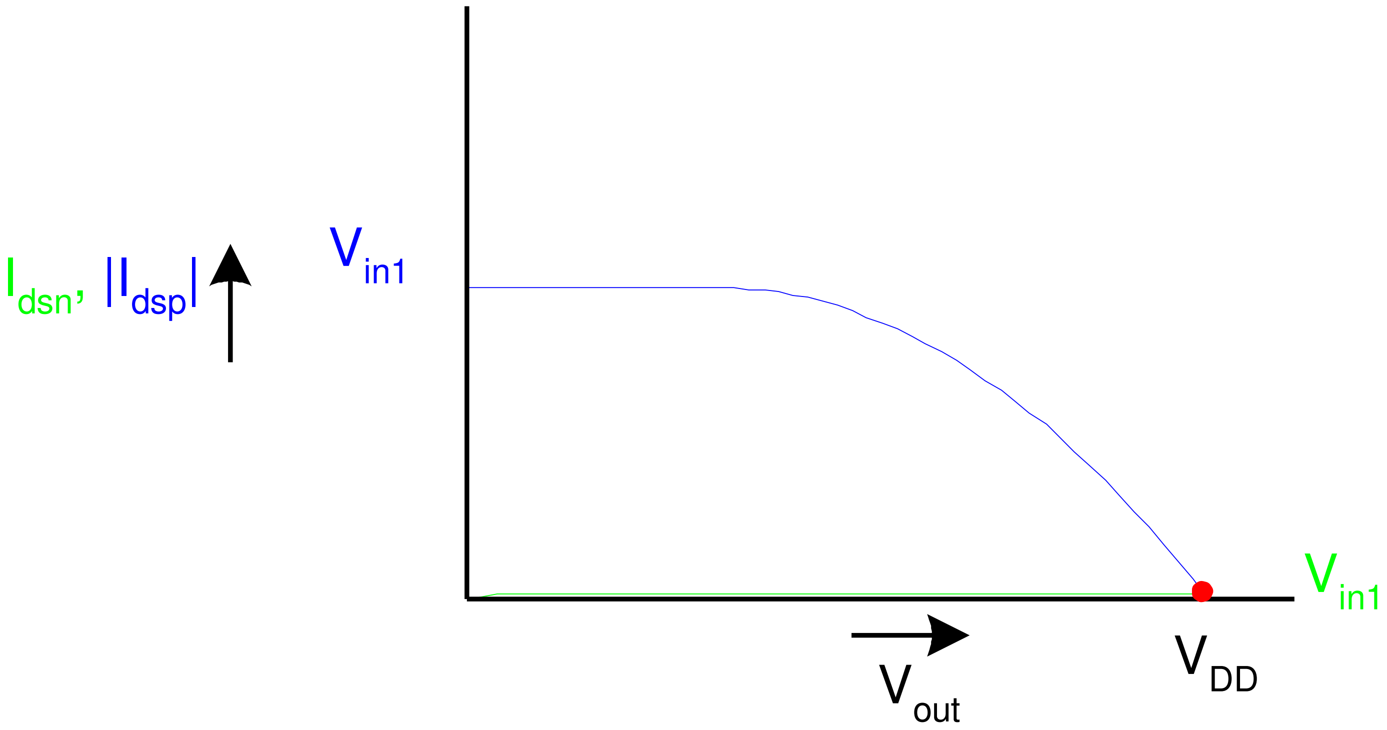
Load Line Analysis

- $V_{in} = 0$



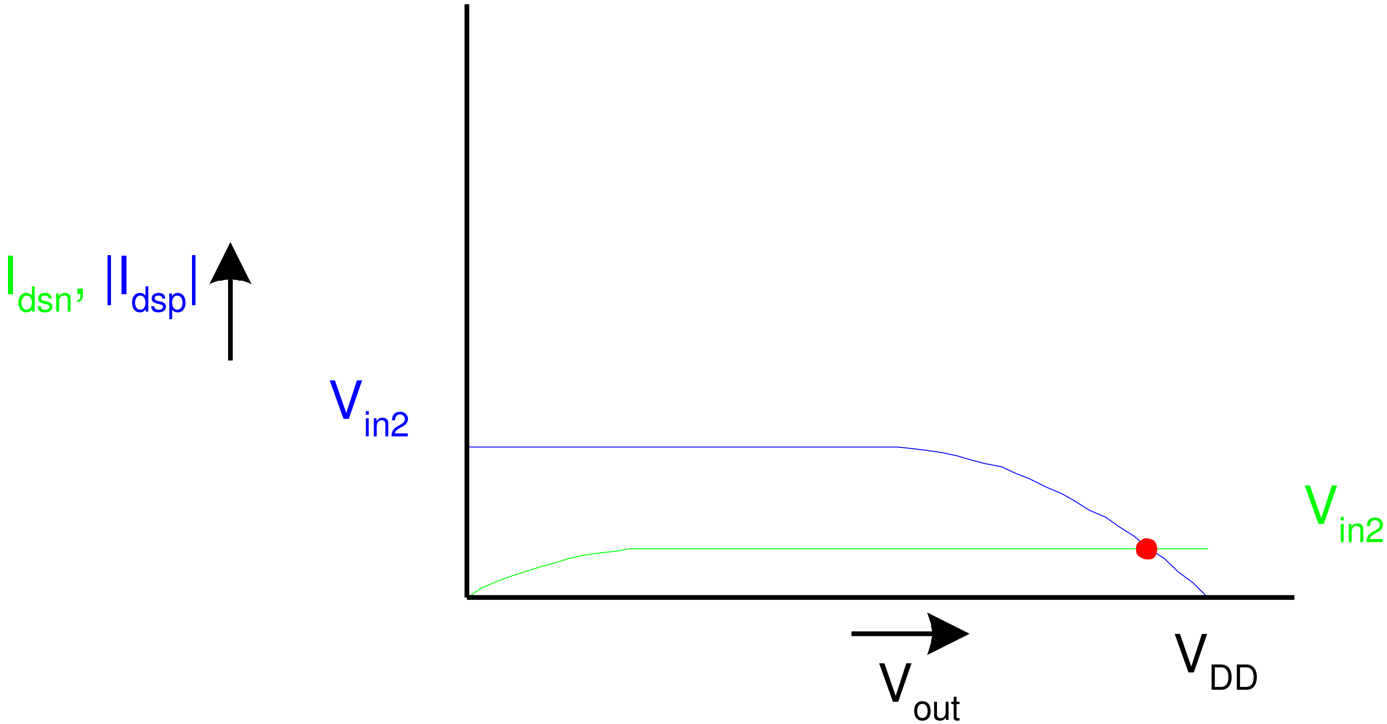
Load Line Analysis

- $V_{in} = 0.2V_{DD}$



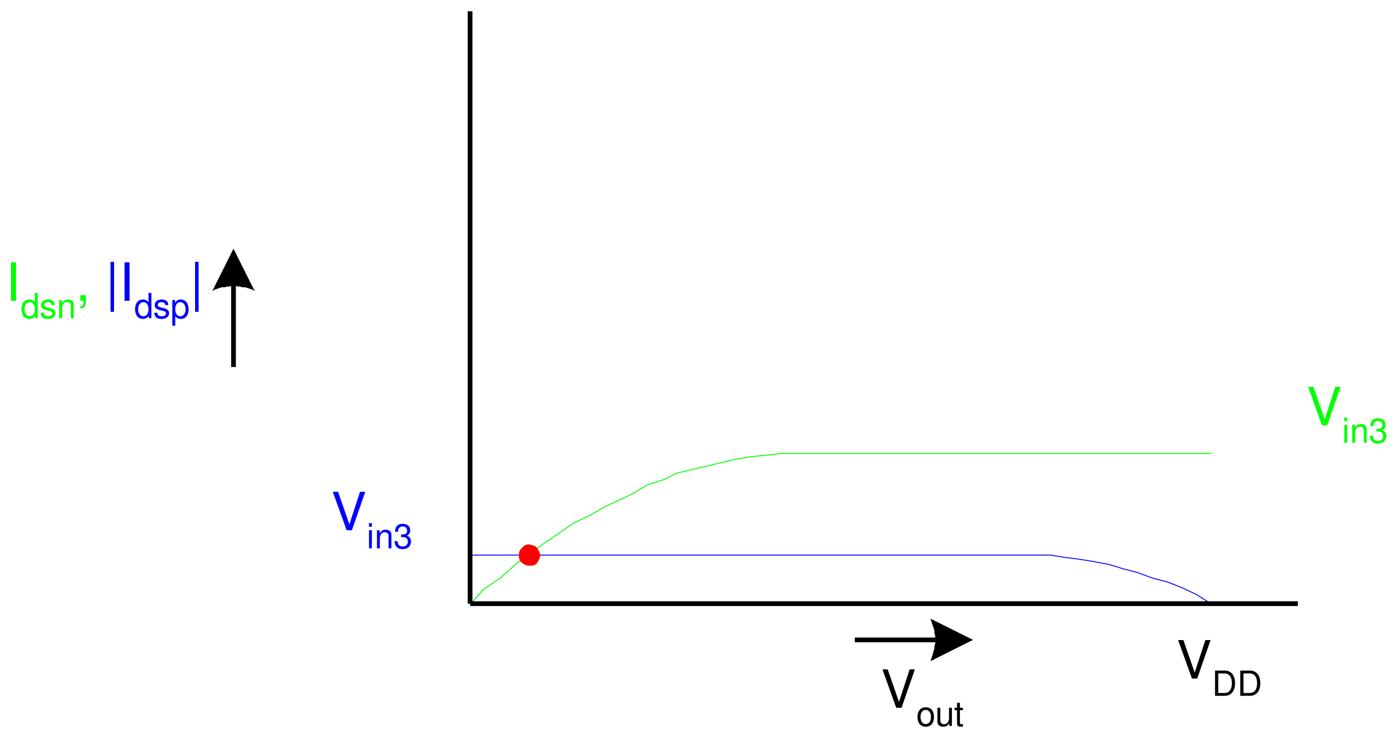
Load Line Analysis

- $V_{in} = 0.4V_{DD}$



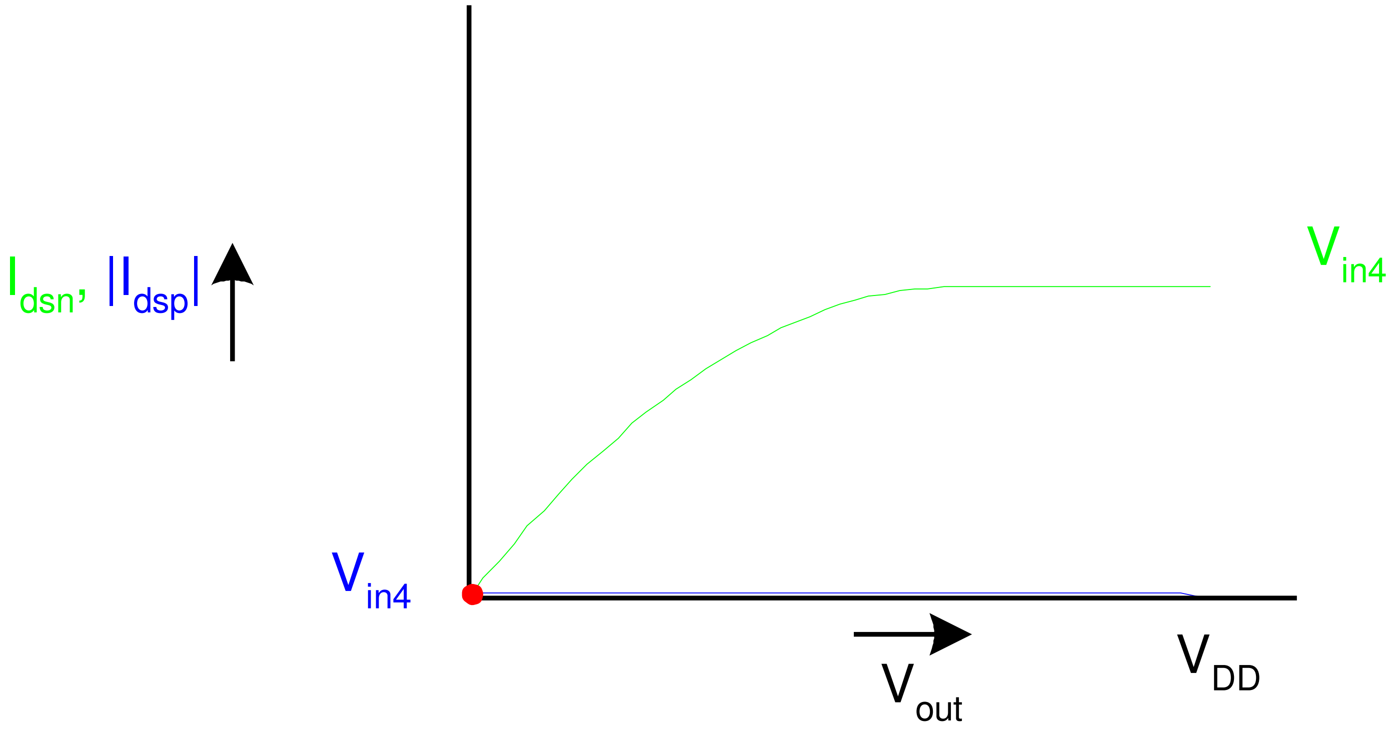
Load Line Analysis

