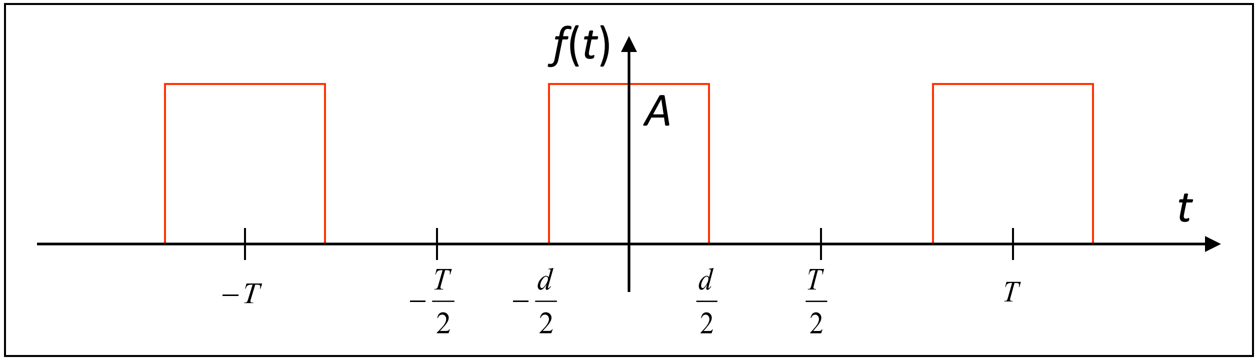
Example



$$c_n = \frac{A}{T}\int_{-d/2}^{d/2} e^{-jn\omega_0 t}\,dt$$

$$= \frac{A}{T}\frac{1}{-jn\omega_0} e^{-jn\omega_0 t}\Big|_{-d/2}^{d/2}$$

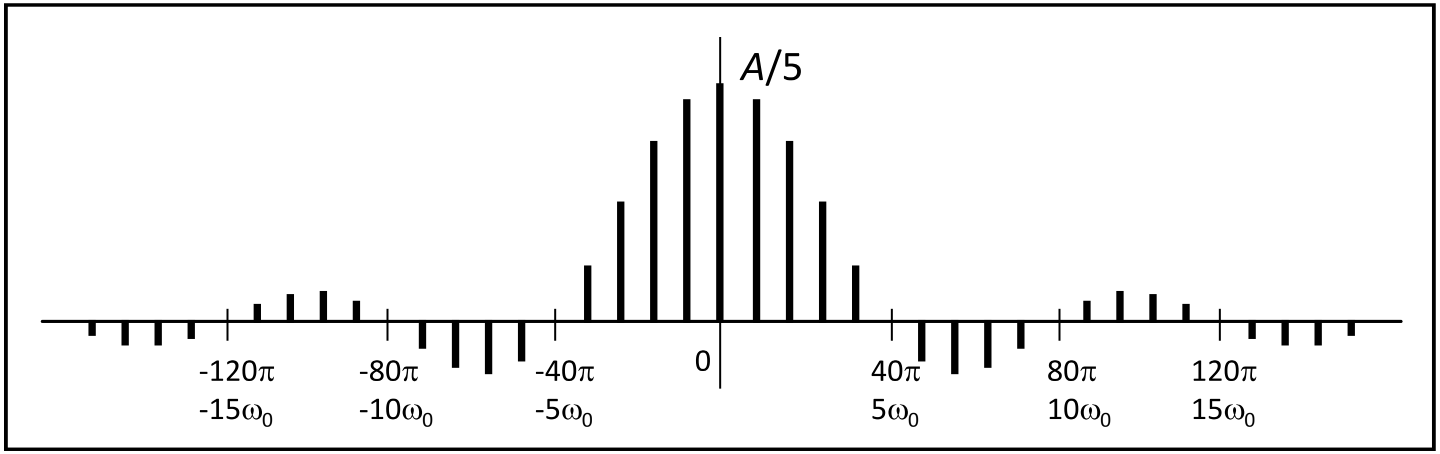
$$= \frac{A}{T}\left(\frac{1}{-jn\omega_0} e^{-jn\omega_0 d/2} - \frac{1}{-jn\omega_0} e^{jn\omega_0 d/2}\right)$$

