Unit-3
Control Charts

In general, how can we monitor quality…?

By observing variation in output measures!

1. Assignable variation: we can assess the cause
2. Common variation: variation that may not be possible to correct (*random variation, random noise*)

Statistical Process Control (SPC)

Every output measure has a target value and a level of "acceptable" variation (upper and lower tolerance limits)

SPC uses samples from output measures to estimate the mean and the variation (standard deviation)

Example

We want beer bottles to be filled with *12 FL OZ* $\pm$ *0.05 FL OZ*

Question:

How do we define the output measures?

In order to measure variation we need…

The average (mean) of the observations:

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

The standard deviation of the observations:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{X})^2}{N}}$$

Average & Variation example

Number of pepperoni's per pizza: 25, 25, 26, 25, 23, 24, 25, 27

Average:

Standard Deviation:

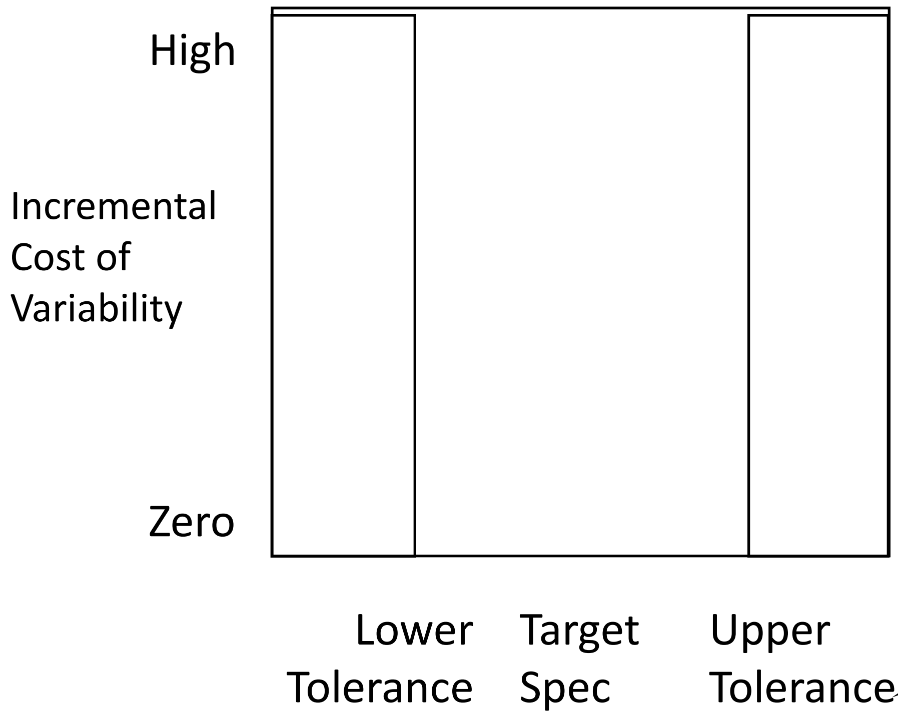
Number of pepperoni's per pizza: 25, 22, 28, 30, 27, 20, 25, 23

Average:

Standard Deviation:

Which pizza would you rather have?

When is a product good enough?