- $V_{in} = 0.6V_{DD}$



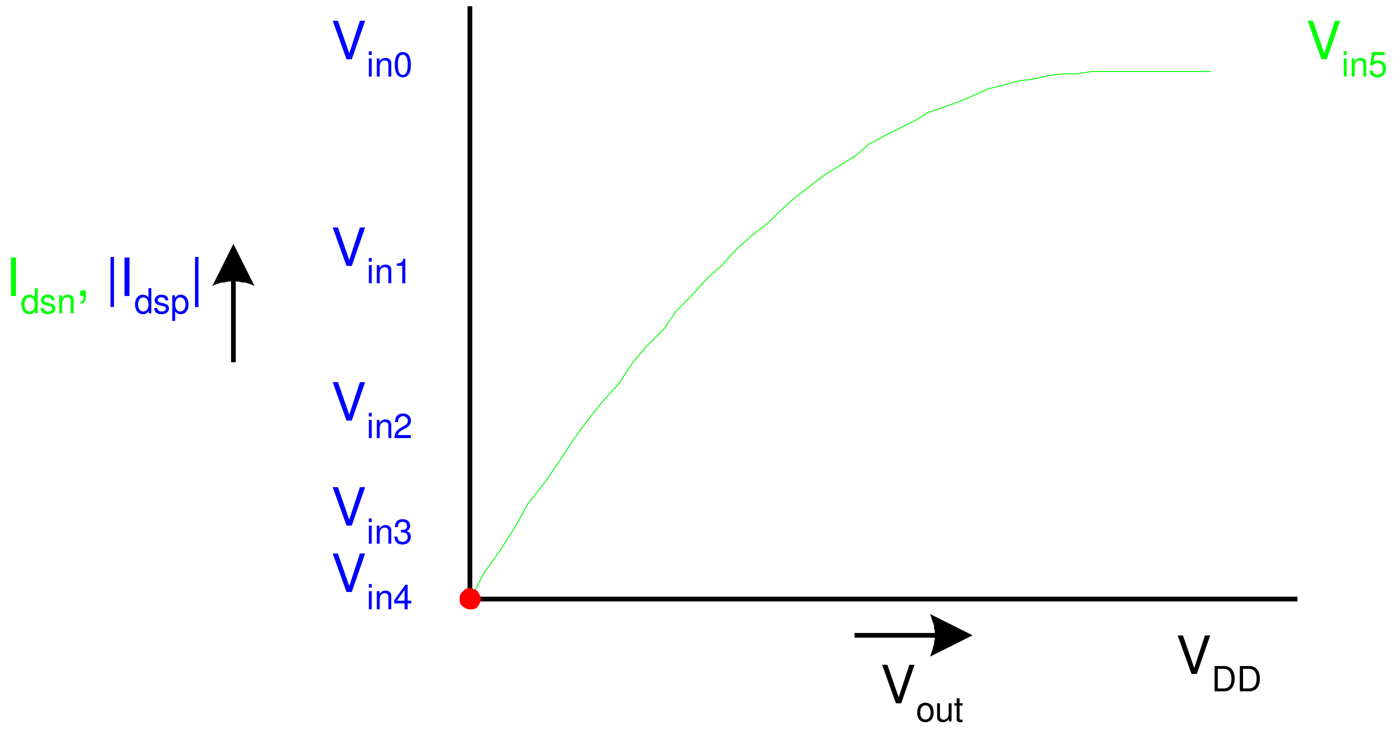
Load Line Analysis

- $V_{in} = 0.8V_{DD}$

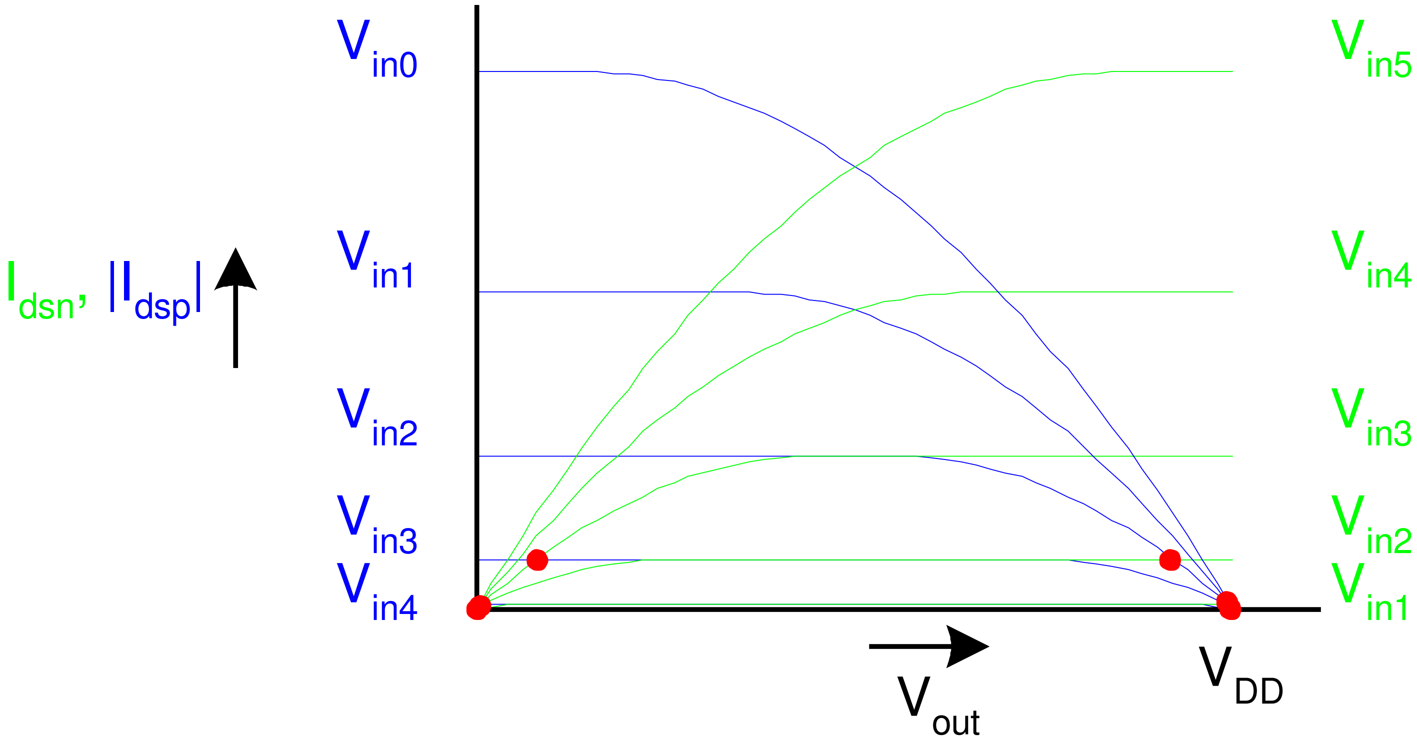


Load Line Analysis

- $V_{in} = V_{DD}$

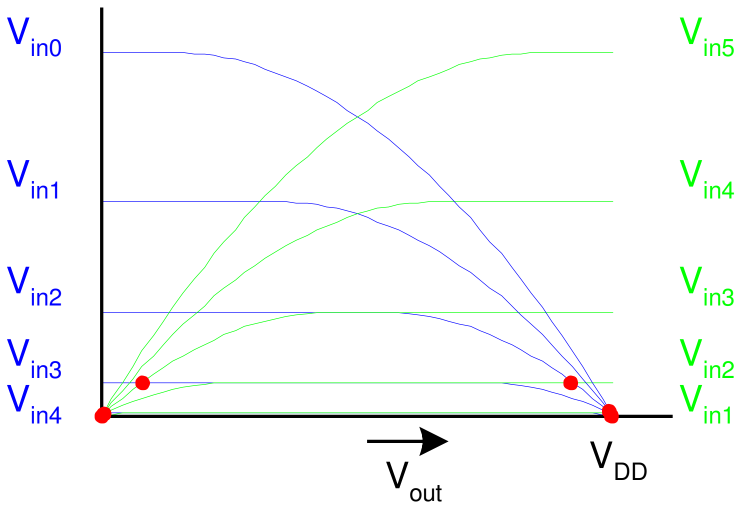


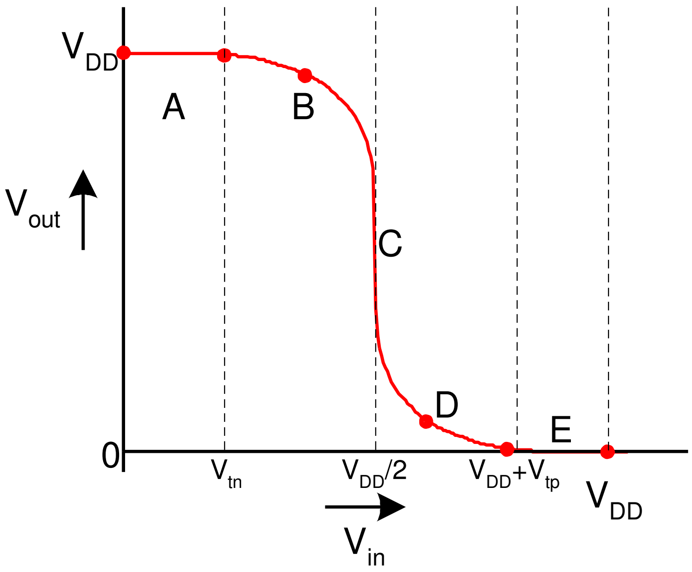
Load Line Summary



DC Transfer Curve

