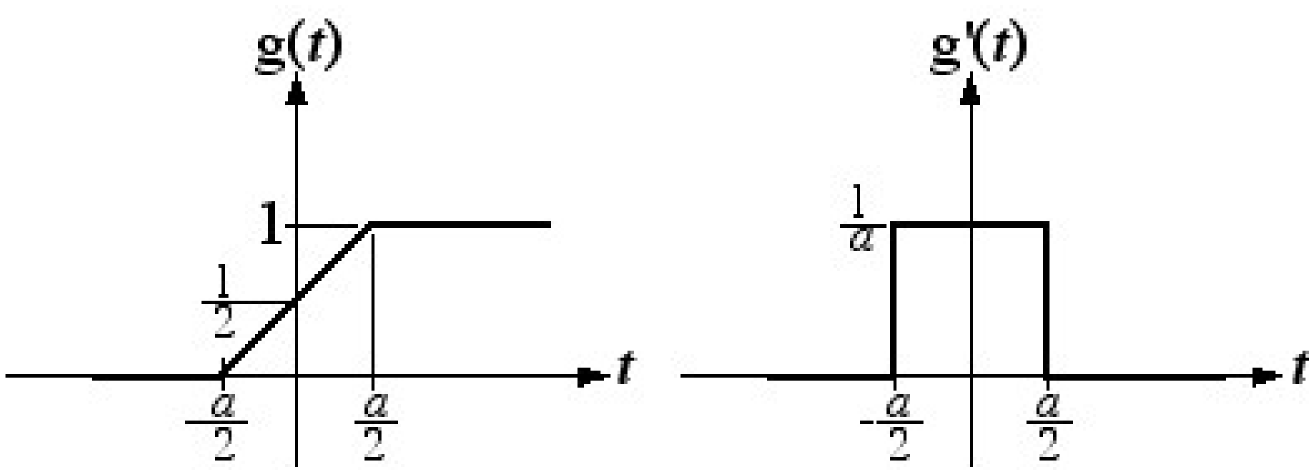
Commonly used signals (in continuous-time as well as in discrete-time)

Unit Impulse Function

As a approaches zero, $\mathrm{g}(t)$ approaches a unit step and $\mathrm{g}'(t)$ approaches a unit impulse

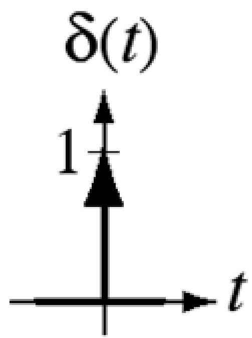


Functions that approach unit step and unit impulse

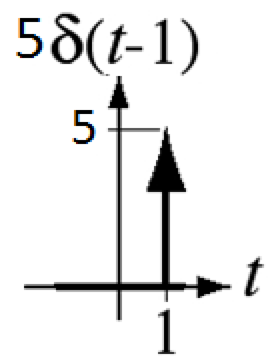
So unit impulse function is the **derivative** of the unit step function or unit step is the integral of the unit impulse function

Representation of Impulse Function

The area under an impulse is called its strength or weight. It is represented graphically by a vertical arrow. An impulse with a strength of one is called a unit impulse.