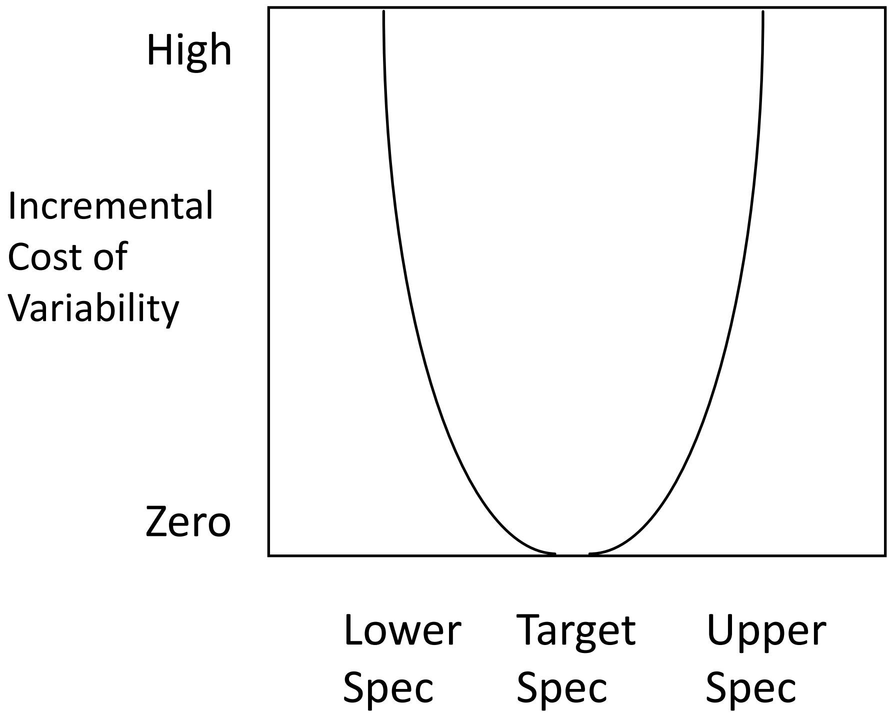


a.k.a
Upper/Lower <u>Design</u> Limits
(UDL, LDL)
Upper/Lower <u>Spec</u> Limits
(USL, LSL)
Upper/Lower <u>Tolerance</u> Limits
(UTL, LTL)

Traditional View

The “Goalpost” Mentality

But are all ‘good’ products equal?



Taguchi’s View
“Quality Loss Function”
(QLF)

LESS VARIABILITY implies BETTER PERFORMANCE !

Capability Index (C_{pk})

It shows how well the performance measure fits the design specification based on a given tolerance level

A process is $k\sigma$ capable if

$$\bar{X} + k\sigma \leq UTL \quad \text{and} \quad \bar{X} - k\sigma \geq LTL$$

$$1 \leq \frac{UTL - \bar{X}}{k\sigma} \quad \text{and} \quad \frac{\bar{X} - LTL}{k\sigma} \geq 1$$

Capability Index (C_{pk})

Another way of writing this is to calculate the capability index:

$$C_{pk} = \min\left\{ \frac{\bar{X} - LTL}{k\sigma}, \frac{UTL - \bar{X}}{k\sigma} \right\}$$

$C_{pk} < 1$ means process is not capable at the $k\sigma$ level

$C_{pk} >= 1$ means process is capable at the $k\sigma$ level

Accuracy and Consistency

We say that a process is accurate if its mean is close to the target T.

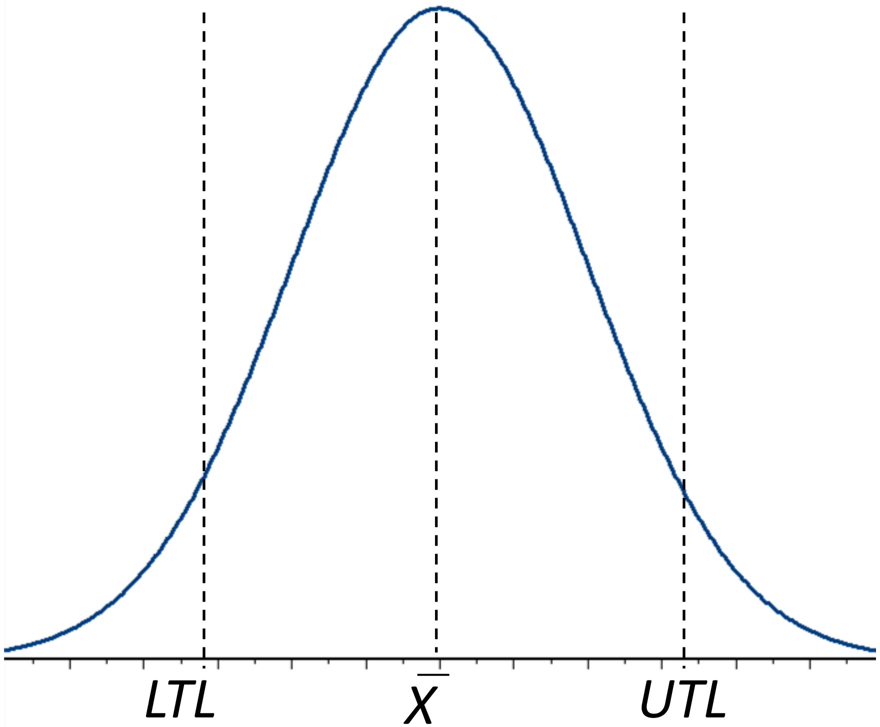
We say that a process is consistent if its standard deviation is low.