- Transcribe points onto V_{in} vs. V_{out} plot

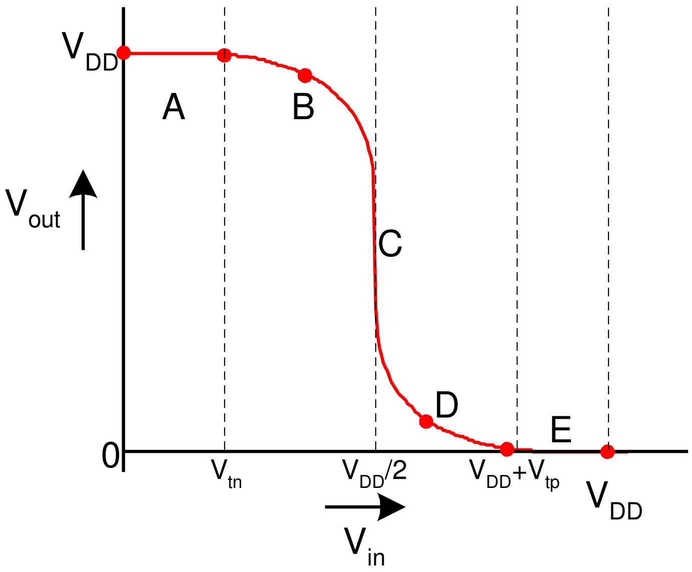




Operating Regions

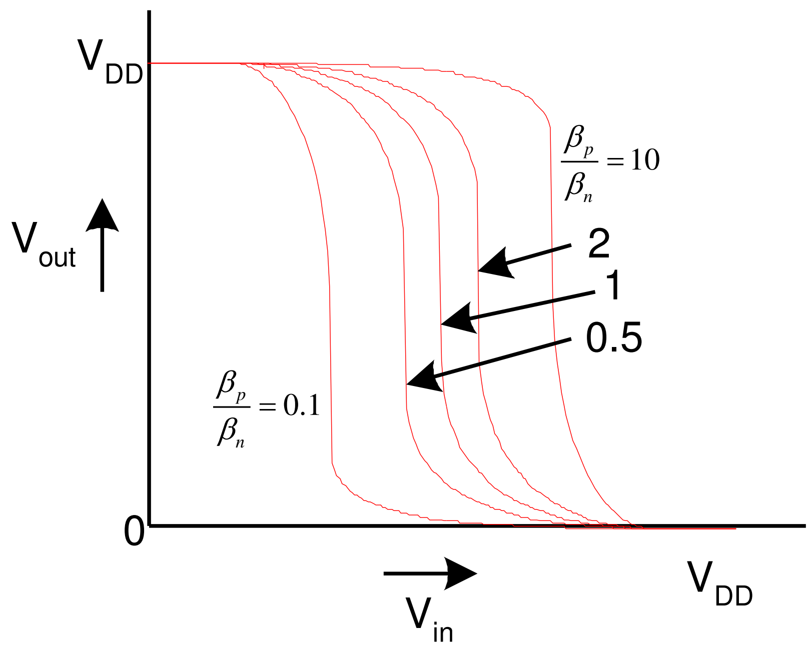
- Revisit transistor operating regions

Region	nMOS	pMOS
A	Cutoff	Linear
B	Saturation	Linear
C	Saturation	Saturation
D	Linear	Saturation
E	Linear	Cutoff