Representation of Unit Impulse



Shifted Impulse of Amplitude 5

Properties of the Impulse Function

The Sampling Property

$$\int_{-\infty}^{\infty} g(t)\,\delta(t-t_0)\,dt = g(t_0)$$

The Scaling Property

$$\delta\big(a(t-t_0)\big) = \frac{1}{|a|}\,\delta(t-t_0)$$

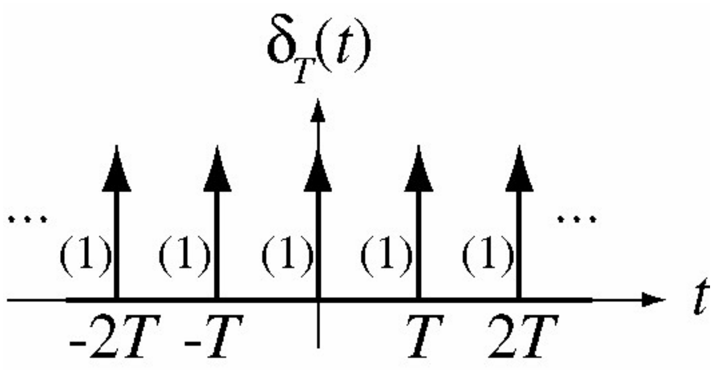
The Replication Property

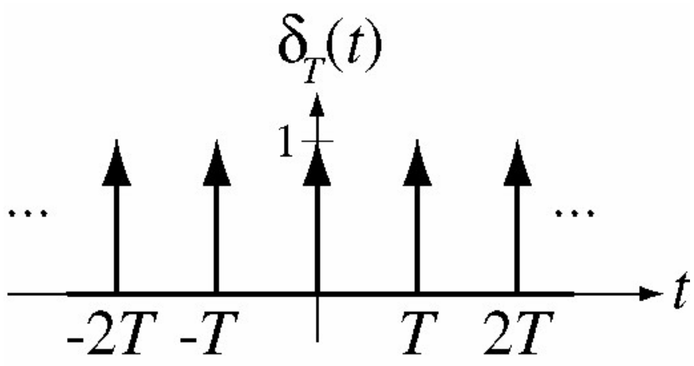
$$g(t) \otimes \delta(t) = g(t)$$

Unit Impulse Train

The unit impulse train is a sum of infinitely uniformly-spaced impulses and is given by

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad , \quad n \text{ an integer}$$

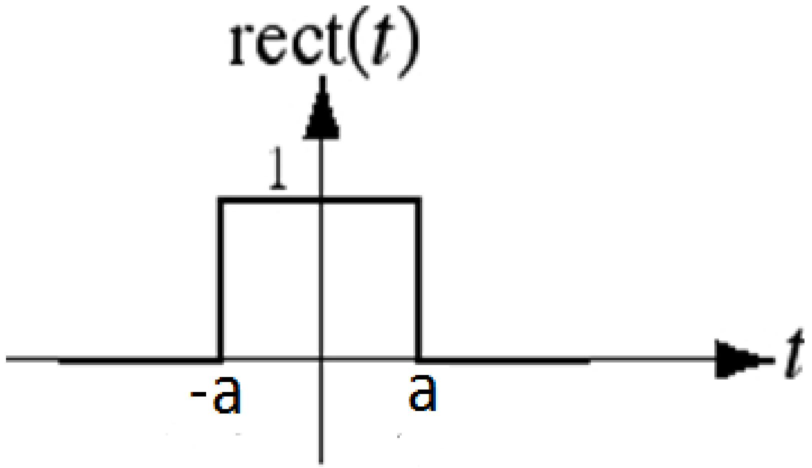




The Unit Rectangle Function

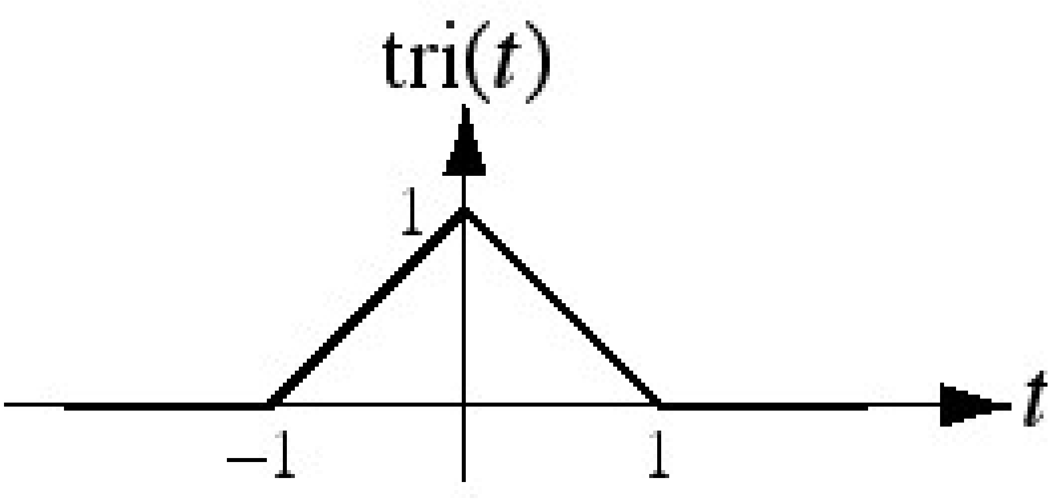
The unit rectangle or gate signal can be represented as combination of two shifted unit step signals as shown