Example 1: Capability Index (C_{pk})

$\bar{X} = 10$ and $\sigma = 0.5$

$LTL = 9$

$UTL = 11$



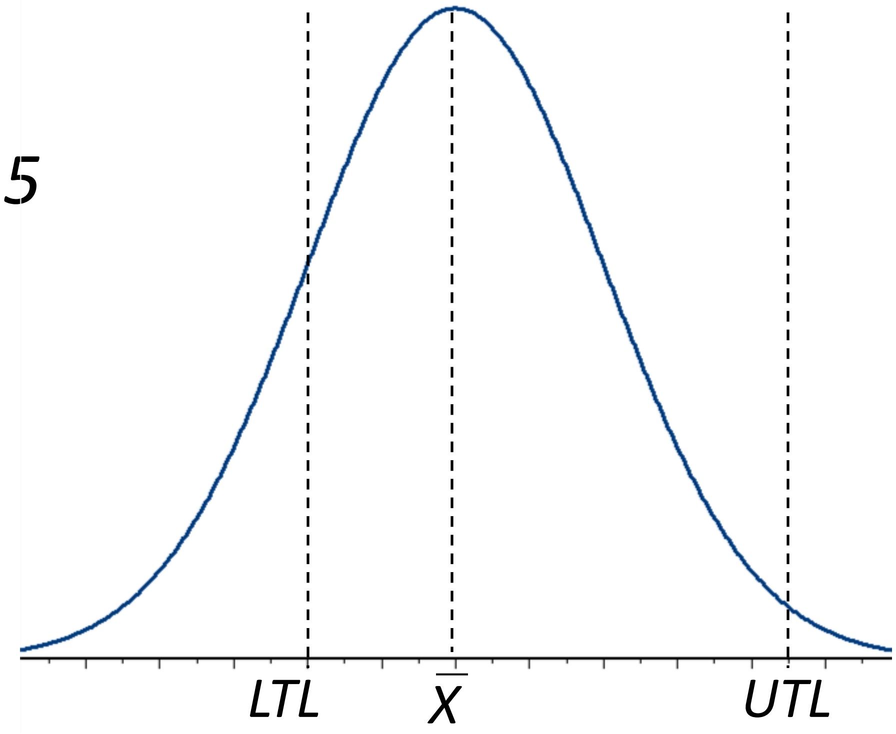
$$C_{pk} = \min\left\{\frac{10-9}{3\times 0.5} \quad \text{or} \quad \frac{11-10}{3\times 0.5}\right\} = 0.667$$

Example 2: Capability Index (C_{pk})

$\overline{X} = 9.5$ and $\sigma = 0.5$

$LTL = 9$

$UTL = 11$

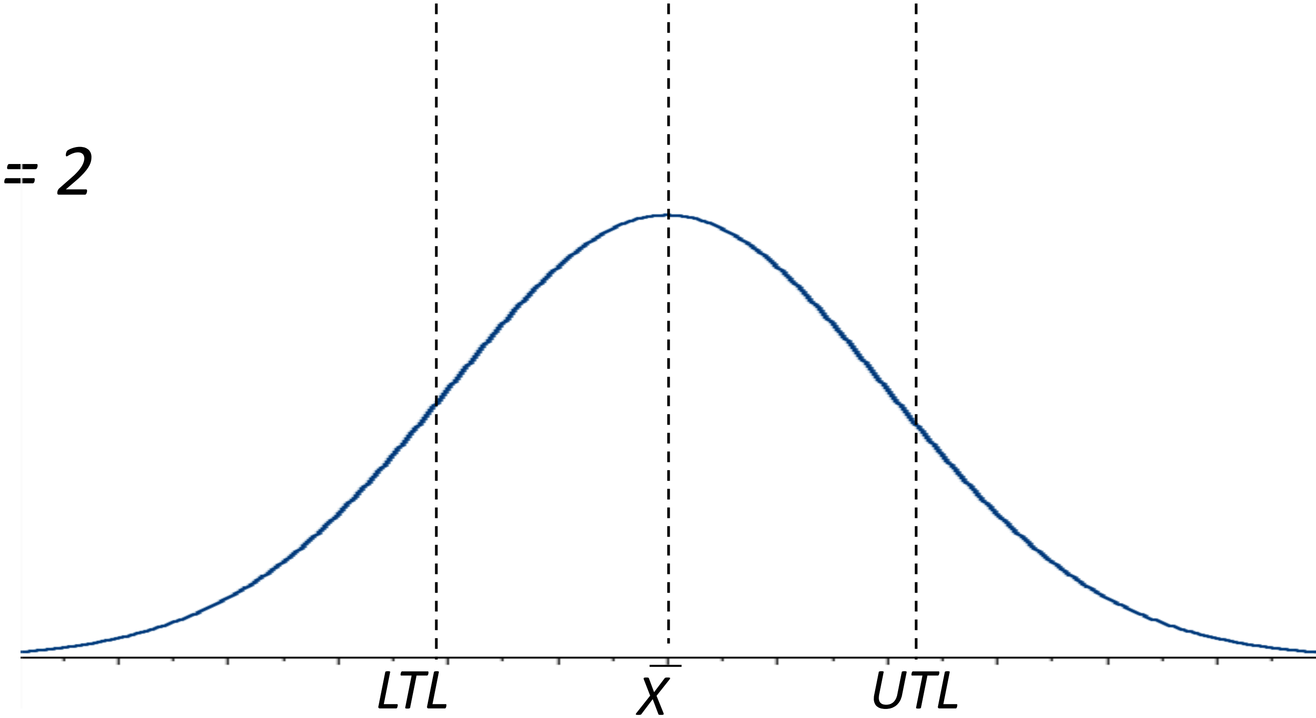


Example 3: Capability Index (C_{pk})

