Fourier Series

Periodic Functions

The Mathematic Formulation

- Any function that satisfies

$$f(t) = f(t + T)$$

where T is a constant and is called the *period* of the function.

Example:

$$f(t)=\cos\frac{t}{3}+\cos\frac{t}{4}$$ Find its period.

$$f(t)=f(t+T) \Rightarrow \cos\frac{t}{3}+\cos\frac{t}{4}=\cos\frac{1}{3}(t+T)+\cos\frac{1}{4}(t+T)$$

Fact: $\cos\theta=\cos(\theta+2m\pi)$

$$\frac{T}{3}=2m\pi \Rightarrow T=6m\pi$$

$$\frac{T}{4}=2n\pi \Rightarrow T=8n\pi$$

$$\Rightarrow T=24\pi$$ smallest *T*

Example:

$f(t) = \cos \omega_1 t + \cos \omega_2 t$ Find its period.

$f(t) = f(t+T) \Rightarrow \cos \omega_1 t + \cos \omega_2 t = \cos \omega_1 (t+T) + \cos \omega_2 (t+T)$

$\omega_1 T = 2m\pi$

$\omega_2 T = 2n\pi$

$\Rightarrow \dfrac{\omega_1}{\omega_2} = \dfrac{m}{n} \Rightarrow \dfrac{\omega_1}{\omega_2}$ must be a rational number

Example:

$$f(t) = \cos 10t + \cos(10+\pi)t$$

Is this function a periodic one?

$$\frac{\omega_1}{\omega_2} = \frac{10}{10+\pi}$$

not a rational number