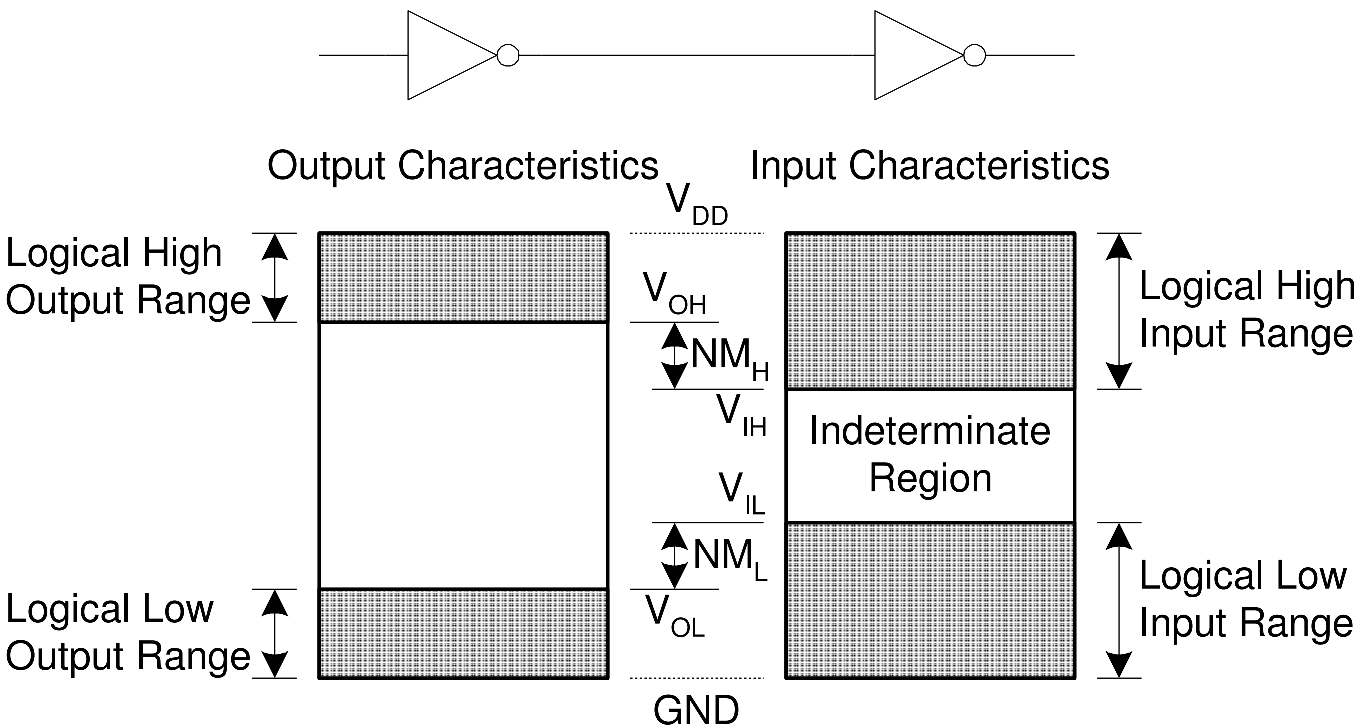
Beta Ratio

- If $\beta_p / \beta_n \neq 1$, switching point will move from $V_{DD}/2$
- Called *skewed* gate
- Other gates: collapse into equivalent inverter



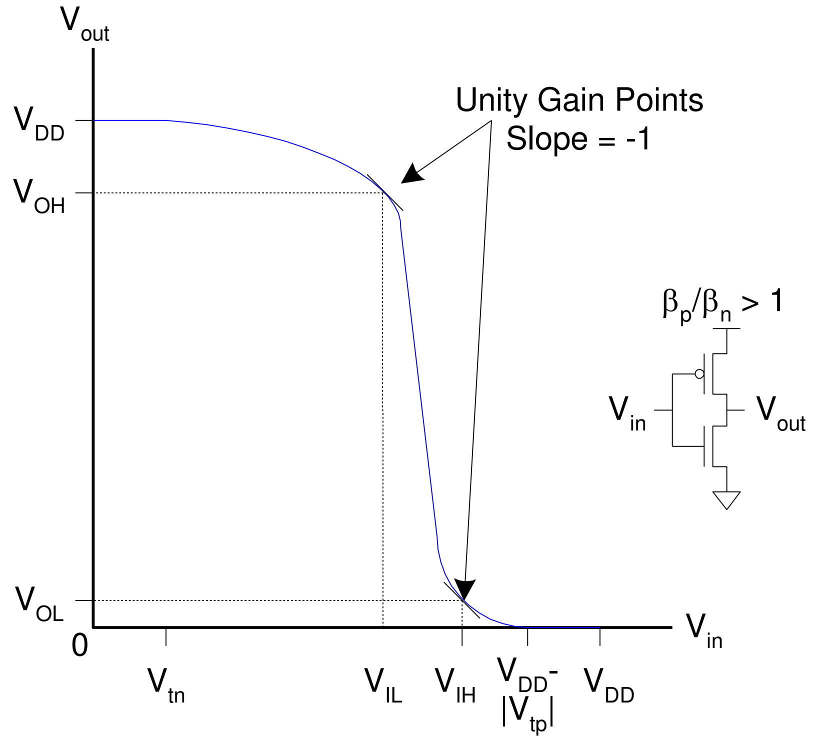
Noise Margins

- How much noise can a gate input see before it does not recognize the input?