$\overline{X} = 10$ and $\sigma = 2$

$LTL = 9$

$UTL = 11$



Example

Consider the capability of a process that puts pressurized grease in an aerosol can. The design specs call for an average of 60 pounds per square inch (psi) of pressure in each can with an upper tolerance limit of 65psi and a lower tolerance limit of 55psi. A sample is taken from production and it is found that the cans average 61psi with a standard deviation of 2psi.

1. Is the process capable at the 3σ level?
2. What is the probability of producing a defect?

Solution

LTL = 55 UTL = 65   σ = 2 $\bar{X} = 61$

$$C_{pk} = \min(\frac{\bar{X} - LTL}{3\sigma}, \frac{UTL - \bar{X}}{3\sigma})$$

$$C_{pk} = \min(\frac{61 - 55}{6}, \frac{65 - 61}{6}) = \min(1, 0.6667) = 0.6667$$

No, the process is not capable at the 3σ level.

Solution

$$
\begin{aligned}
P(\text{defect}) &= P(X<55) + P(X>65) \\
&= P(X<55) + 1 - P(X<65) \\
&= P(Z<(55-61)/2) + 1 - P(Z<(65-61)/2) \\
&= P(Z<-3) + 1 - P(Z<2) \\
&= G(-3)+1-G(2) \\
&= 0.00135 + 1 - 0.97725 \text{ (from standard normal table)} \\
&= 0.0241
\end{aligned}
$$

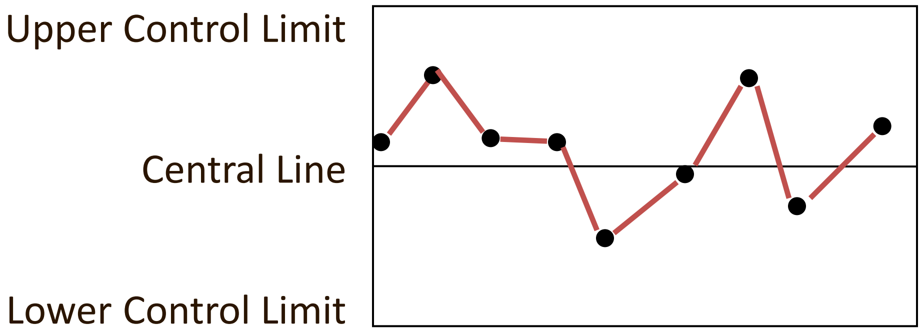
2.4% of the cans are defective.

Example (contd)

Suppose another process has a sample mean of 60.5 and a standard deviation of 3.

Which process is more accurate? This one.
Which process is more consistent? The other one.

Control Charts



Control charts tell you when a process measure is exhibiting abnormal behavior.

Two Types of Control Charts

- $\overline{X}$/R Chart:

  This is a plot of *averages* and *ranges* over time (used for performance measures that are *variables*)

- p Chart:

  This is a plot of *proportions* over time (used for performance measures that are *yes/no attributes*)

Statistical Process Control with *p* Charts

When should we use *p* charts?

1. When decisions are simple "yes" or "no" by inspection
2. When the sample sizes are large enough (>50)

Sample (day)	Items	Defective	Percentage
1	200	10	0.050
2	200	8	0.040
3	200	9	0.045
4	200	13	0.065
5	200	15	0.075
6	200	25	0.125
7	200	16	0.080

Statistical Process Control with *p* Charts

Let's assume that we take *t* samples of size *n* …

$$\overline{p} = \frac{\text{total number of "defects"}}{(\text{number of samples}) \times (\text{sample size})}$$

