

Fourier Series

Periodic Functions

The Mathematic Formulation

- Any function that satisfies

$$f(t) = f(t + T)$$

where T is a constant and is called the *period* of the function.

Example:

$$f(t) = \cos \frac{t}{3} + \cos \frac{t}{4} \quad \text{Find its period.}$$

$$f(t) = f(t+T) \longrightarrow \cos \frac{t}{3} + \cos \frac{t}{4} = \cos \frac{1}{3}(t+T) + \cos \frac{1}{4}(t+T)$$

Fact: $\cos \theta = \cos(\theta + 2m\pi)$

$$\begin{array}{l} \frac{T}{3} = 2m\pi \\ \frac{T}{4} = 2n\pi \end{array} \longrightarrow \begin{array}{l} T = 6m\pi \\ T = 8n\pi \end{array} \longrightarrow T = 24\pi \quad \text{smallest } T$$

Example:

$$f(t) = \cos \omega_1 t + \cos \omega_2 t \quad \text{Find its period.}$$

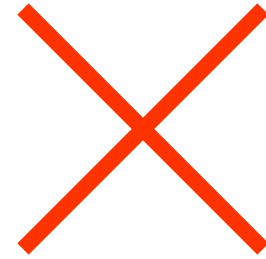
$$f(t) = f(t+T) \quad \longrightarrow \quad \cos \omega_1 t + \cos \omega_2 t = \cos \omega_1 (t+T) + \cos \omega_2 (t+T)$$

$$\begin{array}{l} \omega_1 T = 2m\pi \\ \omega_2 T = 2n\pi \end{array} \quad \longrightarrow \quad \frac{\omega_1}{\omega_2} = \frac{m}{n} \quad \longrightarrow \quad \frac{\omega_1}{\omega_2} \quad \text{must be a rational number}$$

Example:

$$f(t) = \cos 10t + \cos(10 + \pi)t$$

Is this function a periodic one?



$$\frac{\omega_1}{\omega_2} = \frac{10}{10 + \pi} \quad \text{not a rational number}$$