Logic Levels

- To maximize noise margins, select logic levels at
  - unity gain point of DC transfer characteristic



Transient Response

- DC analysis tells us V_{out} if V_{in} is constant
- Transient analysis tells us $V_{out}(t)$ if $V_{in}(t)$ changes
  - Requires solving differential equations

- Input is usually considered to be a step or ramp
  - From 0 to V_{DD} or vice versa

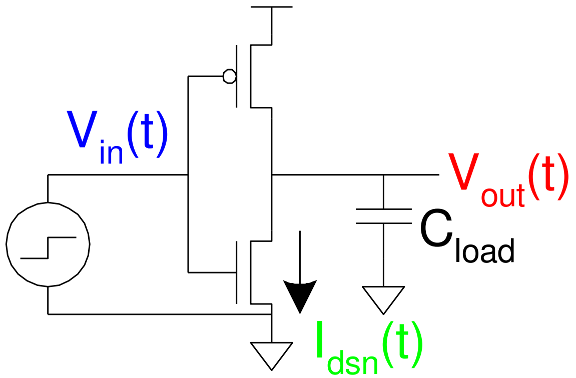
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) =$$

$$V_{out}(t < t_0) =$$

$$\frac{dV_{out}(t)}{dt} =$$



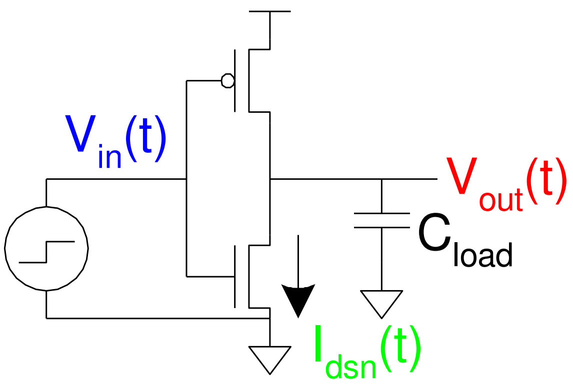
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) =$$

$$\frac{dV_{out}(t)}{dt} =$$



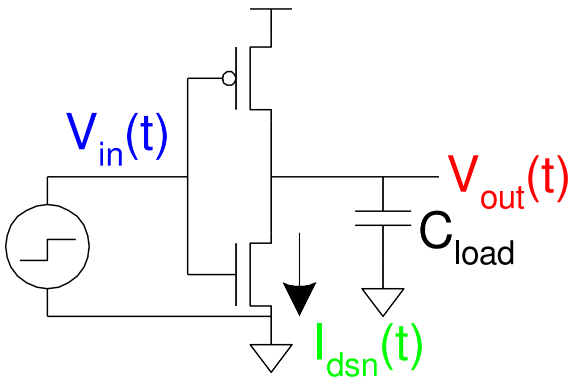
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} =$$



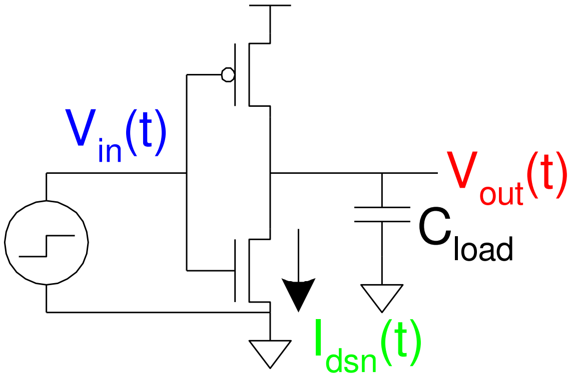
Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

$$\frac{dV_{out}(t)}{dt} = -\frac{I_{dsn}(t)}{C_{load}}$$



$$I_{dsn}(t) = \begin{cases} & t \le t_0 \\ & V_{out} > V_{DD} - V_t \\ & V_{out} < V_{DD} - V_t \end{cases}$$

Inverter Step Response

- Ex: find step response of inverter driving load cap

$$V_{in}(t) = u(t - t_0)V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

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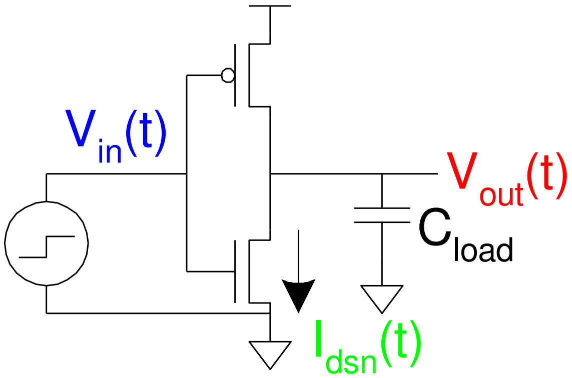
$$I_{dsn}(t) = \begin{cases} 0 & t \le t_0 \\ \frac{\beta}{2}\left(V_{DD} - V\right)^2 & V_{out} > V_{DD} - V_t \\ \beta\left(V_{DD} - V_t - \frac{V_{out}(t)}{2}\right)V_{out}(t) & V_{out} < V_{DD} - V_t \end{cases}$$



Inverter Step Response

- Ex: find step response of inverter driving load cap

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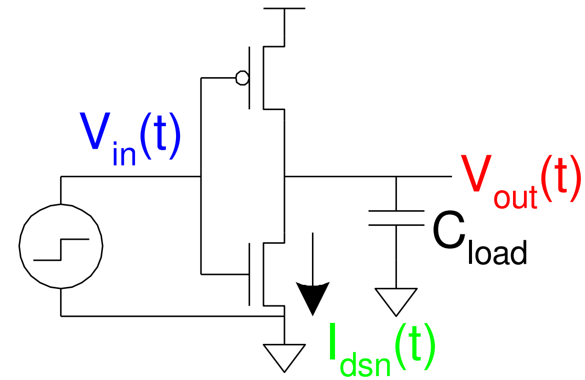
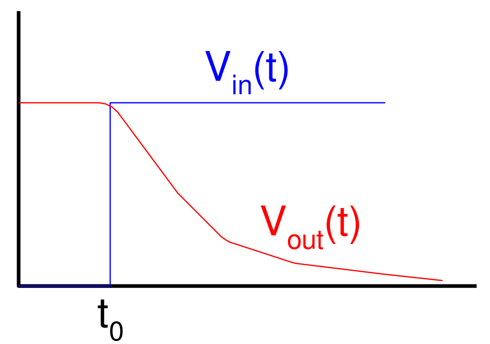