$$s_{\overline{p}} = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

$$UCL = \overline{p} + z s_{\overline{p}}$$

$$LCL = \overline{p} - z s_{\overline{p}}$$

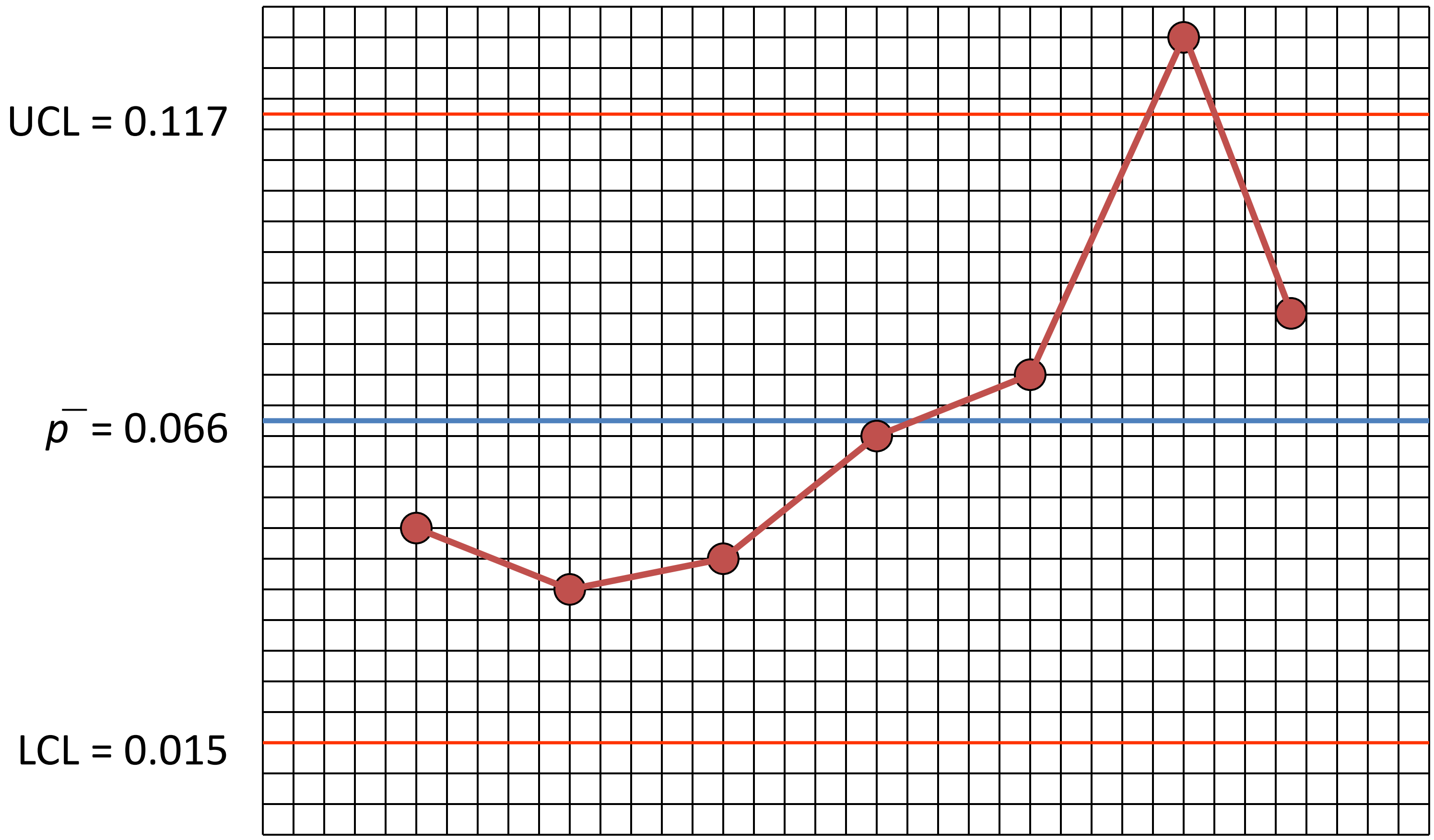
Statistical Process Control with *p* Charts

$$\overline{p} = \frac{80}{6 \times 200} = \frac{1}{15} = 0.066$$

$$s_{\overline{p}} = \sqrt{\frac{0.066(1-0.066)}{200}} = 0.017$$

$$UCL = 0.066 \ + \ 3 \times 0.017 \ = \ 0.117$$
$$LCL = 0.066 \ - \ 3 \times 0.017 \ = \ 0.015$$

Statistical Process Control with *p* Charts



Statistical Process Control with *X/R* Charts

When should we use *X/R* charts?

1. It is not possible to label “good” or “bad”
2. If we have relatively smaller sample sizes (<20)

Statistical Process Control with *X/R* Charts

Take t samples of size n (sample size should be 5 or more)

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\overline{X}$ is the mean for each sample

$$R = \max\{x_i\} - \min\{x_i\}$$

R is the range between the highest and the lowest for each sample

Statistical Process Control with *X/R* Charts

$$\overline{\overline{X}} = \frac{1}{t}\sum_{j=1}^{t}\overline{X}_j$$

$\overline{\overline{X}}$ is the average of the averages.

