

# **Analogous System**

# Linear Mechanical Elements

Description	Trans Mech
Damper (a.k.a. Dashpot or Linear Friction)	$f = \pm B(v_1 \pm v_2)$
Power dissipation in Damper	$P = fv = f^2 \frac{1}{B} = v^2 B$
Spring	$f = \pm K(x_1 \pm x_2)$
Energy stored in spring	$E = \frac{1}{2} K(\Delta x)^2 \text{ or } E = \frac{1}{2} \frac{1}{K} f^2$
Mass	$f = M \frac{dv}{dt} \text{ or } \frac{dv}{dt} = f / M, \text{ where } f \text{ is the sum of all forces,}$ <p style="text-align: center;">each taken with the appropriate sign.</p>
Energy stored in mass	$E = \frac{1}{2} Mv^2$

# Force-voltage and force-current analogy

It is possible to make electrical and mechanical systems using *analogs*. An analogous electrical and mechanical system will have differential equations of the same form. There are two analogs that are used to go between electrical and mechanical systems.

# The analogous quantities are given below

## Key Concept: Analogous Quantities

Electrical Quantity	Mechanical Analog I (Force-Current)	Mechanical Analog II (Force Voltage)
Voltage, $e$	Velocity, $v$	Force, $f$
Current, $i$	Force, $f$	Velocity, $v$
Resistance, $R$	Lubricity, $1/B$ (Inverse friction)	Friction, $B$
Capacitance, $C$	Mass, $M$	Compliance, $1/K$ (Inverse spring constant)
Inductance, $L$	Compliance, $1/K$ (Inverse spring constant)	Mass, $M$
Transformer, $N_1:N_2$	Lever, $L_1:L_2$	Lever, $L_1:L_2$

To see the analogies more clearly, examine the following table that shows the constitutive relationships for the various analogous quantities. The entries for the mechanical analogs are formed by substituting the analogous quantities into the equations for the electrical elements. For example the electrical version of Ohm's law is  $e = iR$ . The Mechanical I analog stipulates that  $e$  is replaced by  $v$ ,  $i$  by  $f$  and  $R$  by  $1/B$ , which yields  $v = f/B$ .

Electrical Equation	Mechanical Analog I (Force-Current)	Mechanical Analog II (Force Voltage)
$e = iR$	$v = \frac{f}{B}$	$f = vB$
$e = L \frac{di}{dt}$ $i = \frac{1}{L} \int e \cdot dt$	$v = \frac{1}{K} \frac{df}{dt}$ $f = K \int v \cdot dt = K \cdot x$	$f = M \frac{dv}{dt} = M \cdot a$ $v = \frac{1}{M} \int f \cdot dt$
$e = \frac{1}{C} \int i \cdot dt$ $i = C \frac{de}{dt}$	$v = \frac{1}{M} \int f \cdot dt$ $f = M \frac{dv}{dt} = M \cdot a$	$f = K \int v \cdot dt = K \cdot x$ $v = \frac{1}{K} \frac{df}{dt}$
power = $e \cdot i$	power = $v \cdot f$	power = $f \cdot v$
Transformer $\frac{e_1}{e_2} = \frac{N_1}{N_2} = \frac{i_2}{i_1}$	Lever $\frac{v_1}{v_2} = \frac{L_1}{L_2} = \frac{f_2}{f_1}$	Lever $\frac{f_1}{f_2} = \frac{L_2}{L_1} = \frac{v_2}{v_1}$
capacitor energy $\frac{1}{2} C \cdot e^2$	mass energy $\frac{1}{2} M \cdot v^2$	spring energy $\frac{1}{2} K \cdot x^2 = \frac{1}{2} K \cdot \left(\frac{f}{K}\right)^2 = \frac{1}{2} \frac{f^2}{K}$
inductor energy $\frac{1}{2} L \cdot i^2$	spring energy $\frac{1}{2} K \cdot x^2 = \frac{1}{2} K \cdot \left(\frac{f}{K}\right)^2 = \frac{1}{2} \frac{f^2}{K}$	mass energy $\frac{1}{2} M \cdot v^2$