Delay Definitions

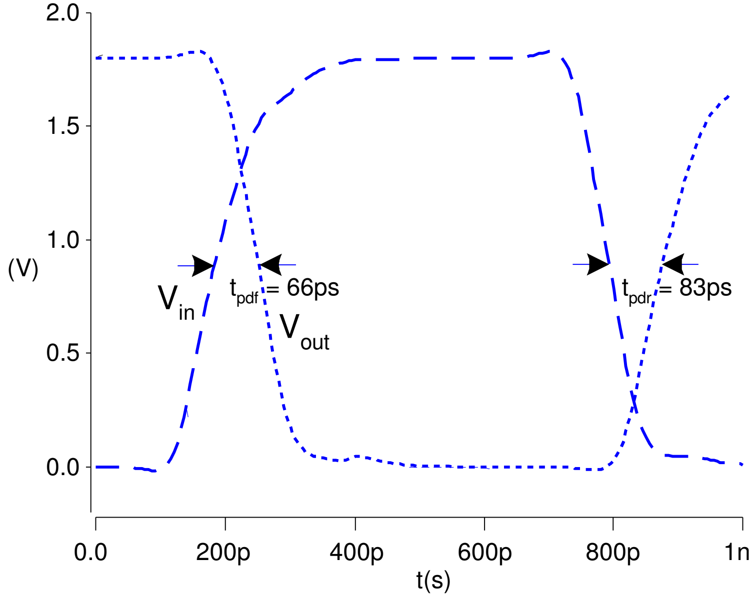
- $\mathbf{t_{pdr}}$: *rising propagation delay*
  - From input to rising output crossing $V_{DD}/2$
- $\mathbf{t_{pdf}}$: *falling propagation delay*
  - From input to falling output crossing $V_{DD}/2$
- $\mathbf{t_{pd}}$: *average propagation delay*
  - $t_{pd} = (t_{pdr} + t_{pdf})/2$
- $\mathbf{t_r}$: *rise time*
  - From output crossing 0.2 V_{DD} to 0.8 V_{DD}
- $\mathbf{t_f}$: *fall time*
  - From output crossing 0.8 V_{DD} to 0.2 V_{DD}

Delay Definitions

- $\mathbf{t_{cdr}}$: *rising contamination delay*
  - From input to rising output crossing $V_{DD}/2$
- $\mathbf{t_{cdf}}$: *falling contamination delay*
  - From input to falling output crossing $V_{DD}/2$
- $\mathbf{t_{cd}}$: *average contamination delay*
  - $t_{pd} = (t_{cdr} + t_{cdf})/2$

Simulated Inverter Delay

- Solving differential equations by hand is too hard
- SPICE simulator solves the equations numerically
  - Uses more accurate I-V models too!
- But simulations take time to write

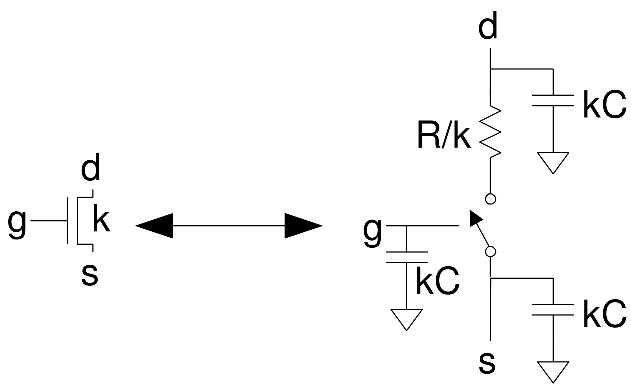


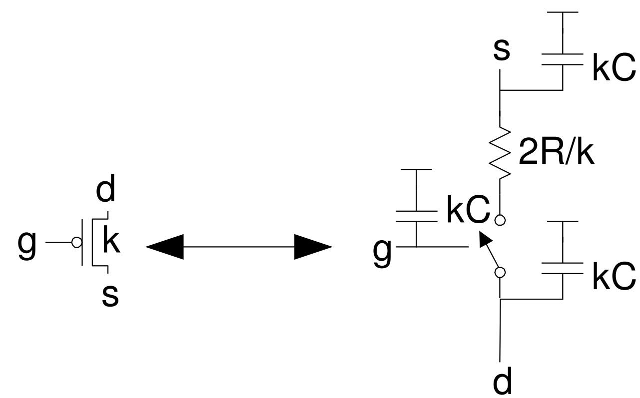
Delay Estimation

- We would like to be able to easily estimate delay
  - Not as accurate as simulation
- The step response usually looks like a 1st order RC response with a decaying exponential.
- Use RC delay models to estimate delay
  - C = total capacitance on output node
  - Use *effective resistance* R
  - So that $t_{pd} = RC$
- Characterize transistors by finding their effective R
  - Depends on average current as gate switches

RC Delay Models

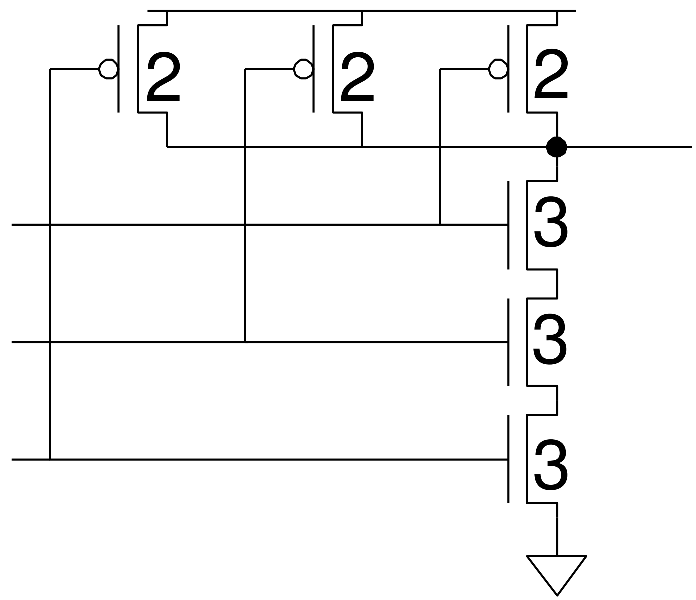
- Use equivalent circuits for MOS transistors
  - Ideal switch + capacitance and ON resistance
  - Unit nMOS has resistance R, capacitance C
  - Unit pMOS has resistance 2R, capacitance C
- Capacitance proportional to width
- Resistance inversely proportional to width





Example: 3-input NAND

- A 3-input NAND with transistor widths chosen to achieve effective rise and fall resistances equal to a unit inverter (R).

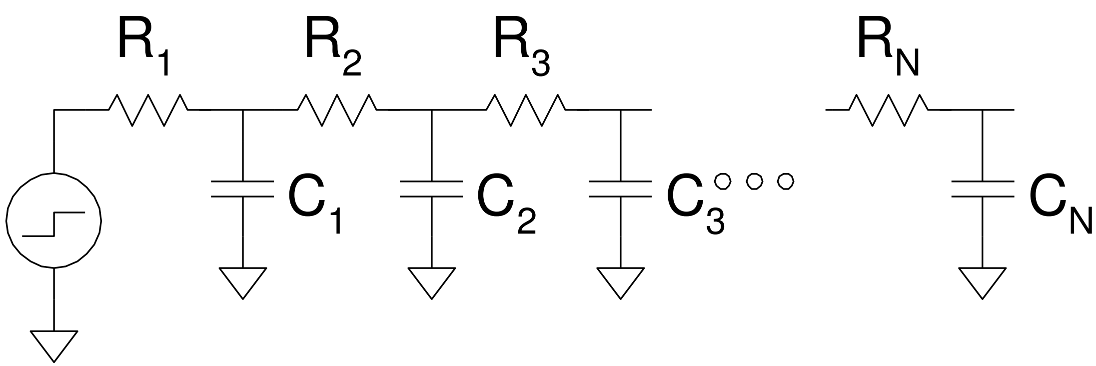


Elmore Delay

- ON transistors look like resistors
- Pullup or pulldown network modeled as *RC ladder*
- Elmore delay of RC ladder