$$\overline{R} = \frac{1}{t}\sum_{j=1}^{t}R_j$$

$\overline{R}$ is the average of the ranges

Statistical Process Control with *X/R* Charts

define the upper and lower control limits…

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$$

$$UCL_{\overline{R}} = D_4\overline{R}$$

$$LCL_{\overline{R}} = D_3\overline{R}$$

Read A_2, D_3, D_4 from Table TN 8.7

Example: SPC for bottle filling…

Sample	Observation (x_i)					Average	Range (R)
1	11.90	11.92	12.09	11.91	12.01		
2	12.03	12.03	11.92	11.97	12.07		
3	11.92	12.02	11.93	12.01	12.07		
4	11.96	12.06	12.00	11.91	11.98		
5	11.95	12.10	12.03	12.07	12.00		
6	11.99	11.98	11.94	12.06	12.06		
7	12.00	12.04	11.92	12.00	12.07		
8	12.02	12.06	11.94	12.07	12.00		
9	12.01	12.06	11.94	11.91	11.94		
10	11.92	12.05	11.92	12.09	12.07		

Example: SPC for bottle filling…

Calculate the average and the range for each sample…

Sample	Observation (x_i)					Average	Range (R)
1	11.90	11.92	12.09	11.91	12.01	11.97	0.19
2	12.03	12.03	11.92	11.97	12.07	12.00	0.15
3	11.92	12.02	11.93	12.01	12.07	11.99	0.15
4	11.96	12.06	12.00	11.91	11.98	11.98	0.15
5	11.95	12.10	12.03	12.07	12.00	12.03	0.15
6	11.99	11.98	11.94	12.06	12.06	12.01	0.12
7	12.00	12.04	11.92	12.00	12.07	12.01	0.15
8	12.02	12.06	11.94	12.07	12.00	12.02	0.13
9	12.01	12.06	11.94	11.91	11.94	11.97	0.15
10	11.92	12.05	11.92	12.09	12.07	12.01	0.17

Then…

$$\overline{\overline{X}} = 12.00$$

is the *average* of the *averages*

$$\overline{R} = 0.15$$

is the *average* of the *ranges*

Finally…

Calculate the upper and lower control limits?

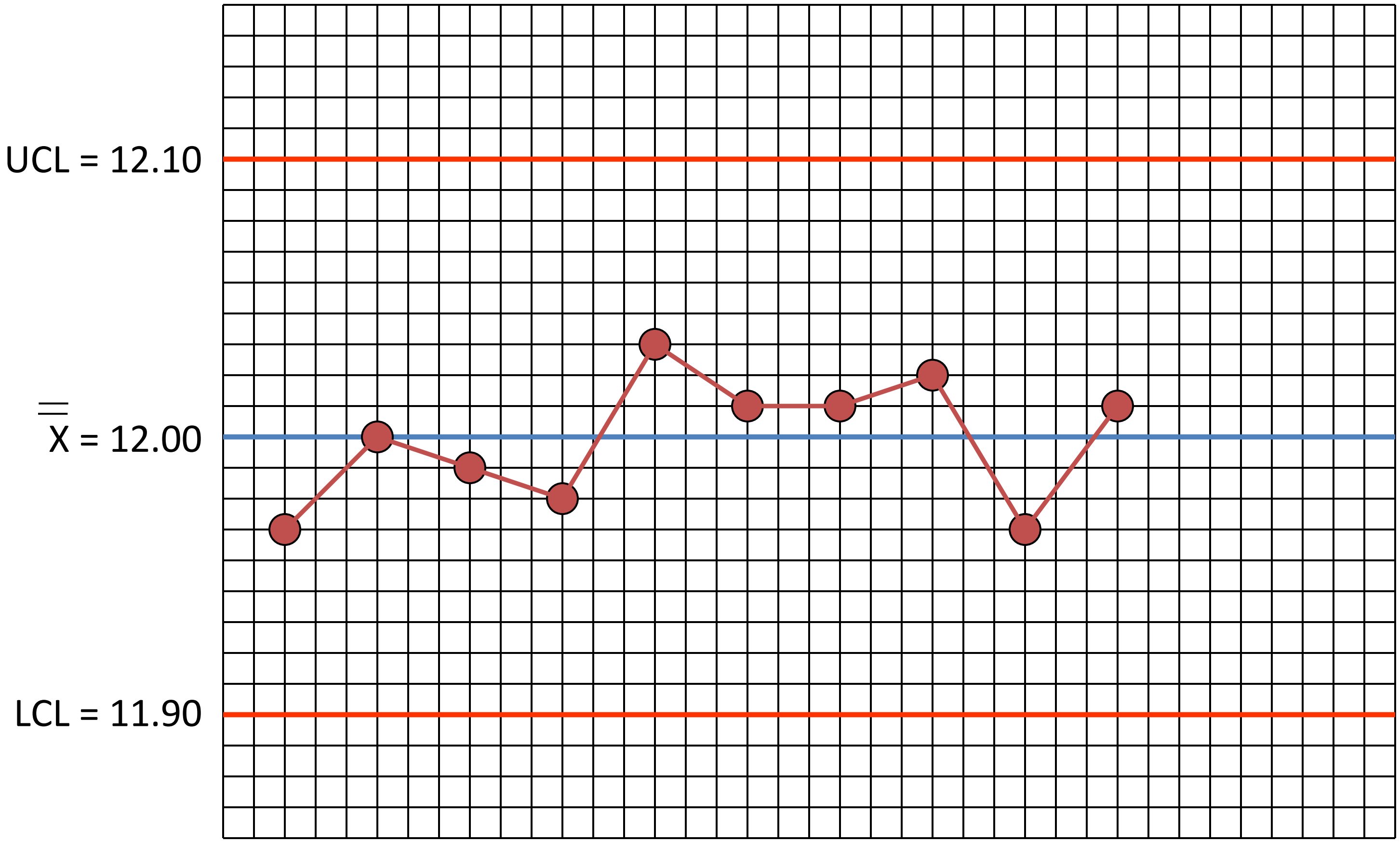
$$UCL_{\overline{X}} = 12.00 + 0.58 \times 0.15 = 12.09$$