$$= \frac{A}{T}\frac{1}{-jn\omega_0}(-2j\sin n\omega_0 d/2)$$

$$= \frac{A}{T}\frac{1}{\frac{1}{2}n\omega_0}\sin n\omega_0 d/2$$

$$= \frac{Ad}{T}\frac{\sin\left(\frac{n\pi d}{T}\right)}{\left(\frac{n\pi d}{T}\right)}$$

Example

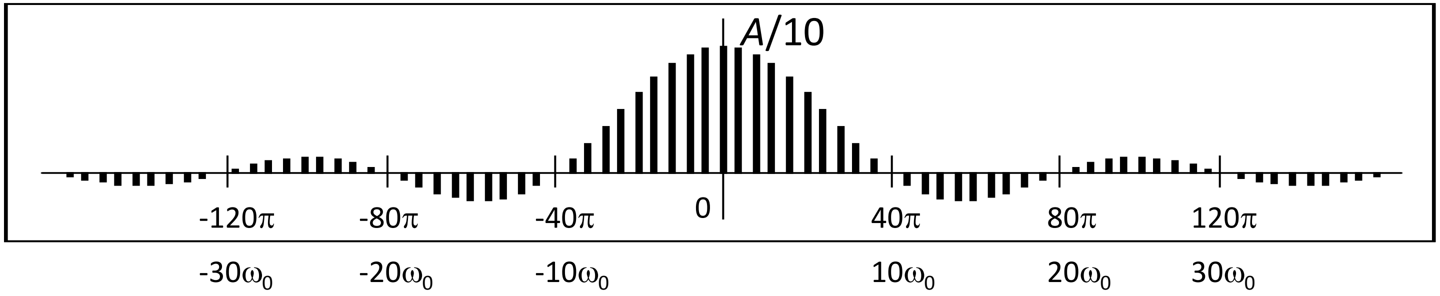


$$c_n = \frac{Ad}{T} \frac{\sin\left(\frac{n\pi d}{T}\right)}{\left(\frac{n\pi d}{T}\right)}$$

$$d = \frac{1}{20}, \quad \boxed{T = \frac{1}{4},} \quad \frac{d}{\mathrm{T}} = \frac{1}{5}$$

$$\omega_0 = \frac{2\pi}{T} = 8\pi$$

Example



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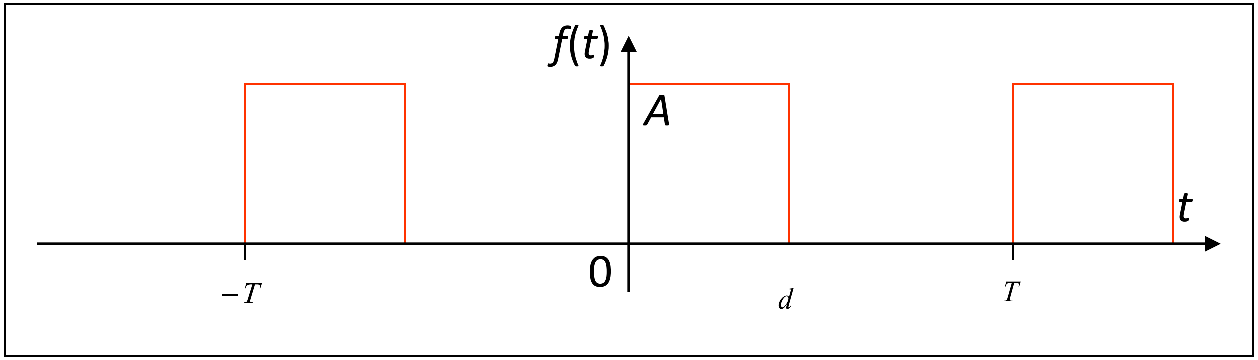