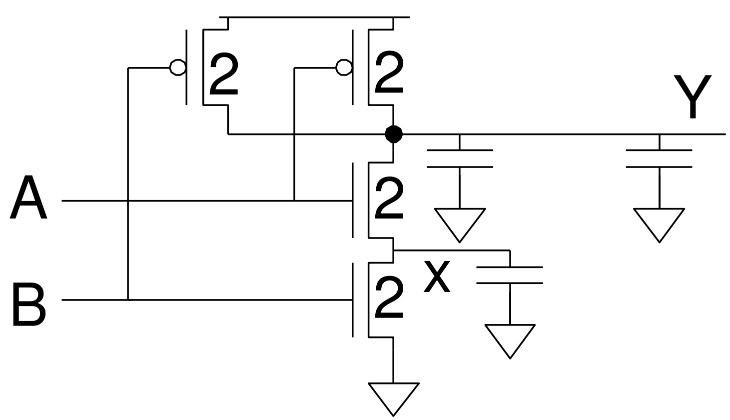
$$t_{pd} \approx \sum_{\text{nodes } i} R_{i-to-source} C_i$$

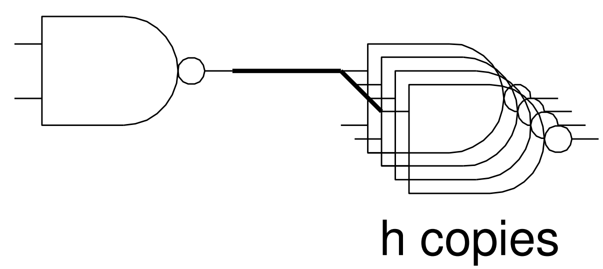
$$= R_1 C_1 + \left(R_1 + R_2\right) C_2 + \ldots + \left(R_1 + R_2 + \ldots + R_N\right) C_N$$



Example: 2-input NAND

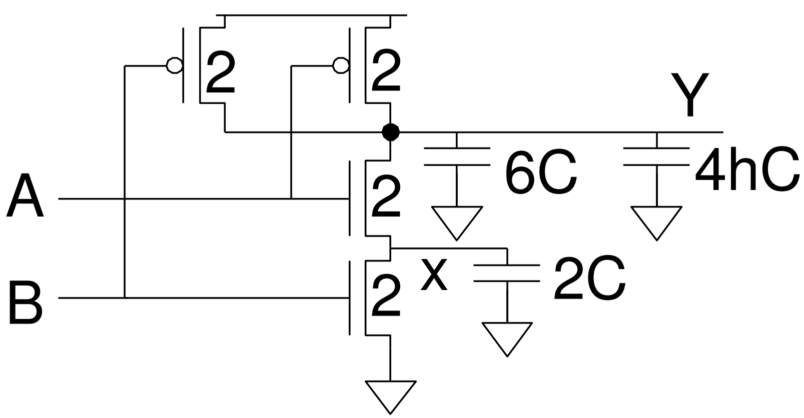
- Estimate worst-case rising and falling delay of 2-input NAND driving *h* identical gates.

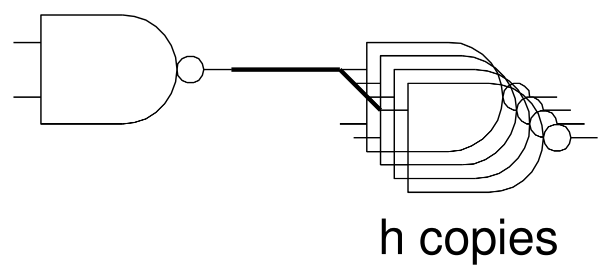




Example: 2-input NAND

- Estimate rising and falling propagation delays of a 2-input NAND driving *h* identical gates.





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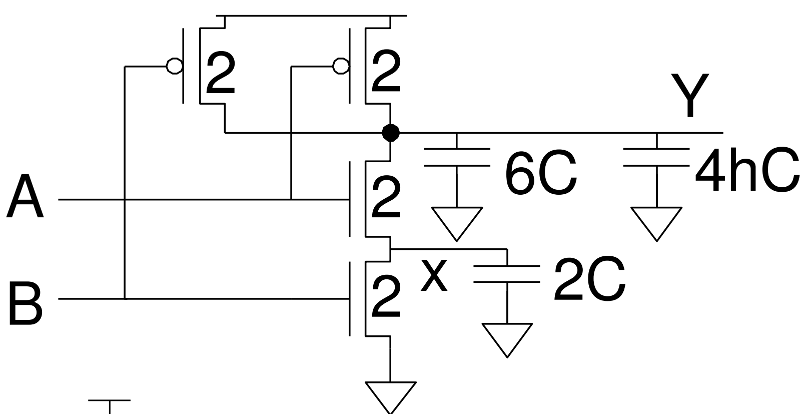
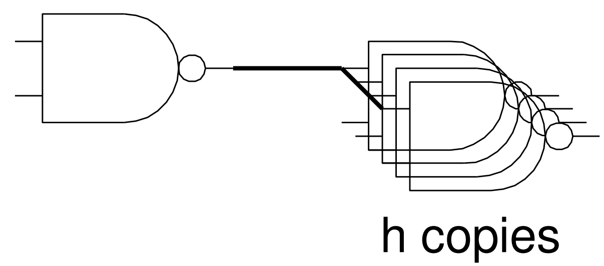
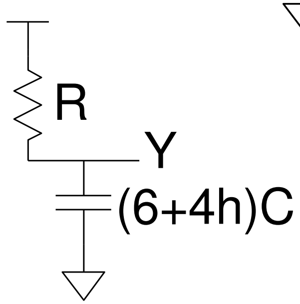


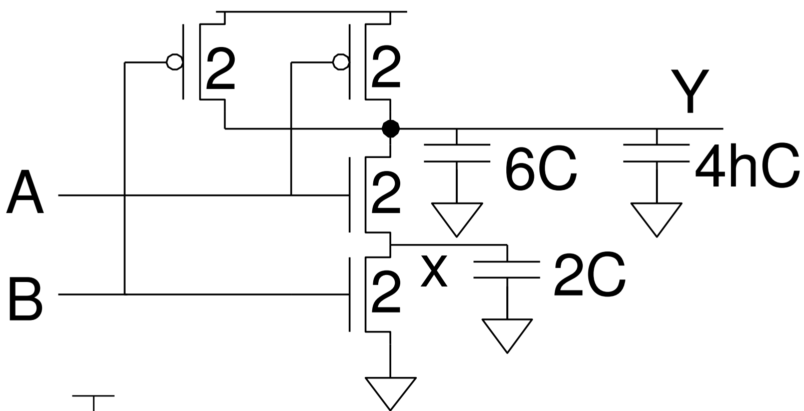


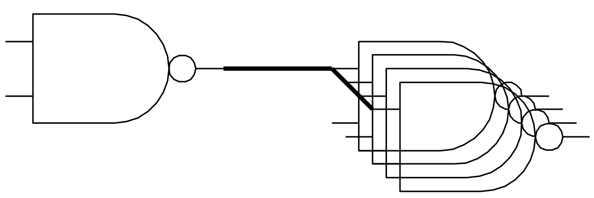


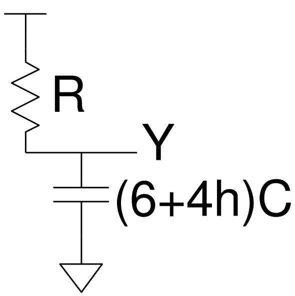
$$t_{pdr} =$$

Example: 2-input NAND

- Estimate rising and falling propagation delays of a 2-input NAND driving *h* identical gates.