$$LCL_{\overline{X}} = 12.00 - 0.58 \times 0.15 = 11.91$$

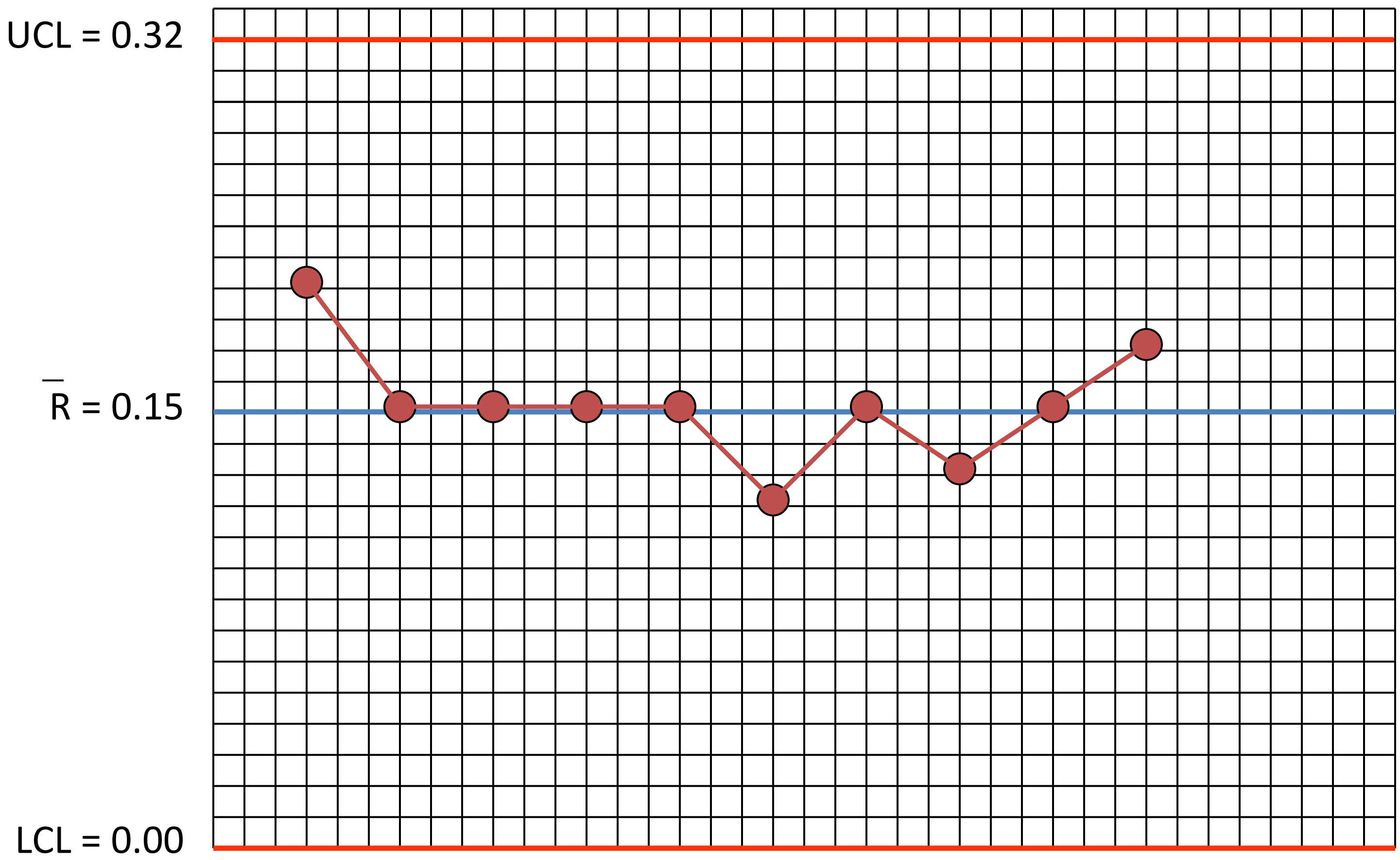
$$UCL_{\overline{R}} = 2.11 \times 0.15 = 1.22$$