$$rect(t) = u(t+a)-u(t-a)$$



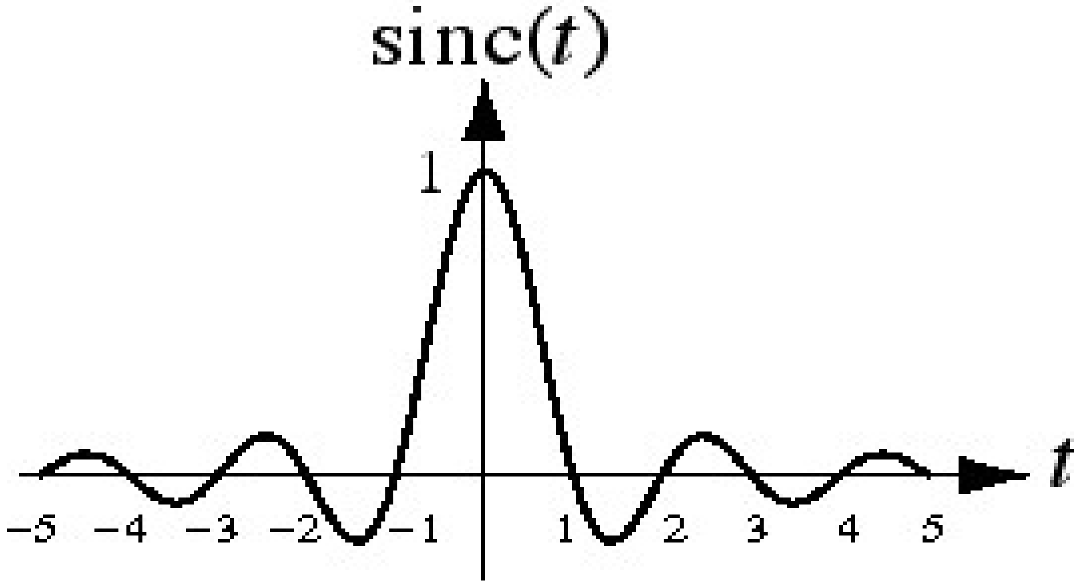
The Unit Triangle Function

A triangular pulse whose height and area are both one but its base width is not, is called unit triangle function. The unit triangle is related to the unit rectangle through an operation called **convolution**.



Sinc Function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



Discrete-Time Signals

- **Sampling** is the acquisition of the values of a continuous-time signal at discrete points in time
- x(t) is a continuous-time signal, x[n] is a discrete-time signal

$$\mathrm{x}[n] = \mathrm{x}(nT_s) \text{ where } T_s \text{ is the time between samples}$$

Discrete Time Exponential and Sinusoidal Signals

- DT signals can be defined in a manner analogous to their continuous-time counter part