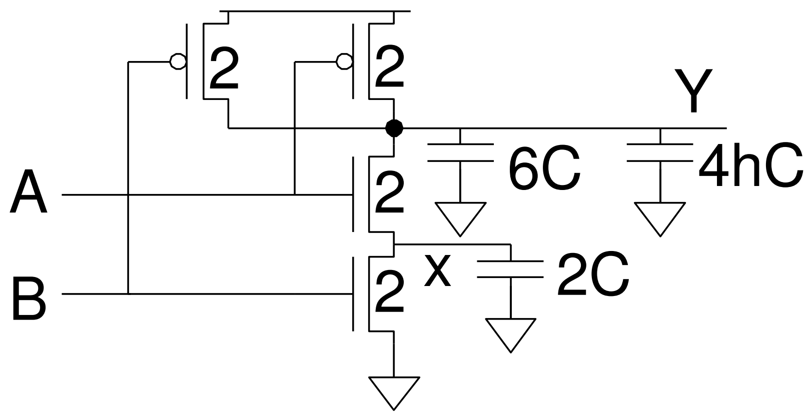


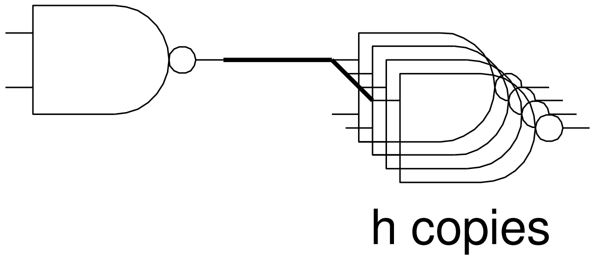


$$t_{pdr} = (6 + 4h)RC$$

Example: 2-input NAND

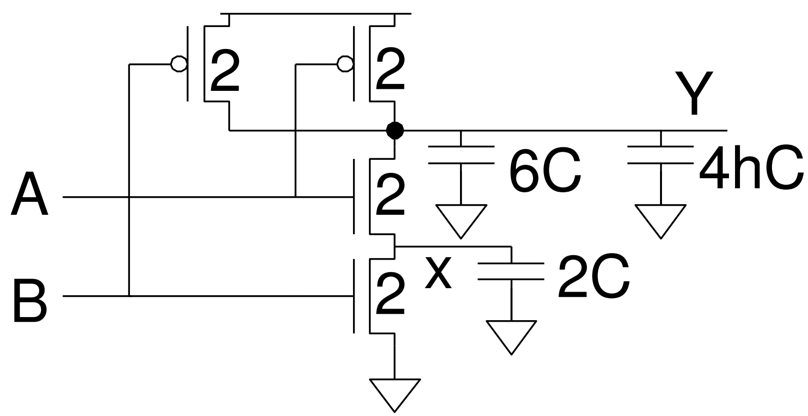
- Estimate rising and falling propagation delays of a 2-input NAND driving *h* identical gates.

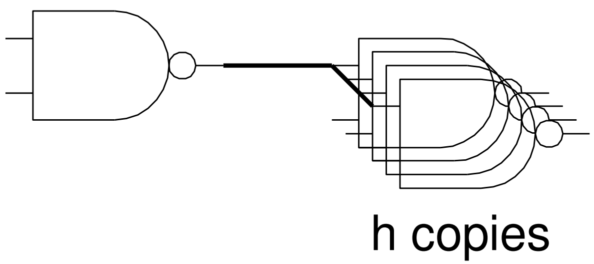


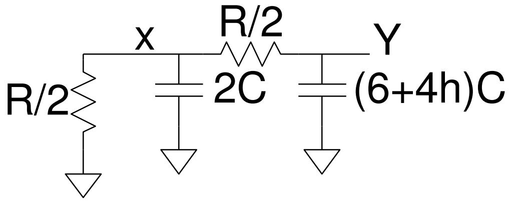


Example: 2-input NAND

- Estimate rising and falling propagation delays of a 2-input NAND driving *h* identical gates.



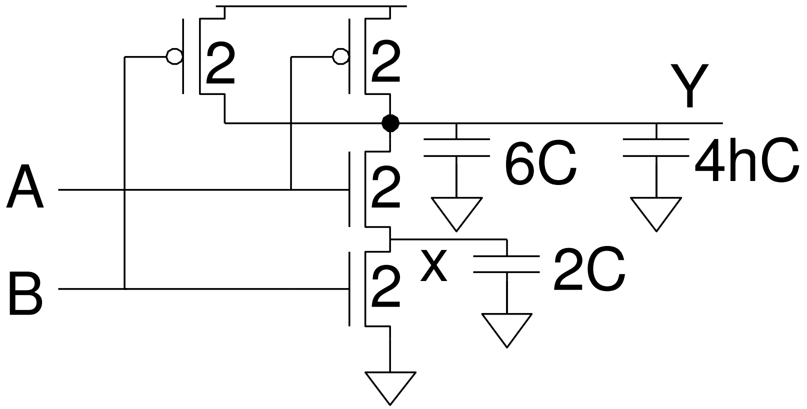


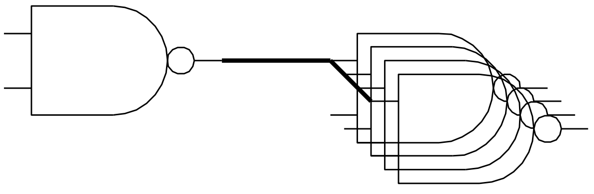


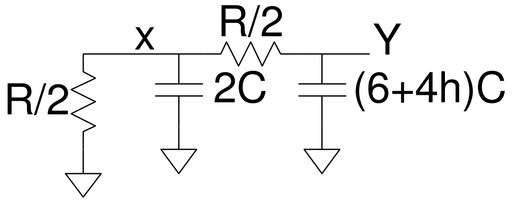
$$t_{pdf} =$$

Example: 2-input NAND

- Estimate rising and falling propagation delays of a 2-input NAND driving *h* identical gates.







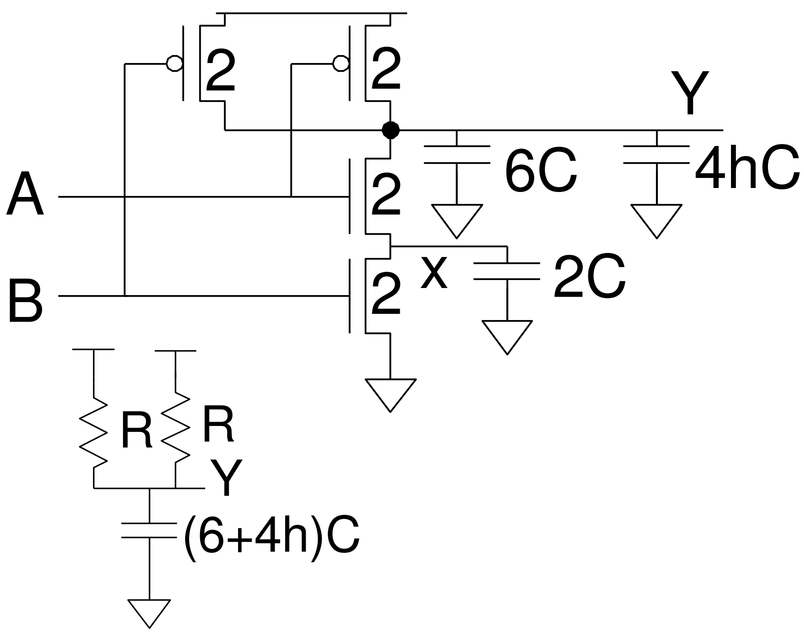
$$t_{pdf} = (2C)\left(\tfrac{R}{2}\right) + \left[(6+4h)C\right]\left(\tfrac{R}{2} + \tfrac{R}{2}\right)$$
$$= (7+4h)RC$$

Delay Components

- Delay has two parts
  - *Parasitic delay*
    - 6 or 7 RC
    - Independent of load
  - *Effort delay*
    - 4h RC
    - Proportional to load capacitance

Contamination Delay

- Best-case (contamination) delay can be substantially less than propagation delay.
- Ex: If both inputs fall simultaneously



$$t_{cdr} = \left(3 + 2h\right) RC$$