$$LCL_{\overline{R}} = 0 \times 0.15 = 0$$

The X Chart

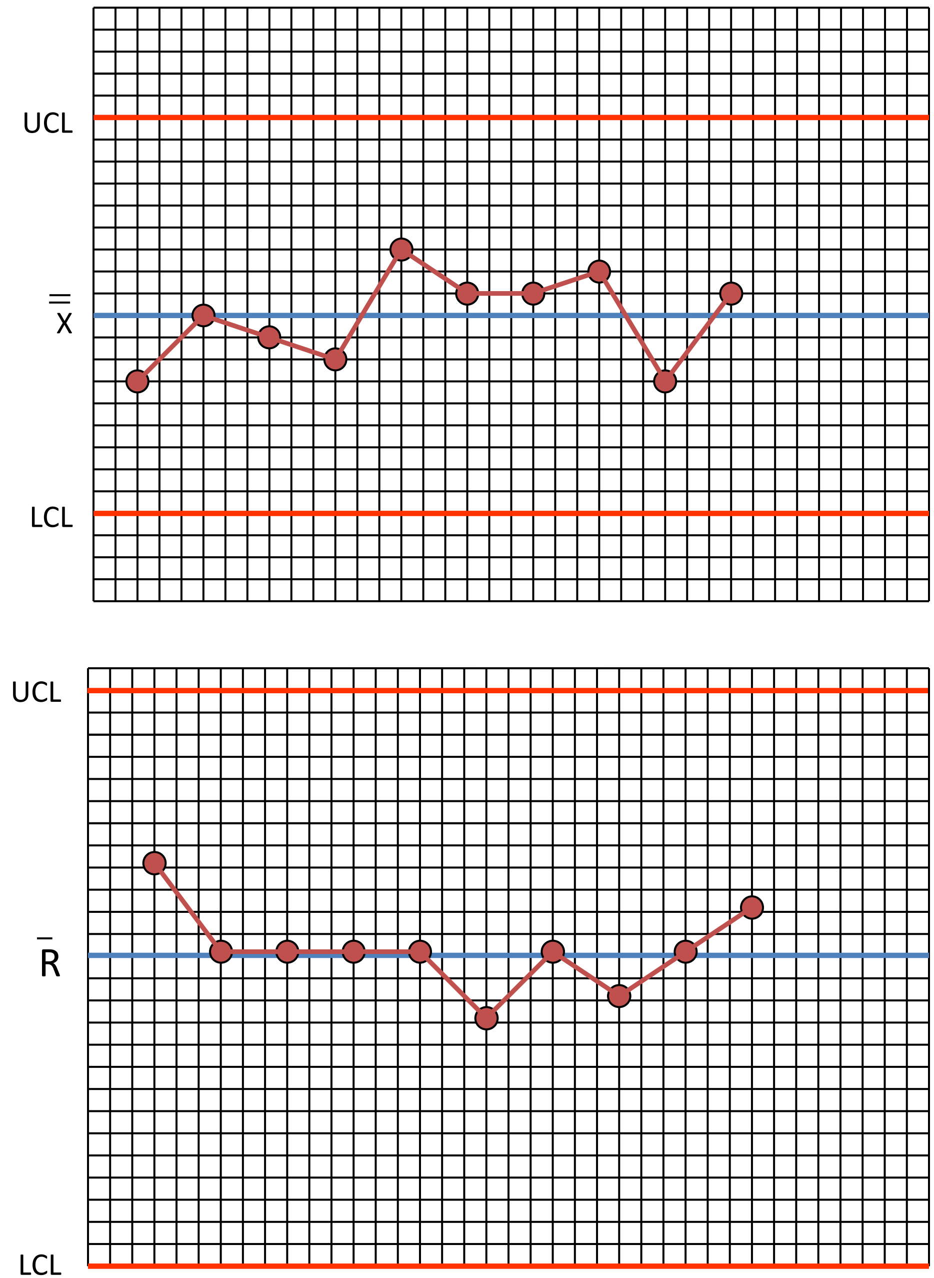


The R Chart



The X/R Chart

What can you conclude?



Thank you