$x[n] = A \sin(2\Pi n/N_o+\theta)$

$= A \sin(2\Pi F_o n+\theta)$

Discrete Time Sinusoidal Signal

Discrete Time Exponential Signal

$x[n] = a^n$

n = the discrete time

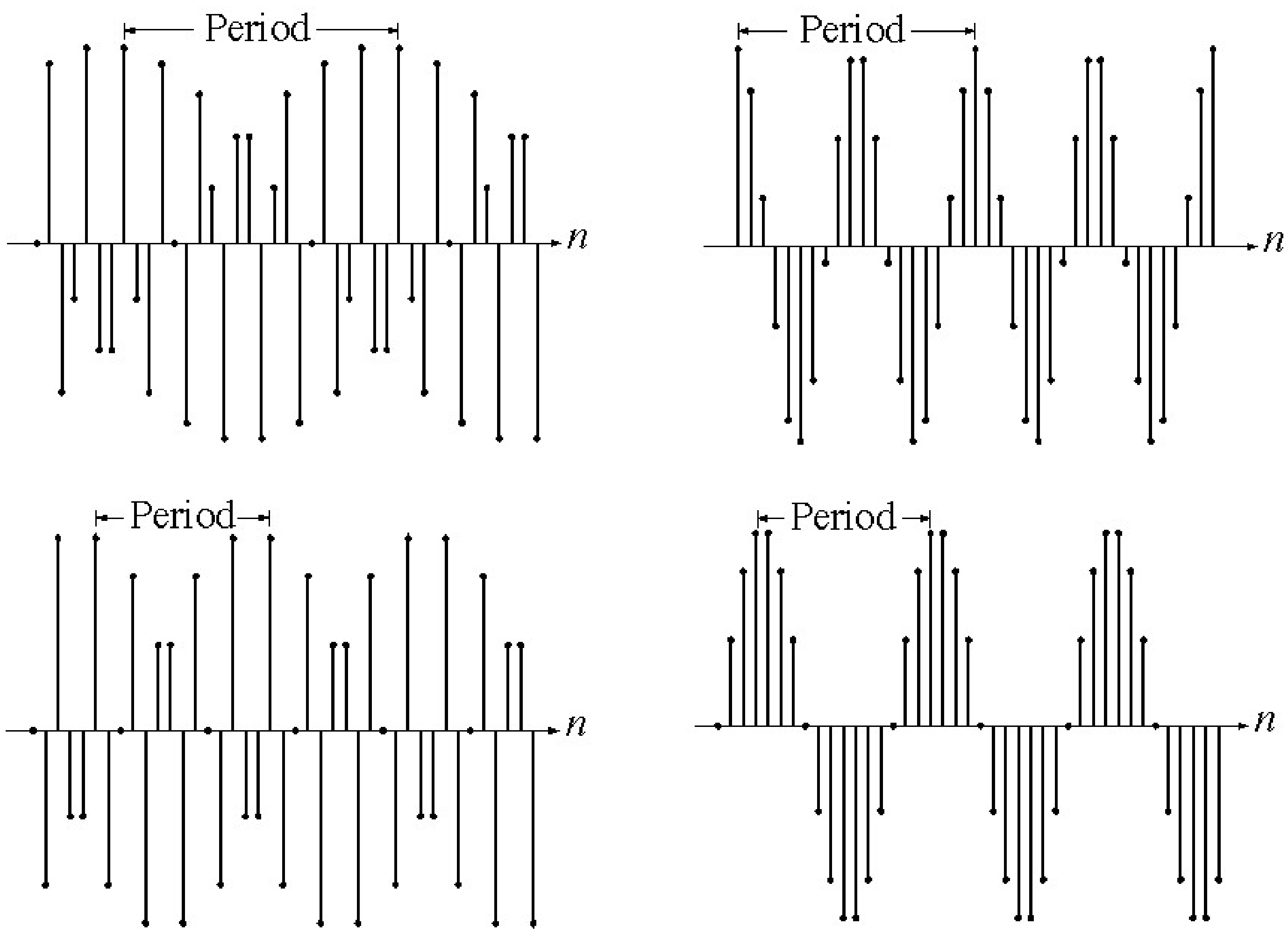
A = amplitude

θ = phase shifting radians,

N_o = Discrete Period of the wave

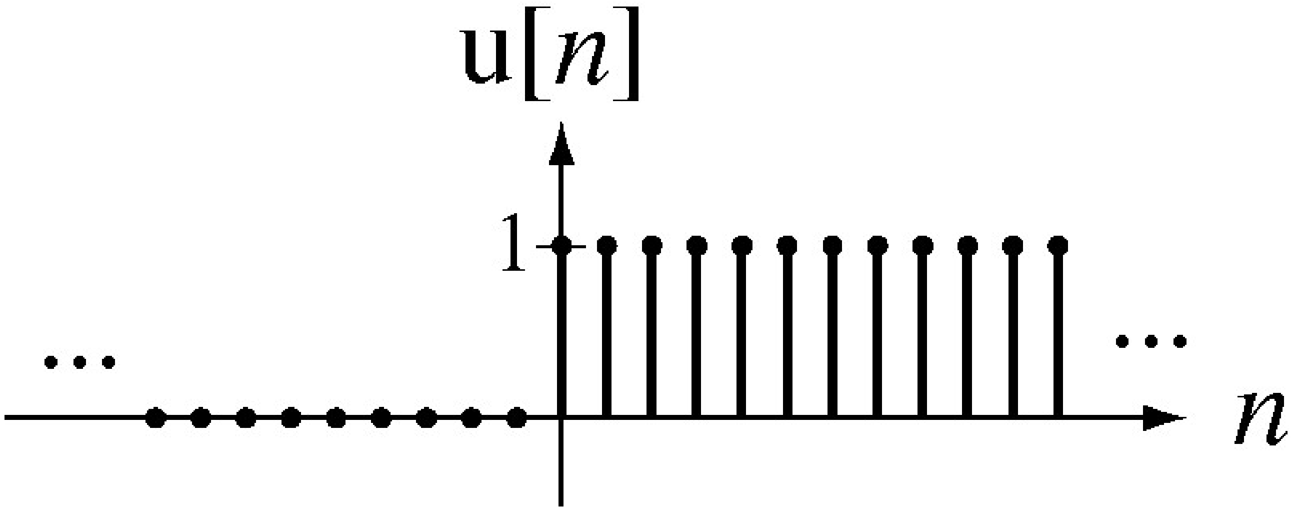
$1/N_0 = F_o = \Omega_o/2\Pi$ = Discrete Frequency