$$c_n = \frac{A}{T}\int_0^d e^{-jn\omega_0 t}\,dt$$

$$= \frac{A}{T}\frac{1}{-jn\omega_0}e^{-jn\omega_0 t}\Big|_0^d$$

$$= \frac{A}{T}\left(\frac{1}{-jn\omega_0}e^{-jn\omega_0 d} - \frac{1}{-jn\omega_0}\right)$$

$$= \frac{A}{T}\frac{1}{jn\omega_0}(1 - e^{-jn\omega_0 d})$$

$$= \frac{A}{T}\frac{1}{jn\omega_0}e^{-jn\omega_0 d/2}(e^{jn\omega_0 d/2} - e^{-jn\omega_0 d/2})$$

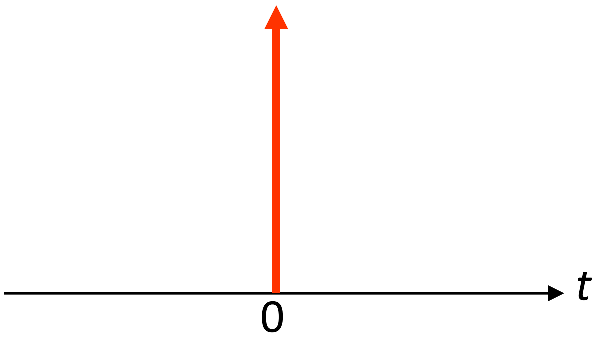
$$= \frac{Ad}{T}\frac{\sin\left(\frac{n\pi d}{T}\right)}{\left(\frac{n\pi d}{T}\right)}e^{-jn\omega_0 d/2}$$

Fourier Series

Impulse Train

Dirac Delta Function

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$



Also called *unit impulse function*.

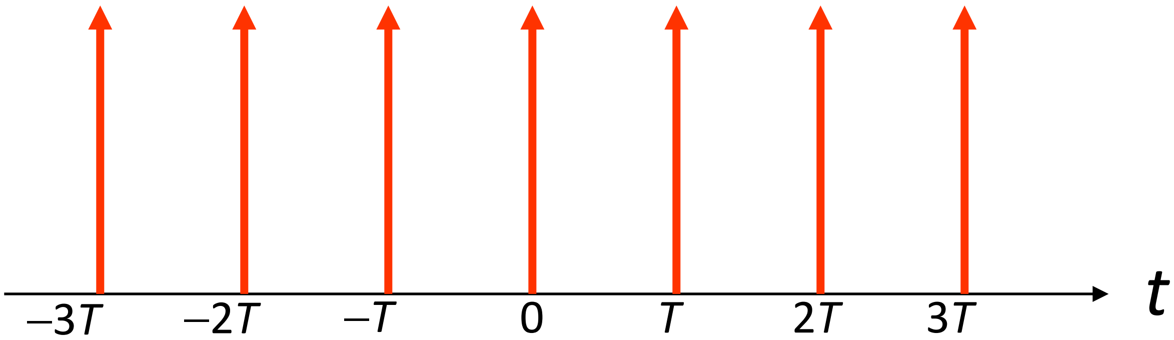
Property

$$\int_{-\infty}^{\infty} \delta(t)\phi(t)\,dt = \phi(0)$$

$\phi(t)$: Test Function

$$\int_{-\infty}^{\infty} \delta(t)\phi(t)\,dt = \int_{-\infty}^{\infty} \delta(t)\phi(0)\,dt = \phi(0)\int_{-\infty}^{\infty} \delta(t)\,dt = \phi(0)$$

Impulse Train



$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

Fourier Series of the Impulse Train

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} \delta_T(t)dt = \frac{2}{T}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} \delta_T(t)\cos(n\omega_0 t)dt = \frac{2}{T}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} \delta_T(t)\sin(n\omega_0 t)dt = 0$$

$$\delta_T(t) = \frac{1}{T} + \frac{2}{T}\sum_{n=-\infty}^{\infty} \cos n\omega_0 t$$