Discrete Time Sinusoidal Signals



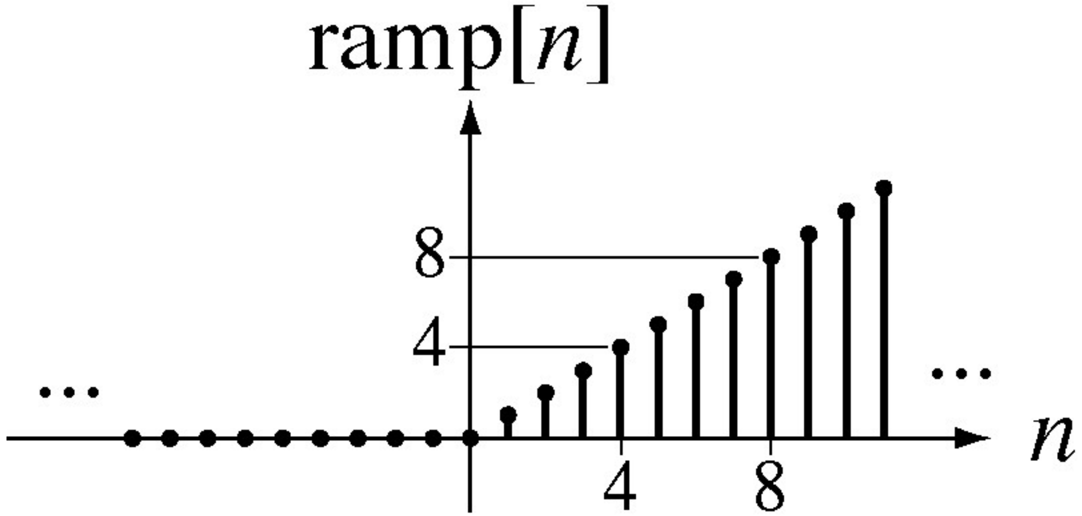
Discrete Time Unit Step Function or Unit Sequence Function

$$\mathrm{u}[n] = \begin{cases} 1 \ , & n \geq 0 \\ 0 \ , & n < 0 \end{cases}$$



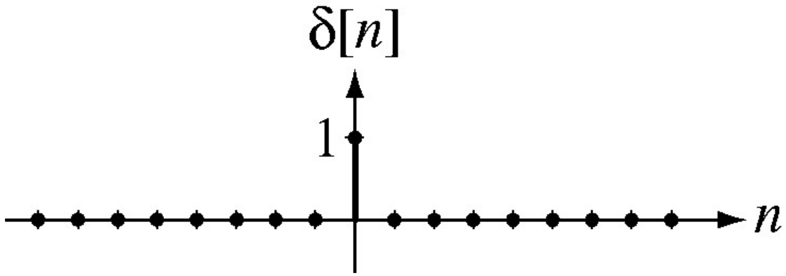
Discrete Time Unit Ramp Function

$$\text{ramp}[n] = \begin{Bmatrix} n & , & n \geq 0 \\ 0 & , & n < 0 \end{Bmatrix} = \sum_{m=-\infty}^{n} \text{u}[m-1]$$



Discrete Time Unit Impulse Function or Unit Pulse Sequence

$$\delta[n] = \begin{cases} 1 \;, & n = 0 \\ 0 \;, & n \neq 0 \end{cases}$$



$\delta[n] = \delta[an]$ for any non-zero, finite integer a.

Unit Pulse Sequence Contd.

- The discrete-time unit impulse is a function in the ordinary sense in contrast with the continuous-time unit impulse.
- It has a sampling property.
- It has no scaling property i.e.

  $\delta[n] = \delta[an]$ for any non-zero finite integer ‘a’